ABSTRACT

Title of dissertation: ESSAYS ON FIRM GROWTH, FIRM INNOVATION, AND INTERNATIONAL TRADE

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This dissertation studies the effect of competition on firms' decisions for heterogeneous innovation, and its implication for the recent decline in business dynamism in the U.S. in the context of increasing competitive pressure by foreign firms due to globalization. In Chapter 1, I theoretically investigate the effect of competition on firm innovation by developing a discrete-time endogenous growth model where multi-product firms do two types of innovation subject to friction in technology spillovers. Firms improve their existing products through internal innovation while getting into product markets outside of firms' scope through external innovation. Novel friction I consider is that it takes time to learn others' technology during external innovation, which I denote as an imperfect technology spillover. In contrast to existing models, this friction allows incumbent firms to defend themselves from competitors by building technological barriers through internal innovation. I calibrate the model and run counterfactual exercises of increasing competition, where competition is from either

outside of the economy, such as foreign exporters, or domestic firm entry. I find that regardless of the source of competition, domestic incumbent firms i) increase their internal innovation for products they have a technological advantage while decreasing it for products with no technological advantage, and ii) decrease their external innovation. This shift of innovation composition lowers firms' investment in overall innovation in the U.S., where firms are creative in the sense that they do a lot of external innovation. However, increasing competition increases firms' investment in overall innovation in an economy where firms do less external innovation. In an economy with high external innovation costs, firms put very little resource for external innovation even before increasing competition, which implies that there is little room for adjustment. Thus, although external innovation is decreased after an increase in competition, this small reduction is more than offset by increased internal innovation for defensive reasons. These findings highlight the importance of examining the composition of innovation as opposed to overall innovation, and sheds light on the reason for the differential effect of the same competition shock, such as Chinese import competition, on firms' overall innovation across different countries identified by recent studies.

In Chapter 2, I empirically test the model predictions derived in Chapter 1, and by building on these findings, I argue that the decline in high-growth firm activity and startup rates in the U.S. is a result of multi-product firms' optimal innovation decisions in response to increasing competitive pressure by foreign firms due to globalization. The three predictions I derive using a simplified three-period version of my model are i) increasing competition makes innovative firms increase their investment in internal innovation for defensive reasons, ii) if innovation intensity is high in the economy, firms do less external innovation, and iii) increasing expected profit makes firms invest more in internal innovation. By using firm-level data from the U.S. Census Bureau integrated with firm-level patent data from the USPTO, I find regression results consistent with the model's predictions. Then, I extend the baseline closed economy model developed in Chapter 1 and build a two-country endogenous growth model to show that increasing competitive pressure by foreign firms contributes to the recent decline in high-growth firm activities and startup rates in the U.S. by inducing innovation-intensive and thus fast-growing firms to invest more in internal innovation for defensive reasons. And because innovative incumbents in each product market are now good at protecting their markets with heightened technological barriers, all types of firms find it difficult to enter others' markets through external innovation. Thus, the startup rate falls, and all firms reduce their investment in external innovation. This shift in innovation cuts the employment growth of innovation-intensive firms, as external innovation makes firms grow faster than internal innovation by requiring firms to hire a new set of workers to produce new products.

ESSAYS ON FIRM GROWTH, FIRM INNOVATION, AND INTERNATIONAL TRADE

by

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Dedication

To my father, mother, and Seula.

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Disclaimer

Any opinions and conclusions expressed herein are those of the author and do not necessarily represent the views of the U.S. Census Bureau. All results have been reviewed to ensure that no confidential information is disclosed.

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Chapter 1: A Theory of Competition, Firm Innovation, and Growth under Imperfect Technology Spillovers

1.1 Introduction

Studies of the effect of competition on firm innovation have a long history, as economists br that innovation is a major source of economic growth. Researchers have studied the impact of competition within different product markets and across countries with varying degrees of development, both empirically and theoretically. The results, however, are inconclusive. As documented in Gilbert (2006), differences in market structure, types of innovation, and degree of protection for innovation cause incentives for innovation to move in different directions and offset each other.

In this chapter, I theoretically investigate the effect of competition on firm innovation by developing a discrete-time endogenous growth model where multi-product firms do two types of innovation—internal and external—subject to imperfect technology spillovers in the form of lagged learning of others' technology, extending the Akcigit and Kerr (2018) framework. Aided by this model, I decompose the overall changes in innovation in response to increasing competition into changes in the level and composition of the two types of innovation. I show that competition can either increase or decrease overall innovation, because i) competition affects the two types of innovation differently, and ii) factors such as the innovation cost structure determine the relative changes in the two types of innovation in response to competition.

In the real world, firms are multi-product firms, and they grow both by expanding their existing markets and entering other product markets. Thus, firms' growth paths depend on their product portfolio choices. In my model, I allow multi-product firms to choose their product portfolio through two types of innovation. Firms use internal innovation to improve their existing product quality and production processes, and use external innovation to enter new markets outside of their existing scope and drive incumbent firms out.[1]

Also, in the real world, firms can defend their product markets from competitors by improving their products further. My study shows that this channel is important in understanding firm growth and firm entry. If own product improvement can be an effective tool for blocking competitors from either entering into or expanding in a firm's existing product market, then internal innovation affects not only an individual firm's growth path but also firm entry in each product market. Existing models, however, assume either that firms have a single product, or that they can't use innovation defensively.

In existing models that allow multi-product firms to grow through product scope expansion (e.g., Klette and Kortum (2004) and Akcigit and Kerr (2018)),

[1] A real-world example of external innovation is Apple developing iPhone and getting into the cell phone industry back in 2007 when its major business was on computer manufacturing. A real-world example of internal innovation is Apple improving and producing iPhone 11 from iPhone 10.

firms cannot protect their markets because any firm can learn and copy other firms' frontier technology immediately. Thus, escape-competition through improving one's own technology is not possible. Existing models with step-by-step innovation, such as Aghion et al. (2001) and Akcigit et al. (2018), allow for certain forms of escape-competition, but assume single-product firms. This lack of realism in existing models limits their ability to analyze the effect of competition on firm innovation and firm growth. To move forward, I allow multi-product firms to defend their product markets through internal innovation by introducing a friction in learning others' technology, which I label as imperfect technology spillovers.

When a firm attempts to enter another firm's market and take over through external innovation, it first needs to learn the technology of the incumbent firm so that it can then improve on top. Realistically, however, there are lags in learning others' technology. In my model economy, imperfect technology spillovers take the form of lagged learning, in which potential rival firms can only learn the product-specific technology of incumbent firms with a one-period lag.[2] Thus, internal innovation is built on the current frontier technology, while external innovation is built on lagged technology. Imperfect spillovers generate a technology gap, defined as the gap between the current period frontier technology for a given product that only the incumbent can use, and the one-period lagged technology that potential rival firms can learn through R&D.

Then, incumbent firms can use this time lag to improve their technology further through internal innovation for defensive reasons, which makes it harder for

[2]This is equivalent to saying that it takes one period to learn others' technology.

competitors to catch up with their technology and steal their business. In other words, incumbent firms can build a technological advantage in their markets. In such an environment, individual firms use internal innovation not only to improve the profitability of their products but also to escape competition. In this sense, my framework brings together quality-ladder innovation models and step-by-step innovation models. The flip side is that defensive internal innovation by incumbents makes it difficult to take over another firm's market through external innovation, as firms need to overcome the technological advantage of incumbent firms. This technological barrier can become higher if competition increases, because competition incentivizes incumbents to do more internal innovation.

The introduction of imperfect technology spillovers is the key theoretical contribution that allows us to distinguish the effect of competitive pressure on internal versus external innovation. In addition, imperfect technology spillovers imply a novel technological-barrier effect, in which factors that affect defensive internal innovation also affect the probability of successful external innovation and business takeover in the economy.

To my knowledge, this is the first theoretical model of defensive innovation that allows individual multi-product firms to grow both by improving in their existing markets and by taking other firms' markets, through two different types of innovation. Allowing for both internal and external innovation is important for understanding the effect of competition on firms' innovation decisions, as well as firm-level and aggregate economic growth. Firms have different incentives for different types of innovation, and they can use these two types of innovation strategically to increase their profits and the probability of survival. Also, Akcigit and Kerr (2018) show that external innovation contributes more than internal innovation to both firm employment growth and aggregate economic growth. Thus, allowing for only one type of innovation, while ignoring potential compositional changes, could disguise the true effect of competition on innovation.

A simple three-period version of my model analytically shows how both types of innovation respond to increasing competition by decomposing firms' innovation incentives into three terms, namely the escape-competition effect, the Schumpeterian effect, and the technological barrier effect. I show that the technology gap, which summarizes the technological advantage incumbent firms have in their own market, and determines the gain in future profits from internal innovation, is the key to understanding why some firms increase while others decrease their internal innovation when competition increases. Internal innovation increases expected future profits by improving product quality, thus widening the technology gap and lowering the probability of losing the product line to another firm. For this reason, increasing competition induces firms to increase their internal innovation efforts, which is the escape-competition effect. On the other hand, increased competition raises the aggregate probability of losing a product line (the aggregate creative destruction arrival rate), as there are more firms performing external innovation in the economy. This lowers the expected profits from each product line and discourages firms' internal and external innovation, which is the Schumpeterian effect.

Whether a firm increases or decreases its internal innovation intensity for each of its products depends on which of these two effects dominates. I show that the escape competition effect dominates the Schumpeterian effect for firms that have innovated intensively in recent periods, and therefore are likely to have technological advantage accumulated in their own markets. Thus, increasing competition motivates innovation-intensive, high-growth firms to increase their internal innovation for defensive reasons. These firms become better at protecting themselves from competitors by building technological barriers in their existing markets.[3]

For an individual firm, the aggregate internal innovation intensity and the distribution of the technology gap determine the probability of a successful business takeover for a given amount of external innovation effort. The higher the average technology gap and the more internal innovation effort of incumbent firms, the harder it is to take over another firm's product market. I define this as the technological-barrier effect. Increased competition increases the average value of the technology gap distribution by changing both individual firm's internal innovation decisions and the aggregate external innovation intensity. Thus, increased competition lowers individual firms' optimal external innovation intensity through the Schumpeterian effect and the technological-barrier effect.

To understand the effect of increasing competition on the composition of innovation and the aggregate economy, I calibrate an infinite-horizon version of my model to innovative firms the U.S. manufacturing sector from 1987 to 1997 and perform three counterfactual exercises: i) increased competitive pressure by foreign firms (so that the aggregate creative destruction arrival rate depends in part on for-

[3] For example, as of 2020, we hear that Apple is planning to introduce new iPhones more frequently, twice per year, because competition in the cellphone industry is intensified.

eign firms), ii) increased competitive pressure by foreign firms in an economy where the external innovation cost is much higher than in the U.S., and iii) lower entry costs (specifically, a lower external innovation cost for potential startups).

Because the change in the aggregate creative destruction arrival rate (equivalently, the change in competitive pressure) is held constant, the three counterfactual exercises result in the same change in individual incumbent firms' innovation decisions. That is, incumbent firms increase their internal innovation for existing products for which they have a technological advantage, decrease their internal innovation for products for which they have no technological advantage, and decrease their external innovation.

However, the three exercises result in very different changes in the aggregate economy. Comparing the results from exercises i) and ii), I show that the average firm-level R&D to sales ratio decreases in response to increased foreign competition in the economy calibrated to the U.S., but would increase in an economy with a higher external innovation cost (low creativity). In an economy with high external innovation costs, firms put few resources into external innovation even when competition is low, which implies that there is very little room for further downward adjustment. Thus, although external innovation falls after an increase in competitive pressure, this reduction is more than offset by increased investment for internal innovation for defensive reasons. In the economy calibrated to the U.S., on the other hand, firms are active in doing external innovation, and the decrease due to increasing competition is substantial. Thus, overall innovation falls.

This result sheds light on the differential effect of Chinese import competition on firms' overall innovation across different countries identified by recent studies such as Bloom et al. (2016), Autor et al. (2019), and my own empirical results in Chapter 2. This result also highlights the importance of examining changes in the composition of innovation as opposed to changes in overall innovation for understanding the effect of competition on firm innovation.

In exercise iii), incumbent firms respond to increasing domestic competitive pressure in the same way as under an increase in competitive pressure due to foreign firms. However, firm entry responds differently. The mass of domestic startups decreases in response to an increase in competitive pressure from foreign firms, while it increases in the case of lowered domestic entry costs. This insight can allow researchers to identify whether competitive pressure comes from foreign firms or from the domestic entry margin.

The rest of the paper proceeds as follows. Section 1.2 develops a discrete-time infinite horizon baseline general equilibrium model. Section 1.3 analyzes a simple three-period model to study the proposed mechanism in detail and derive empirically testable predictions. Section 1.4 presents results from quantitative analysis of the baseline model. Section 1.5 concludes.

1.2 Baseline Model

In this section, I introduce a discrete time infinite horizon endogenous growth model with multi-product firms, two types of innovation, imperfect technological spillovers, and an exogenous source of competitive pressure. The exogenous competitive pressure can come from firms in foreign countries if we consider the aggregate economy, or from domestic incumbent firms in other sectors or states if we consider a certain sector or state. The baseline model extends Akcigit and Kerr (2018) in three dimensions: i) I assume imperfect technology spillovers by assuming that R&D expenditure on external innovation only allows rivals to learn the incumbent's technology lagged by one period, ii) I introduce an escape-competition effect, in which incumbent firms' internal innovation decision depends on last period's innovation results, which are summarized by the technology gap $\Delta_{j,t} = \frac{q_{j,t}}{q_{j,t-1}}$, and iii) I allow for shifts of the aggregate creative destruction arrival rate to analyze the effect of increasing competitive pressure on firms' innovation and growth dynamics.

Hereafter, the time subscript is suppressed whenever there is no confusion. Superscript $'$ is used to denote next period variables $(t + 1)$, and subscript -1 is used for last period variables $(t - 1)$. The terms product quality and technology are used interchangeably.

1.2.1 Representative Household

The representative household has a logarithmic utility function and is populated by a measure one continuum of individuals. Each individual supplies one unit of labor each period inelastically and consumes a portion C_t of the economy's final good. Thus the household's lifetime utility is

$$U = \sum_{t=0}^{\infty} \beta^t \log(C_t) \, .$$

Homogeneous workers are employed in the final goods sector (L). Thus in each period, the labor market satisfies

$$L = 1\,. \tag{1.1}$$

1.2.2 Final Good Producer

The final good producer uses labor (L) and a continuum of differentiated products indexed by $j \in [0, 1]$ to produce a final good. Denote $\mathcal{D}$ as the index set for differentiated products produced by domestic firms. Products with $j \notin \mathcal{D}$ are produced by foreign firms (or domestic incumbent firms in other sectors/states), as discussed later. The constant returns to scale production technology w.r.t. labor and differentiated products can be written as

$$Y = \frac{L^\theta}{1-\theta}\left[\int_0^1 q_j^\theta\, y_j^{1-\theta}\, \mathcal{I}_{\{j\in\mathcal{D}\}}\, dj \;+\; \int_0^1 q_j^\theta\, y_j^{1-\theta}\, \mathcal{I}_{\{j\notin\mathcal{D}\}}\, dj\right],$$

where y_j is the quantity of differentiated product j, q_j is its quality, and $\mathcal{I}_{\{\cdot\}}$ are indicator functions. The final good price is normalized to be one in every period without loss of generality. The final good is produced competitively and input prices are taken as given.

1.2.3 Differentiated Products Producers

There is a set of measure $\mathcal{F}_d$ domestic firms and a set of measure $\mathcal{F}_o$ foreign firms with $\mathcal{F}_d + \mathcal{F}_o \in (0, 1)$, which are determined endogenously in equilibrium, produce differentiated products each period and sell their products in domestic markets. Each differentiated product is produced in the producer's own region using domestic resources.[4] Since each operating firm owns at least one product line, and each product line is owned by a single firm, a firm f can be characterized by the collection of its product lines $\mathcal{J}^f = \{j : j \text{ is owned by firm } f\}$. Only the owner of each product line can observe the product-line specific current period technology (product quality) $q_{j,t}$, and the technology gap between t and $t-1$ $\Delta_{j,t} = \frac{q_{j,t}}{q_{j,t-1}}$. Thus each product line can be characterized by its quality and technology gap, (q_j, Δ_j). Each differentiated product $j \in [0,1]$ is produced at a unit marginal cost in terms of the final good.

1.2.4 Innovation by Differentiated Good Producers

The differentiated good producers engage in two types of R&D—internal and external—to increase their profits from products they currently produce, to protect their product markets from competitors, and to expand their businesses, where the R&D output takes the form of improvements in product quality (equivalently, production technology). Innovation outcomes are realized at the beginning of the next period. To allow incumbent firms to protect their own product markets from competitors (the escape-competition effect) and to capture the fact that it is more difficult to take over other firms' product markets when firms are very innovative on average (the technological-barrier effect), I introduce imperfect technological

[4] If competitive pressure is from foreign firms, then firm's own region is own country. If competitive pressure is from other state, then firm's own region is the state firm operates.

spillovers, which are captured by lagged learning: firms that don't own product line j can only learn the incumbent's last period technology, $q_{j,t-1}$. Thus, external innovation builds on the past-period technology. Also, I assume that a domestic firm can learn a foreign firm's lagged technology if and only if that foreign firm sells its products in the domestic market.

In this setup, learning another firm's technology is costly in a sense that i) rivals can only learn incumbent firms' last period technology, and ii) learning involves R&D—only firms with strictly positive R&D expenditure can learn another firm's past technology through undirected learning.[5] For a particular product, the current period technology $q_{j,t}$ and the technology gap $\Delta_{j,t} \equiv \frac{q_{j,t}}{q_{j,t-1}}$ are observable only to the firm operating product line j in that period. However, aggregate variables and the technology gap distribution (the share of product lines with a certain level of technology gap) are publicly observable, and these are the objects individual firms need to know to make their optimal innovation decisions. Thus, a stationary equilibrium is well defined. When two firms' technologies are neck and neck in a particular product line, a coin-toss tiebreaker rule applies as in Acemoglu et al. (2016) to make sure each product is produced by only one firm. An unused technology (idea) is assumed to depreciate by an amount sufficient to ensure that it becomes unprofitable to innovate on top of it next period.[6] Thus, only the winning firm from the coin toss keeps the product line until it is taken over by another firm through creative destruction (external innovation), while the losing firm never tries to enter the same

[5] Firms do not know which product line technology they will learn prior to their learning. This assumption helps keep the model tractable.

[6] If you don't recall your skill or idea frequently, you gradually forget about it. This is in some sense consistent with the literature discussing displaced workers' human capital depreciation.

market through internal innovation. Thus, the undirected nature of external innovation is ensured, and only the firm currently producing a product is allowed to do internal innovation on that product. Finally, to maintain tractability I assume that each firm can do only one external innovation in each period regardless of the total number of product lines the firm owns.

1.2.4.1 Internal Innovation

Successful internal innovation improves the current quality $q_{j,t}$ by $\lambda > 1$. The probability of successful internal innovation, $z_{j,t}$, is determined by the level of R&D expenditure $R^{in}_{j,t}$ in units of the final good:

$$z_{j,t} = \left(\frac{R^{in}_{j,t}}{\widehat{\chi} q_{j,t}}\right)^{\frac{1}{\widehat{\psi}}} ,$$

where $\widehat{\chi} > 0$ and $\widehat{\psi} > 1$. Thus incumbent firm's good j quality realized at the beginning of $t+1$, assuming the firm is not displaced by creative destruction, is:[7]

$$\left\{q^{in}_{j,t+1}\right\} = \begin{cases} \left\{\lambda q_{j,t}\right\} & \text{with probability } z_{j,t} \\ \\ \left\{q_{j,t}\right\} & \text{with probability } 1 - z_{j,t} \ . \end{cases}$$

As time is discrete and firms are multi-product firms, internal innovation outcomes follow a binomial process as in Ates and Saffie (2016).

[7]Hereafter, I write the quality of product j as a point set. This makes it easy to write the case when external innovation fails and firm does not acquire any product lines, which will be written as product quality set to be an empty set.

1.2.4.2 External Innovation

Incumbents and potential startups attempt to take over other incumbents' markets through external innovation. Successful external innovation generates an improvement in product quality of $\eta > 1$ relative to the incumbent's lagged technology, where R&D results are realized at the beginning of next period. I assume $\lambda^2 > \eta > \lambda$. This assumption ensures that firms can protect their own product lines from potential rivals through internal innovation, while $\eta > \lambda$ reflects the idea that external innovation introduces a new way of producing an existing product more efficiently. Thus, external innovation contributes more to both firm employment and aggregate growth than internal innovation, as found empirically in Akcigit and Kerr (2018). Both potential startups' and incumbent firms' external innovations are undirected in a sense that they are realized in any other product line with equal probability.

Existing firms with at least one product line ($n_f > 0$) decide the probability of external innovation x_t by choosing R&D expenditures R_t^{ex} in units of the final good:

$$x_t = \left(\frac{R_t^{ex}}{\widetilde{\chi}\, \bar{q}_t} \right)^{\frac{1}{\widetilde{\psi}}} ,$$

where $\widetilde{\chi} > 0$, and $\widetilde{\psi} > 1$, and $\bar{q}_t$ is the average quality in the country where the firm is located. Thus, for prospective external innovators whose takeover is not pre-empted by an incumbent's successful defensive innovation, the distribution of

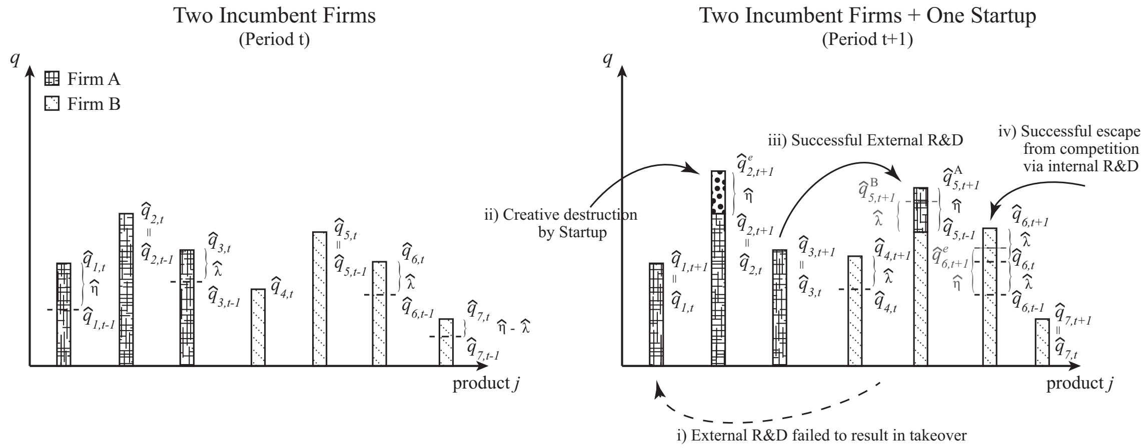

Figure 1.1: Firms' Innovation and Product Quality Evolution Example

quality at the start of the next period is:

$$\left\{q_{j,t+1}^{ex}\right\} = \begin{cases} \left\{\eta q_{j,t-1}\right\} & \text{with probability } x_t \\ \\ \varnothing & \text{with probability } 1 - x_t \,. \end{cases}$$

With probability $1 - x_t$, the external innovation fails, which implies there is zero probability that the firm will take over product line j. In this case, product quality for product line j for the potential entrant does not exist.

To better understand the firm's innovation decisions, and to show how business takeover through external innovation and escape competition through internal innovation work in detail, the following section graphically illustrates specific cases.

1.2.4.3 Business Takeover and Escape Competition, an Illustration

Figure 1.1 illustrates how firms' product quality portfolio and technology gap portfolio evolve over time. Firm A owns the first three product lines and firm B owns the last four product lines in period t. Each bar represents a product and the height of the bar represents the log of product quality for each product, $\widehat{q}_{j,t} \equiv \log(q_{j,t})$. Product line 7 is not innovated by any firm. Thus, its quality in $t+1$ remains the same as that in t, and it is still owned by firm B in $t+1$. In case i), firm B does external innovation in an attempt to take over firm A's product line 1. Firm A took this product line over through successful external innovation at $t-1$, but did not internally innovate at t. So $\Delta_{1,t} = \eta$, and $q^A_{1,t+1} = \eta q_{1,t-1}$ (implying that $\widehat{q}^A_{1,t+1} = \widehat{\eta} + \widehat{q}_{1,t-1}$, where $\widehat{\eta} \equiv \log(\eta)$) for firm A. Firm B, meanwhile, learns $q_{1,t-1}$ in period t and innovates, so that in period $t+1$, it realizes $q^B_{1,t+1} = \eta q_{1,t-1}$, which is the same as $q^A_{1,t+1}$. A coin is tossed and firm A is the winner. Thus, firm A keeps product line 1. Case ii) illustrates how a firm can lose its existing product line through another firm's external innovation (creative destruction). Firm A failed to do internal innovation on product line 2 in periods $t-1$ and t. Thus, at the beginning of period $t+1$, the quality of product line 2 for firm A is equal to $q^A_{2,t+1} = q_{2,t-1}$. A potential startup learns product line 2's last period technology (quality) by investing in R&D in period t and succeeds in externally innovating the product quality. Thus, at the beginning of period $t+1$, the product quality of product line 2 for the potential startup is equal to $q^e_{2,t+1} = \eta q_{2,t-1}$. Since $q^e_{2,t+1} > q^A_{2,t+1}$, the startup takes over product line 2. Case iii) illustrates how incumbent firm A can take over incumbent firm B's product line through external innovation, despite internal innovation by incumbent firm B. Since there was no internal innovation between $t-1$ and t for product line 5, $q_{5,t} = q_{5,t-1}$. Thus, firm A's quality for product line 5 after external innovation is $q^A_{5,t+1} = \eta q_{5,t}$. Firm B internally innovates product line 5 in period

t. Thus, firm B's quality for product line 5 is $q^B_{5,t+1} = \lambda q_{5,t-1}$. Since $\eta > \lambda$, firm A takes over product line 5. Case iv) illustrates how firms can escape from competition (creative destruction) through successful internal innovation. Firm B succeeds in internally innovating its product line 6 for two consecutive periods. Thus, the quality of product line 6 for firm B in period $t+1$ is equal to $q^B_{6,t+1} = \lambda^2 q_{6,t-1}$. Rival firms can increase the quality for product line 6 only up to $q^e_{6,t+1} = \eta q_{6,t-1}$. Since $\lambda^2 > \eta$, firm B successfully protects product line 6 from competitors. These examples show an important feature that is unique to the economy with imperfect technology spillovers. Because incumbents can escape competition through internal innovation, not all firms that succeed in external innovation can successfully take over another firm's business. Thus, the success probability of a business takeover is generally lower than the probability of external innovation, and it depends on the existing technology gap in the target market (product).

1.2.4.4 Product Quality Evolution

As a rival firm can only learn last period's technology, the technology gap, defined as $\Delta_{j,t} = \frac{q_{j,t}}{q_{j,t-1}}$, is the most important factor determining an incumbent firm's success/failure at protecting its product line through internal innovation. The technology gap summarizes the technological advantage incumbent firms have in their own markets. In this model, there are four possible values for the technology gap:

Lemma 1. *There can be only four values for the technology gap in this economy,*

$\Delta^1 = 1$, $\Delta^2 = \lambda$, $\Delta^3 = \eta$, *and* $\Delta^4 = \frac{\eta}{\lambda}$, *and product lines with* Δ^3 *and* Δ^4 *can occur only through external innovation.*

Proof: See Appendix A.1.2.1.

To describe the evolution of product quality and the implied probabilities of retaining or losing a product from the perspective of an incumbent firm, consider a product line j with quality $q_{j,t}$ and technology gap $\Delta_{j,t}$ owned by a firm f. Denote z_j^ℓ as the probability of internal innovation for product line j when its technology gap is equal to $\frac{q_{j,t}}{q_{j,t-1}} = \Delta^\ell$. Suppose product line j has technology gap $\Delta_{j,t} = \Delta^1$. If the firm is successful at internal innovation with probability z_j^1, its product quality next period is $q_{j,t+1}^{in} = \lambda q_{j,t-1}$; otherwise, $q_{j,t+1}^{in} = q_{j,t-1}$.

If creative destruction arrives at rate $\overline{x}$—where $\overline{x}$ is the probability that an individual product market is faced with a rival that succeeded in external innovation—then the product quality of the rival will equal $q_{j,t+1}^{en} = \eta q_{j,t-1}$. Since $q_{j,t+1}^{en} > \lambda q_{j,t-1} > q_{j,t-1}$, the rival takes over product line j regardless of the firm's success at internal innovation. Thus with probability $\overline{x}$, firm f loses product line j next period.

With the same arguments, product quality for product line j for firm f next period and the transition probabilities for all cases can be defined as:

$$\Big\{q_{j,t+1} \,\Big|\, \Delta_{j,t} = \Delta^1\Big\} = \begin{cases} \varnothing & \text{, with prob. of } \overline{x} \\ \big\{q_{j,t}\big\} & \text{, with prob. of } (1-\overline{x})(1-z_j^1) \\ \big\{\lambda q_{j,t}\big\} & \text{, with prob. of } (1-\overline{x})z_j^1 \end{cases} \tag{1.2}$$

$$\left\{q_{j,t+1} \mid \Delta_{j,t} = \Delta^2\right\} = \begin{cases} \varnothing & \text{, with prob. of } \overline{x}(1 - z_j^2) \\ \{q_{j,t}\} & \text{, with prob. of } (1-\overline{x})(1 - z_j^2) \\ \{\lambda q_{j,t}\} & \text{, with prob. of } z_j^2 \end{cases} \tag{1.3}$$

$$\left\{q_{j,t+1} \mid \Delta_{j,t} = \Delta^3\right\} = \begin{cases} \varnothing & \text{, with prob. of } \tfrac{1}{2}\overline{x}(1 - z_j^3) \\ \{q_{j,t}\} & \text{, with prob. of } \left(1-\tfrac{1}{2}\overline{x}\right)(1 - z_j^3) \\ \{\lambda q_{j,t}\} & \text{, with prob. of } z_j^3 \end{cases} \tag{1.4}$$

$$\left\{q_{j,t+1} \mid \Delta_{j,t} = \Delta^4\right\} = \begin{cases} \varnothing & \text{, with prob. of } \overline{x}\left(1 - \tfrac{1}{2}z_j^4\right) \\ \{q_{j,t}\} & \text{, with prob. of } (1-\overline{x})(1 - z_j^4) \\ \{\lambda q_{j,t}\} & \text{, with prob. of } \left(1-\tfrac{1}{2}\overline{x}\right) z_j^4 \end{cases} \tag{1.5}$$

where product quality set equal to $\varnothing$ means that firm f loses product line j next period, and the $\frac{1}{2}$ terms in the probabilities are due to the coin-toss tiebreaker rule for neck and neck cases. Thus for any Δ^ℓ except for Δ^1, firms can lower the probability of losing its product lines by investing more in internal innovation, where the magnitude of the decrease in probability of losing the product depends on the technology gap. For this reason, firms have incentive to increase their internal innovation intensity (R&D investment that increases the probability of internal innovation) when they are faced with more competition, as represented by a higher creative destruction arrival rate $\overline{x}$.

The conditional takeover probability — the probability of product takeover, conditional on successful external innovation — can be computed as follows. If a rival firm succeeds in externally innovating a product line with technology gap Δ^1, then it takes over this product line with probability one. For a product line with technology gap Δ^2, this probability is equal to $1 - z^2$; for technology gap Δ^3 it is $\frac{1}{2}(1 - z^3)$, and for technology gap Δ^4 it is $1 - \frac{1}{2}z^4$.[8] Thus with a technology gap distribution (share of product lines with technology gap Δ^ℓ) $\left\{\mu(\Delta^\ell)\right\}_{\ell=1}^{4}$, the conditional takeover probability is equal to

$$\overline{x}_{takeover} = \mu(\Delta^1) + (1 - z^2)\mu(\Delta^2) + \frac{1}{2}(1 - z^3)\mu(\Delta^3) + \left(1 - \frac{1}{2}z^4\right)\mu(\Delta^4)\,.$$

The higher the overall innovation (both internal and external) intensity, the wider is the average technology gap in the economy. Thus it becomes more difficult for rival firms to take over other firms' product markets. This conditional takeover probability defines the technological barrier channel through which either incumbent firms' increasing internal innovation intensity, or an increase in the overall external innovation intensity in the economy (reflected as an increase in the aggregate creative destruction arrival rate) could lower domestic firms' incentive for external innovation, which results in lower firm growth rates. This technological barrier effect is distinct from the well-known Schumpeterian effect, by which firms' innovation incentives decline due to lowered expected future profits conditional on successful innovation. Higher overall innovation intensity in the economy will likely lower $\overline{x}_{takeover}$, as the share of product lines with technology gap Δ^1 (where the probability of product takeover is the highest) will decrease, while at least some of the z^ℓ for $\ell = 2, 3, 4$ will increase. Since all firms, including potential startups, know the level of $\overline{x}_{takeover}$, firms will optimally choose to lower their external inno-

[8] Here I assume internal innovation intensity z depends only on technology gap Δ^ℓ. In the next section, I prove this is the case.

vation intensity when $\overline{x}_{takeover}$ falls, unless expected profits from external innovation increase enough to offset the loss from a lowered conditional takeover probability.

Note that with technology gap distribution $\left\{\mu\left(\Delta^{\ell}\right)\right\}_{\ell=1}^{4}$, the unconditional probability of a firm failing in an attempted product takeover—the probability of not winning the product line, either due to failure of external innovation (which occurs with probability of $1-x$) or escape-competition by incumbent firms—is

$$\begin{aligned}(1-x) + xz^2\mu(\Delta^2) + x\frac{1}{2}(1+z^3)\mu(\Delta^3) + x\frac{1}{2}z^4\mu(\Delta^4) \\ = 1 - x\left[1 - \left(z^2\mu(\Delta^2) + \frac{1}{2}(1+z^3)\mu(\Delta^3) + \frac{1}{2}z^4\mu(\Delta^4)\right)\right].\end{aligned}$$

Given the above definition of the conditional takeover probability $\overline{x}_{takeover}$, the previous expression can be written as $1 - x\,\overline{x}_{takeover}$. Denote $x_{takeover} \equiv x\,\overline{x}_{takeover}$ as the unconditional probability of successful product takeover.

The probability distribution of the evolution of product quality from the perspective of a rival firm can be similarly defined and is described in Appendix A.1.2.2.

1.2.5 Potential Startups

The economy has a fixed mass of potential domestic startups $\mathcal{E}_d$, and an exogenously determined mass of foreign firms trying to start businesses in domestic markets.[9] To start a business, a potential startup invests in external R&D and, if successful, takes over a product line from an incumbent firm. Similar to incumbent

[9]Strictly speaking, only a portion of the aggregate creative destruction arrival rate accounted by outside firms is exogenously determined in this economy. However, this is effectively the same as having the exogenously determined mass of outside firms trying to start businesses in domestic markets, as it will become clear in the following sections.

firms, potential startups decide the probability of external innovation x_e by choosing R&D expenditures R_e^{ex} in units of the final good:

$$x_e = \left(\frac{R_e^{ex}}{\widetilde{\chi}_e \, \bar{q}} \right)^{\frac{1}{\widetilde{\psi}_e}} ,$$

where $\widetilde{\chi}_e > 0$, and $\widetilde{\psi}_e > 1$, and $\bar{q}$ is the average quality in the country where the potential startup is located.

Let $V(\{(q_j, \Delta_j)\})$ denote the value of a firm that has one product line with product quality q_j and technology gap Δ_j. Then a potential startup's expected profits from entering through R&D is

$$\Pi^e = \beta \mathbb{E}\Big[V(\{(q_j', \Delta_j')\}) \,\big|\, x_e \Big] - \widetilde{\chi}_e (x_e)^{\widetilde{\psi}_e} \overline{q} \, ,$$

where the expectation conditioning on x_e is taken over the distribution of incumbents' product quality q_j and technology gap Δ_j due to the undirected nature of external innovation. Potential startups choose the probability of external innovation x_e that maximizes the expected profits from entry. Since there is no ex-ante heterogeneity among potential startups, all of them choose the same optimal probability of external innovation x_e^*. Thus, the mass of potential domestic startups that succeed in external innovation and try to take over incumbent firms' product markets is $\mathcal{E}_d x_e^*$.

1.2.6 Exogenous Competitive Pressure and Creative Destruction

As briefly explained in the previous section, the aggregate creative destruction arrival rate is the probability (frequency) that in each product market an individual incumbent faces a rival (a domestic startup, a domestic incumbent or a foreign firm) that succeeded in external innovation. Conditional on external innovation, whether the incumbent is replaced by the rival firm depends on the technology gap and internal innovation of the incumbent.

Each firm can externally innovate at most one product line each period, and there is a continuum of unit mass of product lines (markets). Thus, the total mass of firms that succeed in external innovation is equal to the total mass of product markets for which the incumbent faces a rival firm. Since external innovation is undirected, this implies that the probability an individual product market incumbent is faced with competition from an another firm—the aggregate creative destruction arrival rate—is equal to the total mass of firms that succeed in external innovation. Denote $\overline{x}_d$ as the total mass of domestic firms that succeed in external innovation, and denote $\overline{x}_o$ as the foreign firm counterpart. Then the aggregate creative destruction arrival rate $\overline{x}$ is

$$\overline{x} = \overline{x}_d + \overline{x}_o \, .$$

Increased competitive pressure from foreign firms comes from the increased mass of foreign firms trying to start businesses in domestic markets $\mathcal{E}_o$, which increases the mass of foreign firms operating in domestic markets $\mathcal{F}_o$. Thus increasing competitive pressure is defined as an exogenous increase in $\overline{x}_o$ in this model economy.

1.2.7 Equilibrium

I now turn to describing optimal decisions for each agent and the Markov Perfect Equilibrium of the economy, where optimal decisions depend only on individual characteristics, aggregate variables, and the technology gap distribution.

1.2.7.1 Optimal Production and Employment

The solution for the final good producer's profit maximization problem defines the final good producer's optimal demands for labor and differentiated products. Denote p_j as the price for differentiated product j, and w as the wage rate in the domestic economy. Then the inverse demand for differentiated product j is

$$p_j = q_j^{\theta} L^{\theta} y_j^{-\theta} \ . \tag{1.6}$$

In deriving demand for product j I assume that each product is supplied by a single firm. However, past incumbent firms in domestic markets that lost technological leadership to the current leader could in principle try to produce and sell their products through limit pricing, as the marginal cost of production is equal across all firms. To avoid such cases and to simplify the model, I assume the following two-stage price-bidding game:

Assumption 1. *In a given product line j in a given economy, the current incum-*

bents and any former incumbents in the same line enter a two-stage price-bidding game. In the first stage, each firm pays a fee of $\varepsilon > 0$. In the second stage, all firms that paid the fee announce their prices.

This assumption ensures that only the technological leader enters the first stage and announces its price in equilibrium.

Differentiated products producers (both domestic and foreign) take their products demand curve from the final good producer (1.6) as given and maximize profit (revenue net of production cost) for individual product line $j \in \mathcal{J}^f$:

$$\pi(q_j) = \max_{y_j \geqslant 0} \left\{ L^{\theta} q_j^{\theta} y_j^{1-\theta} - y_j \right\} \ .$$

Note that since each differentiated product is produced at unit marginal cost in terms of the final good, the differentiated product producers' problem for supplying to the domestic market is the same for both domestic firms and foreign firms. The FOC of this problem yields optimal level of differentiated product j production:

$$y_j = (1-\theta)^{\frac{1}{\theta}} L q_j \ , \tag{1.7}$$

and by plugging this into the final good producer's differentiated product j demand (1.6), we get the monopoly price

$$p_j = \frac{1}{1-\theta} \ , \tag{1.8}$$

which is a markup $\frac{1}{1-\theta}$ over the unit marginal cost. Using (1.7), we get the profit from individual differentiated product production, which is linear in its quality, holding aggregate variables fixed:

$$\pi(q_j) = \underbrace{\theta(1-\theta)^{\frac{1-\theta}{\theta}} L}_{\equiv \pi} q_j \,.$$

From the final good producer's problem, an equilibrium wage rule is given by

$$w = \theta(1-\theta)^{\frac{1-2\theta}{\theta}} \overline{q} \,, \tag{1.9}$$

which depends only on the average product quality in the economy. Since

$$L = 1 \tag{1.10}$$

in equilibrium, the optimal level of differentiated product j production becomes

$$y_j = (1-\theta)^{\frac{1}{\theta}} q_j \tag{1.11}$$

and the scaling part of the profit from differentiated product production becomes

$$\pi = \theta(1-\theta)^{\frac{1-\theta}{\theta}} \,.$$

Finally, using (1.10) and (1.11), equilibrium final good production can be written as

$$Y = (1-\theta)^{\frac{1-2\theta}{\theta}} \overline{q}\,, \tag{1.12}$$

which grows at the same rate as average (total) product quality.

1.2.7.2 Value Function for Incumbent Firm in the Differentiated Product Market

In this section, I solve for a differentiated product firm's optimal R&D decision. Define $\Phi^f \equiv \{(q_j, \Delta_j)\}_{j\in\mathcal{J}^f}$ as a multi-set of product quality and technology gap pairs currently owned by differentiated products producer f, where (q_j, Δ_j) defines product line j completely. Then firm f's value function can be written as

$$V\left(\Phi^f\right) = \max_{\substack{x\in[0,\bar{x}],\\ \{z_j\in[0,\bar{z}]\}_{j\in\mathcal{J}^f}}} \left\{ \sum_{j\in\mathcal{J}^f} \left[\pi q_j - \hat{\chi} z_j^{\hat{\psi}} q_j\right] - \bar{q}\widetilde{\chi}x^{\widetilde{\psi}} + \widetilde{\beta}\,\mathbb{E}\left[V\left(\Phi^{f\prime} \mid \Phi^f\right) \Big| \{z_j\}_{j\in\mathcal{J}^f}, x\right]\right\}\,,$$

where πq_j is revenue net of production cost. Thus the first three terms define current profits of a firm with product quality and technology gap portfolio Φ^f, and the last term is discounted expected future value, where the conditional expectation is taken over the success or failure of internal and external innovation, creative destruction arrival, winning or losing coin-tosses (c-t), the current period product quality distribution, and the current period technology gap distribution. $\widetilde{\beta}$ is the stochastic discount factor, which is constant over time as there is no uncertainty in this economy.

Proposition 1. *For a given technology gap distribution* $\big\{\mu\big(\Delta^\ell\big)\big\}_{\ell=1}^4$*, the value function of a firm with product quality and technology gap portfolio* $\Phi^f \equiv \big\{(q_j, \Delta_j)\big\}_{j\in\mathcal{J}^f}$ *is of the form:*

$$V(\Phi^f) = \sum_{\ell=1}^{4} A_\ell \left(\sum_{j\in\mathcal{J}^f|\Delta_j=\Delta^\ell} q_j \right) + \; B\,\overline{q}\,,$$

where

$$A_1 = \pi - \widehat{\chi}(z^1)^{\widehat{\psi}} + \widetilde{\beta}\Big[A_1(1-\overline{x})(1-z^1) + \lambda A_2(1-\overline{x})z^1\Big] \tag{1.13}$$

$$A_2 = \pi - \widehat{\chi}(z^2)^{\widehat{\psi}} + \widetilde{\beta}\Big[A_1(1-\overline{x})(1-z^2) + \lambda A_2 z^2\Big] \tag{1.14}$$

$$A_3 = \pi - \widehat{\chi}(z^3)^{\widehat{\psi}} + \widetilde{\beta}\left[A_1\left(1-\frac{1}{2}\overline{x}\right)(1-z^3) + \lambda A_2 z^3\right] \tag{1.15}$$

$$A_4 = \pi - \widehat{\chi}(z^4)^{\widehat{\psi}} + \widetilde{\beta}\left[A_1(1-\overline{x})(1-z^4) + \lambda A_2\left(1-\frac{1}{2}\overline{x}\right)z^4\right] \tag{1.16}$$

$$B = \frac{1}{1-\widetilde{\beta}(1+g)}\left[x\widetilde{\beta}A_{takeover} - \widetilde{\chi}x^{\widetilde{\psi}}\right]\,, \tag{1.17}$$

and optimal innovation probabilities are

$$z^1 = \left[\frac{\widetilde{\beta}\left[(1-\overline{x})\lambda A_2 - (1-\overline{x})A_1\right]}{\widehat{\psi}\widehat{\chi}}\right]^{\frac{1}{\widehat{\psi}-1}} \tag{1.18}$$

$$z^2 = \left[\frac{\widetilde{\beta}\left[\lambda A_2 - (1-\overline{x})A_1\right]}{\widehat{\psi}\widehat{\chi}}\right]^{\frac{1}{\widehat{\psi}-1}} \tag{1.19}$$

$$z^3 = \left[\frac{\widetilde{\beta}\left[\lambda A_2 - \left(1-\frac{1}{2}\overline{x}\right)A_1\right]}{\widehat{\psi}\widehat{\chi}}\right]^{\frac{1}{\widehat{\psi}-1}} \tag{1.20}$$

$$z^4 = \left[\frac{\widetilde{\beta}\left[\lambda\left(1-\frac{1}{2}\overline{x}\right)A_2-\left(1-\overline{x}\right)A_1\right]}{\widehat{\psi}\widehat{\chi}}\right]^{\frac{1}{\widehat{\psi}-1}} \tag{1.21}$$

$$x = \left[\frac{\widetilde{\beta}A_{takeover}}{\widetilde{\psi}\widetilde{\chi}}\right]^{\frac{1}{\widetilde{\psi}-1}} . \tag{1.22}$$

g in the expression for B is the average product quality growth rate in the economy, and $A_{takeover}$ in the expressions for B and x is the ex-ante value of a product line obtained from successful takeover, which is defined as:

$$A_{takeover} \equiv \frac{1}{2}(1-z^3)A_1\mu(\Delta^3) + \left(1-\frac{1}{2}z^4\right)A_2\lambda\mu(\Delta^4) \\ + A_3\eta\mu(\Delta^1) + (1-z^2)A_4\frac{\eta}{\lambda}\mu(\Delta^2) \,.$$

Proof: See Appendix A.1.3.1

As the expression shows, the determinants of $A_{takeover}$ include factors that determine the conditional takeover probability $\overline{x}_{takeover}$.

A_ℓ is the sum of discounted expected profits from owning a product line with technology gap equal to Δ^ℓ, normalized by the current period product quality. The first two terms in (1.13) through (1.16) are the normalized instantaneous profits, and the terms inside the brackets are the normalized option value from internal innovation. If a firm succeeds in internally innovating its product and still owns that product next period, then the normalized value of that product is equal to A_2, as the next period technology gap is equal to Δ^2. If the firm fails to internally innovate its product but still owns that product next period, then the normalized value of that product is equal to A_1 as the next period technology gap is equal to Δ^1. B is the sum of discounted expected profits from owning an additional product through external innovation, normalized by the average product quality. To understand this variable more clearly, rewrite it as

$$B\overline{q} = x\widetilde{\beta}A_{takeover}\overline{q} - \widetilde{\chi}x^{\widetilde{\psi}}\overline{q} + \widetilde{\beta}(1+g)B\overline{q}\,.$$

After investing $\widetilde{\chi}x^{\widetilde{\psi}}\overline{q}$ in external innovation in the current period, the firm receives the discounted expected profit $A_{takeover}\overline{q}$ if external innovation succeeds with probability x next period. The firm owns at least one product line next period if current period external innovation is successful. Thus, it will invest in external innovation next period and receive an expected profit of $B\overline{q}'$ two periods later, where $\overline{q}' = (1+g)\overline{q}$. Thus, (1.17) shows that B is the annuity value of an infinite stream of constant payoffs $x\widetilde{\beta}A_{takeover} - \widetilde{\chi}x^{\widetilde{\psi}}$ at a constant discount rate $\widetilde{\beta}(1+g)$, the growth rate adjusted time discount factor.

For all of the optimal probabilities of internal innovation, the first term inside the brackets in the numerator (after $\widetilde{\beta}$) is the option value from successful internal innovation, which increases quality by λ. The second term is the option value from no internal innovation, which makes next period's technology gap equal to one. Thus, the higher the option value for successful internal innovation, the higher is the optimal probability of internal innovation, holding $\overline{x}$ fixed. For this reason, the optimal probability of internal innovation for each product line depends on its technology gap. Intuitively, a wider technology gap should up to a point increase firms' internal innovation investment, as this implies that escape from competition is easier. However, past some point a wider technology gap should dissuade incumbent firms from investing in internal innovation, since it is much harder for other firms to take over a product line with a very high technology gap. Thus. there is a low probability that an incumbent firm will lose a product line when it has very high technology gap. Corollary 1 formalizes this argument.

Corollary 1. *In an equilibrium where* $\{z^\ell\}_{\ell=1}^4$ *are well defined, the probabilities of internal innovation satisfy* $z^2 > z^3 > z^4 > z^1$.

Proof: See Appendix A.1.3.2

Thus, for a product line with the widest technology gap $\Delta^3 = \eta$, firms invest less in internal innovation than they do for a product line with $\Delta^2 = \lambda$, as there is a lower probability they will lose the product line even if they don't improve its quality—firms with technology gap Δ^3 lose a product line only when they are in a neck and neck case and lose the coin toss. Thus, $z^2 > z^3$, even though $\Delta^3 > \Delta^2$.

Since A_1 and A_2 depend on $\overline{x}$, it is difficult to sign the partial derivatives of $\{z^\ell\}_{\ell=1}^4$ w.r.t. $\overline{x}$. But holding the values for A_1 and A_2 fixed, we can determine the signs of the partial derivatives, which defines the escape-competition effect:

Corollary 2. *With* $\widetilde{\psi} \in (1, 2]$, *the escape-competition effect is the highest and positive for product lines with technology gap equal to* Δ^2, *whereas it is the lowest and negative for product lines with technology gap equal to* Δ^1. *The escape-competition effect is positive for the* Δ^3 *case, whereas its sign is ambiguous for the* Δ^4 *case.*

Proof: See Appendix A.1.3.3

As equation (1.2) shows, a firm cannot protect its product line from takeover through internal innovation if its technology gap is equal to Δ^1. This is why z^1 is a decreasing function of the creative destruction arrival rate $\overline{x}$, other things being equal. As equation (1.3) shows, thee impact of internal innovation on the probability of losing their product is greatest in the Δ^2 case. Thus, the escape-competition incentive is the highest for this case. In the Δ^3 case, a marginal increase in z^3 decreases the probability of losing the product by 50% less than in the Δ^2 case. Thus the escape-competition effect is lower. The escape-competition effect for the Δ^4 case is ambiguous as the probability decrease is even lower.

Meanwhile, the term A_2 in the optimal probability of internal innovation reflects the Schumpeterian effect. The lower the expected future profits from keeping the product line through internal innovation, the lower is the incentive to invest in internal innovation.

The optimal probability of external innovation depends on internal innovation intensities, product values ($\{A_\ell\}_{\ell=1}^4$), and the technology gap distribution. The definition of $A_{takeover}$ and equation (1.22) indicate that higher overall innovation intensities (internal and external) in the economy lower the incentive for external innovation for an individual firm in partial equilibrium, holding product values fixed. This is the technological-barrier effect summarized in the conditional takeover probability $\overline{x}_{takeover}$. Holding probabilities of internal innovation and the technology gap distribution fixed, a decrease in product values decreases an individual firm's incentive for external innovation. This is the Schumpeterian effect.

The direction of the changes in the probabilities of internal and external innovation in response to changes in the aggregate creative destruction arrival rate $\overline{x}$ are ambiguous in general equilibrium. They depend on the relative magnitudes and the directions of the escape-competition effect, the Schumpeterian effect, and the technological-barrier effect. Nonetheless, results from the numerical exercise in Section 1.4.3.1 confirm that the partial equilibrium results for given $\{A_\ell\}_{\ell=1}^4$ and B still hold in general equilibrium for a plausible parameterization. Furthermore, $\{A_\ell\}_{\ell=1}^4$ and B also decrease after an exogenous increase in $\overline{x}$.

1.2.7.3 Potential Startups

Recall that a potential startup's expected profits from entering through R&D are

$$\Pi^e = \widetilde{\beta}\mathbb{E}\Big[V(\{(q_j', \Delta_j')\}) \,\Big|\, x_e\Big] - \widetilde{\chi}_e (x_e)^{\widetilde{\psi}_e}\overline{q}\,.$$

By using the value function derived in Proposition 1, the optimal probability of external innovation for potential startups x_e can be computed as

$$x_e = \left(\widetilde{\beta}\frac{A_{takeover} + \overline{x}_{takeover}B(1+g)}{\widetilde{\psi}_e\,\widetilde{\chi}_e}\right)^{\frac{1}{\widetilde{\psi}_e - 1}}\,. \tag{1.23}$$

The proof is in Appendix A.1.4.

As explained in the previous section, the total mass of domestic firms that succeed in external innovation defines the portion of the aggregate creative destruction arrival rate accounted for by domestic firms. Since the optimal probabilities of external innovation for incumbent firms and potential domestic startups are equal to x and x_e respectively, the aggregate creative destruction arrival rate in this economy is defined as

$$\overline{x} = \underbrace{\mathcal{F}_d\, x + \mathcal{E}_d\, x_e}_{\equiv \overline{x}_d} + \overline{x}_o\,. \tag{1.24}$$

Since the mass of domestic incumbent firms $\mathcal{F}_d$ and the probabilities of external innovation x and x_e depend on $\overline{x}$, an exogenous increase in $\overline{x}_o$ doesn't increase $\overline{x}$ by the same amount in equilibrium. Thus, the level of $\overline{x}$ is endogenously determined even when $\overline{x}_o$ changes exogenously.

1.2.8 Growth rate

As equation (1.12) shows, the output growth rate in this model economy is equal to the product quality growth rate g. Proposition 2 shows how this growth rate is defined and decomposes it according to the contributions made by different groups of firms and types of innovation.

Proposition 2. *The growth rate for aggregate variables in a Balanced Growth Path in this economy, g, is defined as*

$$\begin{aligned} g =& \Big[(1-\overline{x})(1-z^1) + \Delta^2(1-\overline{x})z^1 + \Delta^3\overline{x}\Big]\mu(\Delta^1) \\ &+ \Big[(1-\overline{x})(1-z^2) + \Delta^2 z^2 + \Delta^4\overline{x}(1-z^2)\Big]\mu(\Delta^2) + \Big[1 - z^3 + \Delta^2 z^3\Big]\mu(\Delta^3) \\ &+ \Big[(1-\overline{x})(1-z^4) + \Delta^2(z^4 + \overline{x}(1-z^4))\Big]\mu(\Delta^4) - 1\,. \end{aligned} \tag{1.25}$$

Furthermore, g can be decomposed into four components:

$$
\begin{aligned}
1+g =& \Big[(1-\overline{x})(1-z^1) + \Delta^2(1-\overline{x})z^1\Big]\mu(\Delta^1) + \Big[(1-\overline{x})(1-z^2) + \Delta^2 z^2\Big]\mu(\Delta^2) \\
& \underbrace{+\left[\left(1-\frac{1}{2}\overline{x}\right)(1-z^3) + \Delta^2 z^3\right]\mu(\Delta^3) + \left[(1-\overline{x})(1-z^4) + \Delta^2\left(1-\frac{1}{2}\overline{x}\right)z^4\right]\mu(\Delta^4)}_{\textit{internal innovation by both domestic incumbents and foreign firms}} \\
& + \underbrace{\Delta^3\mathcal{F}_d x\mu(\Delta^1) + \Delta^4\mathcal{F}_d x(1-z^2)\mu(\Delta^2) + \frac{1}{2}\mathcal{F}_d x(1-z^3)\mu(\Delta^3) + \Delta^2\mathcal{F}_d x\left(1-\frac{1}{2}z^4\right)\mu(\Delta^4)}_{\textit{external innovation by domestic incumbent firms}} \\
& + \underbrace{\Delta^3\mathcal{E}_d x_e\mu(\Delta^1) + \Delta^4\mathcal{E}_d x_e(1-z^2)\mu(\Delta^2) + \frac{1}{2}\mathcal{E}_d x_e(1-z^3)\mu(\Delta^3) + \Delta^2\mathcal{E}_d x_e\left(1-\frac{1}{2}z^4\right)\mu(\Delta^4)}_{\textit{external innovation by domestic startups}} \\
& + \underbrace{\Delta^3\overline{x}_o\mu(\Delta^1) + \Delta^4\overline{x}_o(1-z^2)\mu(\Delta^2) + \frac{1}{2}\overline{x}_o(1-z^3)\mu(\Delta^3) + \Delta^2\overline{x}_o\left(1-\frac{1}{2}z^4\right)\mu(\Delta^4)}_{\textit{external innovation by foreign firms}}\ .
\end{aligned}
$$

Proof: See Appendix A.1.5.1

1.2.9 Firm Distribution

As the differentiated product firm's decision rules show, the distribution of firms' technology gaps completely describes the distribution of firms in this model economy.[10] In this section, I describe how I keep track of the evolution of this distribution. Denote the technology gap composition for a firm with n_f product lines and with n_f^ℓ products with technology gap equal to Δ^ℓ, $\ell = 1,2,3,4$ as $\mathcal{N} = (n_f, n_f^1, n_f^2, n_f^3, n_f^4)$, and the density of this object as $\mu(\mathcal{N})$.

[10] The technology gap distribution can be computed from this distribution.

1.2.9.1 Technology Gap Portfolio Composition Distribution Transition

Define the technology gap portfolio composition for a firm with $n_f - k$ products with $\Delta = \Delta^1$, k products with $\Delta = \Delta^2$, zero products with $\Delta = \Delta^3$ and zero products with $\Delta = \Delta^4$ as $\widetilde{\mathcal{N}}(n_f, k) \equiv (n_f, n_f - k, k, 0, 0)$, for $k \in [0, n_f] \cap \mathbb{Z}$, $n_f > 0$. Then without considering external innovation, the probability of $\mathcal{N} = \widetilde{\mathcal{N}}(n_f, k)$ becoming $\mathcal{N}' = \widetilde{\mathcal{N}}(n_f, \widetilde{k})$ can be computed as

$$\widetilde{\mathbb{P}}\Big(n_f, \widetilde{k} \mid n_f, k\Big) = \begin{cases} \sum_{\widetilde{k}^1 = \max\{0,\, \widetilde{k}-k\}}^{\min\{n_f - k,\, \widetilde{k}\}} \begin{pmatrix} n_f - k \\ \widetilde{k}^1 \end{pmatrix} \begin{pmatrix} k \\ \widetilde{k} - \widetilde{k}^1 \end{pmatrix} & \\ \quad \times \begin{bmatrix} (1 - \overline{x})^{n_f - (\widetilde{k} - \widetilde{k}^1)} \, (1 - z^1)^{n_f - k - \widetilde{k}^1} \, (z^1)^{\widetilde{k}^1} \\ \times (1 - z^2)^{k - (\widetilde{k} - \widetilde{k}^1)} \, (z^2)^{\widetilde{k} - \widetilde{k}^1} \end{bmatrix} & \text{for } n_f \geqslant 1, \text{ and} \\ & 0 \leqslant \widetilde{k}, k \leqslant n_f \\ 0 & \text{otherwise,} \end{cases}$$

where

$$\begin{pmatrix} n \\ k \end{pmatrix} = \frac{n!}{k!(n-k)!}$$

is a combination of selecting k elements from n elements without repetition, where the order of selection does not matter. Thus changes in the technology gap composition follow a binomial process, which is one of the novel features that Ates and Saffie

(2016) introduced as a discrete time mapping of the continuous time endogenous firm growth literature. The range for $\widetilde{k}^1$ is of the form described as above due to the fact that

i. For $0 \leqslant \widetilde{k} \leqslant \min\{n_f - k, k\}$, the two combinations are well defined for any $\widetilde{k}^1 \in [0, \widetilde{k}] \cap \mathbb{Z}$ and describe all the possible cases.

ii. For $n_f - k \geqslant k$, $\widetilde{k} > k$, $0 \leqslant \widetilde{k} - \widetilde{k}^1$, and $0 \leqslant \widetilde{k}^1 \leqslant n_f - k$ should be satisfied. Thus $\widetilde{k} - k \leqslant \widetilde{k}^1 \leqslant \widetilde{k}$.

iii. For $k \geqslant n_f - k$, $\widetilde{k} > n_f - k$, $0 \leqslant \widetilde{k} - \widetilde{k}^1$, and $0 \leqslant \widetilde{k}^1 \leqslant n_f - k$ should be satisfied. Thus $\max\{0,\ \widetilde{k} - k\} \leqslant \widetilde{k}^1 \leqslant n_f - k$.

Since product lines can have technology gap equal to Δ^3 or Δ^4 only through external innovation, the probability of a technology gap composition $\mathcal{N} = (n_f, n_f^1, n_f^2, n_f^3, n_f^4)$ becoming $\mathcal{N}' = (n_f', n_f^{1\prime}, n_f^{2\prime}, n_f^{3\prime}, n_f^{4\prime})$ for any $n_f' \leqslant n_f + 1$ can be computed using $\widetilde{\mathbb{P}}(n_f, \widetilde{k} | n_f, k)$, and with this, the change in the technology gap portfolio composition distribution can be tracked. The procedure is described in detail in Appendix A.1.6.

1.2.9.2 Technology Gap Distribution

By using the distribution of the firm-level technology gap composition for domestic firms $\mathcal{F}_d\, \mu(\mathcal{N})$, the aggregate distribution for the technology gap for the product lines owned by domestic firms $\{\widetilde{\mu}(\Delta^\ell)\}_{\ell=1}^4$ can be computed as

$$\widetilde{\mu}(\Delta^\ell) = \sum_{n_f=1}^{\overline{n}_f} \sum_{n_f^\ell=0}^{n_f} n_f^\ell\, \mathcal{F}_d\, \mu(n_f, n_f^1, n_f^2, n_f^3, n_f^4)\,. \tag{1.26}$$

Since this distribution is for the product lines owned by domestic firms, it should sum up to the total mass of product lines owned by domestic firms. Denote the total mass of product lines owned by domestic firms as s_d. Lemma 2 describes its relationship with the aggregate creative destruction arrival rate $\overline{x}$ in a stationary equilibrium:

Lemma 2. *In a stationary equilibrium, the total mass of product lines owned by domestic firms is equal to the share of the aggregate creative destruction arrival rate accounted for by domestic firms. That is,*

$$s_d = \frac{\overline{x}_d}{\overline{x}} .$$

Proof: See Appendix A.1.7.1

Thus,

$$\sum_{\ell=1}^{4} \widetilde{\mu}\big(\Delta^{\ell}\big) = \frac{\overline{x}_d}{\overline{x}} .$$

Since domestic incumbent firms and foreign firms operating in domestic markets are symmetric in terms of their R&D and production technology, their technology gap distribution should differ only by a constant multiple. Thus the aggregate technology gap distribution is equal to $\mu\big(\Delta^{\ell}\big) = \frac{\overline{x}}{\overline{x}_d}\widetilde{\mu}\big(\Delta^{\ell}\big)$ for $\ell = 1, \ldots 4$, and sums up to one:

$$\sum_{\ell=1}^{4} \mu\big(\Delta^{\ell}\big) = 1 .$$

1.2.9.3 Aggregate Variables and Balanced Growth Path Equilibrium

Given the optimal innovation decision rules, aggregate domestic R&D expenses can be computed as

$$R_d = \widehat{\chi} \sum_{\ell=1}^{4} \left[\int_0^1 q_j \, \mathcal{I}_{\{\Delta_j = \Delta^\ell \, , \, j \in \mathcal{D}\}} \, dj \right] (z^\ell)^{\widehat{\psi}} + \mathcal{F}_d \widetilde{\chi} \overline{q} x^{\widetilde{\psi}} + \mathcal{E}_d \widetilde{\chi}_e (x_e)^{\widetilde{\psi}_e} \overline{q} \, , \tag{1.27}$$

where $\mathcal{I}_{\{\Delta_j = \Delta^\ell \, , \, j \in \mathcal{D}\}}$ is an indicator function equal to one if product line j belongs to a domestic firm with technology gap equal to Δ^ℓ. Also, using the optimal differentiated product production rule, the total final goods used as inputs by domestic differentiated product firms can be written as

$$\begin{aligned} Y_d &= \int_0^1 y_j \, \mathcal{I}_{\{j \in \mathcal{D}\}} \, dj \\ &= (1-\theta)^{\frac{1}{\theta}} \int_0^1 q_j \, \mathcal{I}_{\{j \in \mathcal{D}\}} \, dj \, . \end{aligned}$$

Since R&D expenses and differentiated product production costs are paid with final goods, aggregate consumption becomes

$$C = Y - R_d - Y_d \, . \tag{1.28}$$

The total differentiated products produced by foreign firms in this economy are

$$Y_o = \int_0^1 p_j y_j \, \mathcal{I}_{\{j \notin \mathcal{D}\}} \, dj$$

$$= (1-\theta)^{\frac{1-\theta}{\theta}} \int_0^1 q_j \, \mathcal{I}_{\{j \notin \mathcal{D}\}} \, dj \, .$$

Since there is no government expenditure, the Gross Domestic Production (GDP) in this economy is

$$GDP = Y - Y_o \, .$$

With these aggregate variables defined, I can define the equilibrium of this economy:

Definition 1 (Balanced Growth Path Equilibrium)**.** *A balanced growth path equilibrium of this economy consists of y_j^*, p_j^*, w^*, L^*, x^*, $\{z^{\ell *}\}_{\ell=1}^4$, $\overline{x}^*$, x_e^*, $\mathcal{F}_d^*$, R_d^*, Y^*, C^*, g^*, $\mu(\mathcal{N})$, $\{\widetilde{\mu}(\Delta^\ell)\}_{\ell=1}^4$ for every $j \in [0,1]$ with q_j such that: (i) y_j^* and p_j^* satisfy* (1.11) *and* (1.8)*; (ii) wage rate w^* satisfies* (1.9)*; (iii) total labor for final good production L^* satisfies* (1.10)*; (iv) the probabilities of internal innovation $\{z^{\ell *}\}_{\ell=1}^4$ satisfy* (1.18)*,* (1.19)*,* (1.20)*, and* (1.21)*, and the probability of external innovation by incumbents x^* satisfies* (1.22)*; (v) the aggregate creative destruction arrival rate $\overline{x}^*$ satisfies* (1.24)*; (vi) the probability of external innovation of potential startups x_e^* satisfies* (1.23)*; (vii) aggregate output Y^* satisfies* (1.12)*; (viii) aggregate R&D expense R_d^* satisfies* (1.27)*; (ix) aggregate consumption C^* satisfies* (1.28)*; (x) the BGP growth rate g^* satisfies* (1.25)*; (xi) the invariant distribution of the technology gap portfolio composition $\mu(\mathcal{N})$ and the total mass of domestic firms $\mathcal{F}_d^*$ satisfy inflow($\mathcal{N}$) = outflow($\mathcal{N}$); and (xii) the invariant technology gap distribution $\{\widetilde{\mu}(\Delta^\ell)\}_{\ell=1}^4$ satisfies* (1.26)*.*

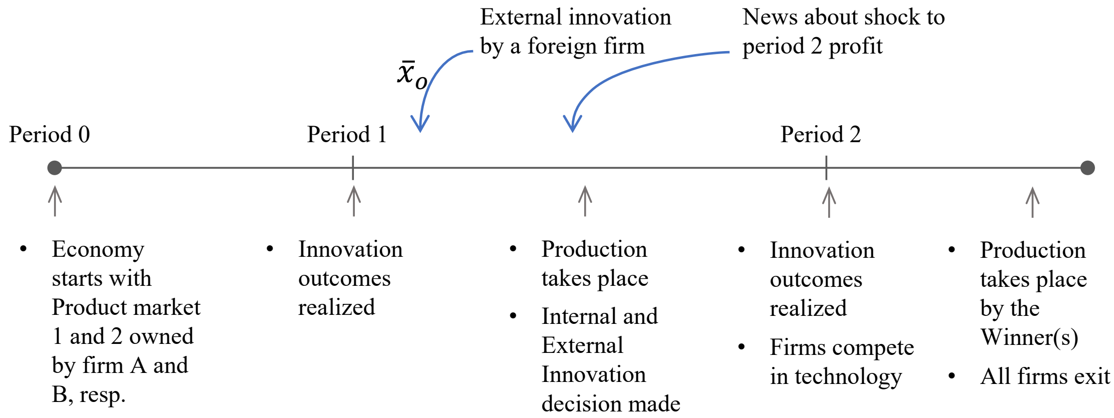

Figure 1.2: Timeline for the Simple Model Economy

1.3 Simple Three-Period Heterogeneous Innovation Model

To understand firms' incentives for internal and external innovation, and to derive empirically testable model predictions, in this section I consider a simplified three-period economy with two product markets and three firms. In period 0, the economy starts with two product markets, market 1 and 2, with initial market-specific technologies $q_{1,0}$ and $q_{2,0}$ and two firms, A and B. Product market 1 is given to firm A and is ready for production. Firm A is also given an initial probability $z_{1,0}$ of internally innovating product 1. Firm B, on the other hand, is given only a probability $x_{2,0}$ of externally innovating product 2. Thus, firm B can start operating and producing in period 1 but not in period 0. If external innovation fails, then firm B still keeps market 2 but produces with initial product quality $q_{2,0}$. Thus, at the beginning of period 1, product qualities in the two markets are equal to:

$$q_{1,1} = \begin{cases} \lambda \, q_{1,0} & \text{with probability } z_{1,0} \\ q_{1,0} & \text{with probability } 1 - z_{1,0} \,, \end{cases}$$

and

$$q_{2,1} = \begin{cases} \eta \, q_{2,0} & \text{with probability } x_{2,0} \\ q_{2,0} & \text{with probability } 1 - x_{2,0} \,. \end{cases}$$

where $\lambda^2 > \eta > \lambda > 1$ are innovation step sizes.

In period 1, the main period of interest, there is a foreign firm (either from a foreign country or different sector/state) that does external innovation hoping to take over the two product markets in period 2. The outside firm succeeds in doing external innovation with probability $\overline{x}_o$ in each product market. Also, there is a news shock about period 2 profit (such as an increase in foreign demand) announced in period 1. Afterwards, the two incumbent firms produce using the given technologies, invest in internal innovation to improve the quality of their own products, and invest in external innovation to take over the other firm's product market. At the beginning of period 2, all innovation outcomes are realized. Then, technological competition in each product market takes place, and the firm with the highest technology in each market produces. The economy ends after period 2. Figure 1.2 summarizes the timing.

In period 1, incumbent firm $i \in \{A, B\}$ invests $R^{in}_{j,1}$ on internal innovation,

$j \in \{1, 2\}$ (e.g., for $i = A$, $j = 1$), implying a success probability $z_{j,1}$ using the R&D production function

$$z_{j,1} = \left(\frac{R^{in}_{j,1}}{\widehat{\chi} q_{j,1}} \right)^{\frac{1}{2}} .$$

Successful internal innovation increases the next-period product quality by $\lambda > 1$. Thus, the period 2 product quality for firm i becomes

$$q^i_{j,2} = \begin{cases} \lambda \, q_{j,1} & \text{with probability } z_{j,1} \\ q_{j,1} & \text{with probability } 1 - z_{j,1} \, . \end{cases}$$

Similarly, firm i invests $R^{ex}_{-j,1}$ to learn the period 0 technology used by firm $-i \neq i$, implying a success probability of external innovation $x_{-j,1}$ using the R&D production function

$$x_{-j,1} = \left(\frac{R^{ex}_{-j,1}}{\widetilde{\chi} q_{-j,0}} \right)^{\frac{1}{2}} ,$$

where $-j$ is owned by $-i$. Successful external innovation increases product quality relative to the past-period quality by $\eta > 1$. Thus, product $-j$'s quality in period 2 for firm i becomes

$$q^i_{-j,2} = \begin{cases} \eta \, q_{-j,0} & \text{with probability } x_{-j,1} \\ \varnothing & \text{with probability } 1 - x_{-j,1} \, , \end{cases}$$

where $\varnothing$ means firm i failed to acquire a production technology for product $-j$.

1.3.1 Optimal Innovation Decisions and Theoretical Predictions

Assume that in each product market j in each period t, firms receive instantaneous profit of $\pi_{j,t}\, q_{j,t}$ where $q_{j,t}$ is the product quality and $\pi_{j,t}$ is a market-period-specific constant known to firms before each period begins. Because there are only two products, incumbents and the foreign firm can perform external innovation on the same product. To keep the model simple, further assume that the foreign firm can do external innovation only if an incumbent fails to do external innovation, following Garcia-Macia et al. (2019). Then the profit maximization problem of firm i that has product market j with quality $q_{j,1}$ in period 1 can be written as

$$V(q_{j,1}) = \max_{\{z_{j,1},\, x_{-j,1}\}} \left\{ \begin{array}{l} \pi_{j,1}q_{j,1} - \widehat{\chi}(z_{j,1})^2 q_{j,1} - \widetilde{\chi}(x_{-j,1})^2 q_{-j,0} \\[2ex] +(1-x_{j,1})(1-\overline{x}_o)\Big[(1-z_{j,1})\pi_{j,2}q_{j,1} + z_{j,1}\pi_{j,2}\lambda q_{j,1}\Big] \\[2ex] +\big[x_{j,1} + (1-x_{j,1})\,\overline{x}_o\big]\Big[z_{j,1}\pi_{j,2}\,\lambda\, q_{j,1}\,\mathcal{I}_{\{\lambda q_{j,1} > \eta q_{j,0}\}} \\ \qquad\qquad\qquad\qquad\quad + \frac{1}{2}(1-z_{j,1})\pi_{j,2}q_{j,1}\,\mathcal{I}_{\{q_{j,1} = \eta q_{j,0}\}}\Big] \\[2ex] +x_{-j,1}\Big[(1-z_{-j,1})\,\pi_{-j,2}\,\eta\, q_{-j,0}\,\mathcal{I}_{\{\eta q_{-j,0} > q_{-j,1}\}} \\ \qquad\quad + z_{-j,1}\,\pi_{-j,2}\,\eta\, q_{-j,0}\,\mathcal{I}_{\{\eta q_{-j,0} > \lambda q_{-j,1}\}} \\ \qquad\quad + \frac{1}{2}(1-z_{-j,1})\pi_{-j,2}\eta q_{-j,0}\,\mathcal{I}_{\{\eta q_{-j,0} = q_{-j,1}\}} \\ \qquad\quad + \frac{1}{2}z_{-j,1}\pi_{-j,2}\eta q_{-j,0}\,\mathcal{I}_{\{\eta q_{-j,0} = \lambda q_{-j,1}\}}\Big] \end{array} \right\},$$

where $\mathcal{I}_{\{\cdot\}}$ is an indicator function that captures the possible relationships between the two technologies among the three firms in period 2 in a given market. The first line shows the period 1 profit net of the total R&D cost. The second line represents the incumbent's period 2 expected profit from market j when the other incumbent and the outside firm fail to externally innovate the market j technology. The third and the fourth line represent the period 2 expected profit from market j when one of the two other firms succeeds in externally innovating the market j technology. The fifth to eighth lines represent the period 2 expected profit from market $-j$ when firm i succeeds in externally innovating the market $-j$ technology. The terms following $\frac{1}{2}$ are for the cases in which two firms can produce the same quality product, so that a coin-toss tiebreaker rule applies.

The interior solutions to this problem are

$$z^*_{j,1} = \begin{cases} \dfrac{\pi_{j,2}}{2\,\widehat{\chi}}\,(\lambda-1)(1-x^*_{j,1})(1-\overline{x}_o) & \text{, when } q_{j,1}=q_{j,0} \\[2ex] \dfrac{\pi_{j,2}}{2\,\widehat{\chi}}\left[\lambda-(1-x^*_{j,1})(1-\overline{x}_o)\right] & \text{, when } q_{j,1}=\lambda\,q_{j,0} \\[2ex] \dfrac{\pi_{j,2}}{2\,\widehat{\chi}}\left[\lambda-\dfrac{1}{2}-\dfrac{1}{2}(1-x^*_{j,1})(1-\overline{x}_o)\right] & \text{, when } q_{j,1}=\eta\,q_{j,0} \end{cases}$$

and

$$x^*_{-j,1} = \begin{cases} \dfrac{\eta\,\pi_{-j,2}}{2\,\widetilde{\chi}} & \text{, when } q_{-j,1}=q_{-j,0} \\[2ex] \dfrac{\eta\,\pi_{-j,2}}{2\,\widetilde{\chi}}\,(1-z^*_{-j,1}) & \text{, when } q_{-j,1}=\lambda\,q_{-j,0} \\[2ex] \dfrac{\eta\,\pi_{-j,2}}{2\,\widetilde{\chi}}\,\dfrac{1}{2}(1-z^*_{-j,1}) & \text{, when } q_{-j,1}=\eta\,q_{-j,0}\,. \end{cases}$$

The above results show that the firm's optimal innovation decisions depend on the (expected) future profit, the technology gap in both its own market and the other firm's market, and other firms' internal and external innovation decisions. From these interior solutions, I draw the following results.

Proposition 3. *For each $q_{j,1}$ and for $\lambda^2 > \eta > \lambda > 1$, we can order internal innovation intensities as*

$$z^*_{j,1}\big|_{q_{j,1}=\lambda q_{j,0}} \quad > \quad z^*_{1,1}\big|_{q_{j,1}=\eta q_{j,0}} \quad > \quad z^*_{j,1}\big|_{q_{j,1}=q_{j,0}} .$$

Furthermore,

$$\left.\frac{\partial z^*_{j,1}}{\partial \overline{x}_o}\right|_{q_{j,1}=\lambda q_{j,0}} \quad > \quad \left.\frac{\partial z^*_{j,1}}{\partial \overline{x}_o}\right|_{q_{j,1}=\eta q_{j,0}} \quad > \quad 0 \quad > \quad \left.\frac{\partial z^*_{j,1}}{\partial \overline{x}_o}\right|_{q_{j,1}=q_{j,0}} .$$

Proof: See Appendix A.2.1

The second part of proposition 3 implies that firms with no technology gap lower their internal innovation investment when they are faced with a higher probability of creative destruction in their own markets, as they cannot increase the probability of escaping competition by improving their products through internal innovation. On the other hand, if a firm has very high technological advantage, then it doesn't increase its internal innovation investment much in response to increased' investment in external innovation by the foreign firm, because the probability of losing its own product market is small. In the intermediate case, firms increase their internal innovation investment more strongly in response to higher external innovation by the foreign firm, as they can substantially lower the probability of losing their markets by doing so.

More successful innovation in period 0 increases the probability of having a high technology gap in period 1, and this helps firms to escape competition. To understand how past innovation intensity affects the firm's current internal innovation decision when the firm is faced with a higher probability of encountering a competitor, $\overline{x}_o$, define the expected value of internal innovation intensity in period 1 as

$$\overline{z}_1^* = z_{1,1}^*\big|_{q_{1,1}=q_{1,0}} \frac{1}{2}(1 - z_{1,0}) \; + \; z_{2,1}^*\big|_{q_{2,1}=q_{2,0}} \frac{1}{2}(1 - x_{2,0}) \; + \; z_{1,1}^*\big|_{q_{1,1}=\lambda q_{1,0}} \frac{1}{2} z_{1,0} \\ + \; z_{2,1}^*\big|_{q_{2,1}=\eta q_{2,0}} \frac{1}{2} x_{2,0} \, ,$$

where $\frac{1}{2}$ comes from the fact that there are two products. Then, proposition 3 gives us:

Corollary 3 (**Escape Competition Effect**). *The impacts of period 0 innovation intensities, $z_{1,0}$ and $x_{2,0}$ on the expected response of internal innovation in period 1 to foreign competition satisfy:*

$$\frac{\partial \overline{z}_1^*}{\partial \overline{x}_o \partial z_{1,0}} > 0 \, , \; \textit{and} \; \frac{\partial \overline{z}_1^*}{\partial \overline{x}_o \partial x_{2,0}} > 0 \, .$$

Proof: See Appendix A.2.2

Corollary 3 implies that intensive innovation in the previous period induces firms to increase the response of their current internal innovation to higher product market competition.

As the optimal decision rule shows, firms' external innovation decision also depends on the past innovation decisions of other firms:

Proposition 4. *For each $q_{j,1}$ and for $\lambda^2 > \eta > \lambda > 1$, we can order external innovation intensities as*

$$x^*_{j,1}\big|_{q_{j,1}=q_{j,0}} \quad > \quad x^*_{j,1}\big|_{q_{j,1}=\lambda q_{j,0}} \quad > \quad x^*_{j,1}\big|_{q_{j,1}=\eta q_{j,0}}$$

Furthermore,

$$\frac{\partial x^*_{j,1}}{\partial \overline{x}_o}\bigg|_{q_{j,1}=q_{j,0}} = 0\,, \quad \frac{\partial x^*_{j,1}}{\partial \overline{x}_o}\bigg|_{q_{j,1}=\lambda q_{j,0}} < 0\,, \ \ and \quad \frac{\partial x^*_{j,1}}{\partial \overline{x}_o}\bigg|_{q_{j,1}=\eta q_{j,0}} < 0\,.$$

Proof: See Appendix A.2.1

Proposition 4 implies that firms do less external innovation if other firms have a higher technology advantage, as it becomes more difficult to take over their markets through external innovation. For product markets with a technological barrier (technology gap > 1), firms also lower their external innovation if the outside firm does more external innovation, as incumbents in these markets will respond by doing more internal innovation with a defensive motive (proposition 3). To understand how the past innovation intensity of other firms affects a firm's current external innovation decision, define the expected value of external innovation intensity in period 1 as

$$\overline{x}^*_1 = x^*_{1,1}\big|_{q_{1,1}=q_{1,0}}\,\frac{1}{2}(1-z_{1,0}) \;+\; x^*_{2,1}\big|_{q_{2,1}=q_{2,0}}\,\frac{1}{2}(1-x_{2,0}) \;+\; x^*_{1,1}\big|_{q_{1,1}=\lambda q_{1,0}}\,\frac{1}{2}z_{1,0}$$

$$+ \; x^*_{2,1}\big|_{q_{2,1}=\eta q_{2,0}} \, \frac{1}{2} x_{2,0} \, .$$

Then, the first part of proposition 4 implies the following:

Corollary 4 (Technological Barrier Effect). *For a given technology $q_{j,1}$ and period 0 innovation intensities, $z_{1,0}$ and $x_{2,0}$, we have*

$$\frac{\partial \overline{x}^*_1}{\partial z_{1,0}} < 0 \;\; , \; and \;\; \frac{\partial \overline{x}^*_1}{\partial x_{2,0}} < 0 \, .$$

Proof: See Appendix A.2.3

Corollary 4 implies that higher technology levels in other markets, due to previous innovation, serve as an effective technological barrier that makes it difficult for a firm to take over another firm's product market. This reduces firms' incentive for external innovation. Because innovation is forward looking, changes in future profit π' are an important factor affecting current period innovation intensity. Proposition 5 summarizes this:

Proposition 5 (Ex-post Schumpeterian Effect)**.** *Given expected period 2 profit $\pi_{j,2}$, we have*

$$\frac{\partial z^*_{j,1}}{\partial \pi_{j,2}} > 0 \, , \quad \forall \, q_{j,1} \;\; , \; and \quad \frac{\partial x^*_{j,1}}{\partial \pi_{j,2}} > 0 \, , \quad for \;\; q_{j,1} = q_{j,0} \, .$$

Signs for $\frac{\partial x^*_{j,1}}{\partial \pi_{j,2}}$ for other technology gaps are ambiguous.

Proof: See Appendix A.2.4

Proposition 5 implies that any factor that affects future profits may affect firms' internal and external innovation. These include market size changes (such as an opportunity to access foreign markets), changes in input costs, and the future survival probability. More specifically, an increase in the expected profit from one's own market induces firms to increase their internal innovation. However, the effect of increasing expected profit in other markets on firms' external innovation is ambiguous for cases with technology gap > 1. This is because incumbents in these markets increase their internal innovation in response to increasing expected profit, and this helps them escape competition. For the case with technology gap $= 1$, incumbents cannot escape competition through internal innovation. Thus, an increase in expected future profit unambiguously increases external innovation for this case. The above results outline various factors affecting internal, external, and total innovation. These predictions can be tested empirically, once we have a well-measured shock to competitive pressure in the data.

1.4 Quantitative Analysis

In this section, I calibrate the model to the average characteristics of the U.S. manufacturing sector from 1987 to 1997, and study how an increase in competitive pressure by foreign firms affects U.S. firms' innovation decisions. Then, I run the same exercise in a model economy where external innovation is much more expensive than the U.S., and compare the results with those from the previous exercise. This comparison highlights how the same competitive pressure shock can lead to a decrease in overall innovation in an economy with high creativity (an economy with less expensive external innovation), and an increase in overall innovation in an economy with low creativity (an economy with expensive external innovation). Lastly, I run an exercise in which I reduce the cost of external innovation by potential startups, which increases competitive pressure by domestic entrants.

1.4.1 Solution Algorithm

Since $\{z^\ell\}_{\ell=1}^4$ are functions of $\overline{x}$; g is a function of $\overline{x}$, $\{z^\ell\}_{\ell=1}^4$, and $\{\mu(\Delta^\ell)\}_{\ell=1}^4$; x is a function of $\overline{x}$ and $\{\mu(\Delta^\ell)\}_{\ell=1}^4$; x_e is a function of $\overline{x}$ and $\{\mu(\Delta^\ell)\}_{\ell=1}^4$; and $\overline{x}$ is a function of $\mathcal{F}_d$, x, and x_e, I solve for an equilibrium of the model by iterating over the value for the aggregate creative destruction arrival rate $\overline{x}$.

1.4.1.0.1 *Solution Algorithm*

i) Guess a value for $\overline{x}$ and the technology gap portfolio composition distribution $\mu(\mathcal{N})$, which imply a technology gap distribution $\{\mu(\Delta^\ell)\}_{\ell=1}^4$ and total mass of domestic firms $\mathcal{F}_d$.

ii) Using the guess for $\overline{x}$, compute $\{A_\ell\}_{\ell=1}^4$, and $\{z^\ell\}_{\ell=1}^4$.

iii) Using the guesses for $\mu(\mathcal{N})$, $\{\mu(\Delta^\ell)\}_{\ell=1}^4$, and $\mathcal{F}_d$,

   a) Compute g, x, B, and x_e.

   b) Compute stationary $\mu_\infty(\mathcal{N})$, thus $\{\mu_\infty(\Delta^\ell)\}_{\ell=1}^4$, using the guesses for

Table 1.1: Parameter Estimates

#	Parameter	Description	Value	Identification
1.	β	time discount rate	0.9615	annual interest rate of 4%
2.	$\widehat{\psi}$	curvature of internal R&D	2	Akcigit and Kerr (2018)
3.	$\widetilde{\psi}$	curvature of external R&D	2	Akcigit and Kerr (2018)
4.	$\widetilde{\psi}^e$	curvature of external R&D, startup	2	Akcigit and Kerr (2018)
5.	θ	quality share in final goods production	0.109	data
6.	$\widehat{\chi}$	scale of internal R&D	0.042	indirect inference
7.	$\widetilde{\chi}$	scale of external R&D	1.184	indirect inference
8.	$\widetilde{\chi}^e$	scale of external R&D, startup	7.696	indirect inference
9.	λ	quality multiplier of internal innovation	0.021	indirect inference
10.	η	quality multiplier of external innovation	0.038	indirect inference
11.	$\overline{x}_o$	exogenous foreign c.d. arrival rate	0.045	indirect inference

$\mu(\mathcal{N})$, innovation decision rules and the relationship

$$\mathcal{F}_{d,n+1}\, \mu_{n+1}(\mathcal{N}) = \mathcal{F}_{d,n}\, \mu_n(\mathcal{N}) + \text{inflow}_n(\mathcal{N}) - \text{outflow}_n(\mathcal{N}) \,.$$

c) Compute g_∞, x_∞, B_∞, and $x_{e\infty}$ using $\mu_\infty(\mathcal{N})$, and $\{\mu_\infty(\Delta^\ell)\}_{\ell=1}^4$.

iv) Compute $\overline{x}' = \mathcal{F}_{d,\infty}\, x_\infty + \mathcal{E}_d\, x_{e\infty}$.

v) If $\overline{x} \neq \overline{x}'$, set $\overline{x} = \overline{x}'$, and $\mu(\mathcal{N}) = \mu_\infty(\mathcal{N})$, use them as new guesses, and return to ii).

vi) iterate ii) to v) until convergence of $\overline{x}$.

1.4.2 Calibration

The eleven structural parameters of the model listed in Table 1.1 are calibrated in two ways. The first group of five parameters is externally calibrated according to the literature and the data. The second group of six parameters is internally calibrated to firm level data and the import penetration ratio in the U.S. manufacturing sector from 1987 to 1997.[11] A sample of firms is drawn from the universe of innovative manufacturing firms in the 1987 through 1997 censuses.[12] The total mass of potential domestic startups ($\mathcal{E}_d$) is set equal to one.

1.4.2.1 Externally Calibrated Parameters

The time discount factor (β) is set equal to 0.9615, which corresponds to an annual interest rate of 4%. The curvatures of the three R&D cost functions ($\widehat{\psi}$, $\widetilde{\psi}$, $\widetilde{\psi}^e$) are taken from Akcigit and Kerr (2018) and their discussion of two lines of literature: one evaluating the empirical relationship between patents and R&D expenditure, and the other evaluating the impact of R&D tax credits on the R&D expenditure of firms. The average profit-to-sales ratio in the model is equal to $\int_f \frac{profit_f}{sales_f} df = \theta$, where profits include R&D expenditures. Thus the quality share in final goods production (θ) is set equal to the corresponding number from the data, which is 10.9% for the 1982-1997 period according to Akcigit and Kerr (2018).

1.4.2.2 Internally Calibrated Parameters

The remaining six parameters are estimated using an indirect inference approach: for each set of six parameter values, I compute six model-generated moments, compare them to the moments from the data, and find a set of parameter

[11] The import penetration ratio in the manufacturing sector is defined as the ratio between the manufacturing imports and the manufacturing value added net of exports plus imports. The manufacturing imports and exports are from World Development Indicators, and the manufacturing value added is from Bureau of Economic Analysis.

[12] Innovative firms are defined as firms with positive R&D expenditure or positive number of patents filing. R&D to sales ratio, firm entry rate, and average sales growth rate are from Akcigit and Kerr (2018), where sample period is from 1982 to 1997. The average number of products is from Bernard et al. (2010), and the high-growth firm growth rate is from Decker et al. (2016).

Table 1.2: Target Moments

Moment	Data	Model	Moment	Data	Model
R&D to sales ratio (%)	4.1	4.1	avg. sales growth rate (%)	1.0	1.0
avg. number of products	3.5	1.5	high-growth firm growth rate (%)	22.8	22.8
firm entry rate (%)	5.8	5.8	import penetration in manuf. (%)	37.4	37.4

values that minimizes the objective function

$$\min \sum_{i=1}^{6} \frac{\left|\text{model moment}_i - \text{data moment}_i\right|}{\frac{1}{2}\left|\text{model moment}_i\right| + \frac{1}{2}\left|\text{data moment}_i\right|},$$

where the six moments are listed in Table 1.2 and discussed in depth next.

The six moments are chosen in consideration of both their importance in answering the main question of this paper, and the relationships among the moments and the parameters coming from the choice of functional forms in the model. Although all the parameter values contribute substantially in determining the value for each model-generated moment, the tight relationship between certain sub-groups of parameters and moments can be noted.

Firms perform internal and external R&D to adjust the number of product lines they operate. Since R&D cost is one of the important factors in determining the level of R&D intensity, and hence the number of product lines the firm owns, I discipline the scale of internal R&D ($\widehat{\chi}$) and the scale of external R&D ($\widetilde{\chi}$) through the R&D to sales ratio and the average number of products firms own.

Potential startups learn and improve existing technologies to enter the market, and the success probability of entry is tightly related to the level of R&D expenditure (cost) they spend. Thus I discipline the scale of external R&D for startups ($\widetilde{\chi}^e$) using the firm entry rate.

Firms grow in terms of both sales and number of employees by improving the qualities of their existing products and/or adding new product lines to their product line portfolios. How fast/slow they can grow depends on how much improvement they can achieve in product quality. Thus I discipline the quality multiplier of internal innovation (λ) and the quality multiplier of external innovation (η) through the average sales growth rate and high-growth firms' (the 90th percentile firm of the firm employment growth distribution) employment growth rate—the key moment in this paper.

Finally, I discipline the initial value for the exogenous foreign creative destruction arrival rate $\overline{x}_o$ using the import penetration ratio in the manufacturing sector, as the exogenous foreign creative destruction arrival rate is tightly related to the share of domestic differentiated product markets occupied by the foreign exporters.

Table 1.2 reports the model generated moments. The model matches the target moments very closely, except for the average number of products. This manifests the drawbacks coming from the assumption that firms can make only one external innovation at a time. It becomes very hard for a firm to add one more product line as its number of product lines increases. Roughly speaking, the probability of adding one more product line for a firm with n_f product lines is equal to $\overline{x}_{takeover}x(1-\overline{x})^{n_f}$, without considering internal innovation. Bar graphs in figure 1.3 with solid lines show the distribution of the number of product lines (product line distribution) and the technology gap distribution computed using the parameter values reported in Table 1.1. As we can see, the product line distribution resembles a Pareto distribu-

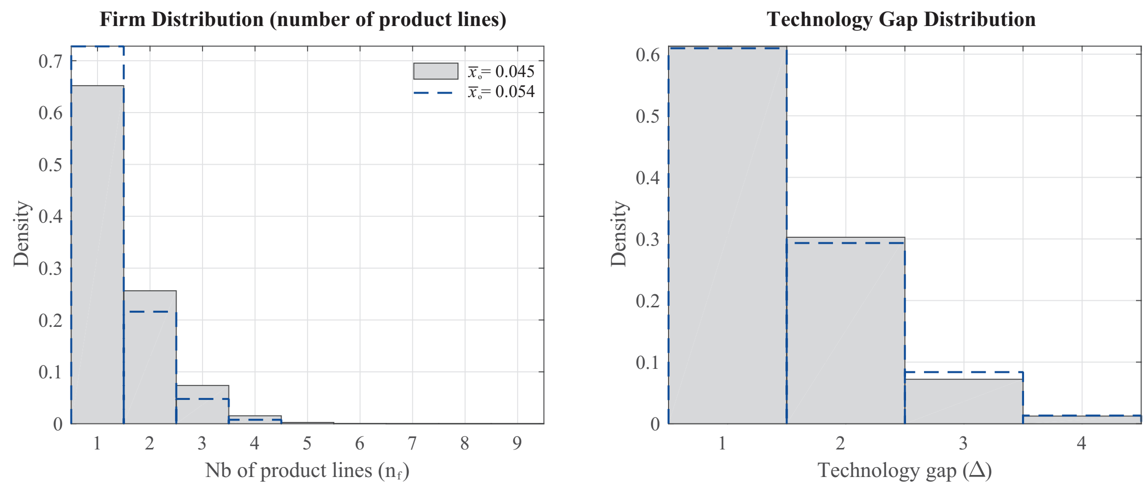

Figure 1.3: Firm Distribution and Technology Gap Distribution Changes

tion. Roughly 60% of the product lines have a technology gap equal to one under the calibrated parameter values. This might be another symptom of problems arising from the assumption of only one external innovation at a time, and it influences the level of the technological-barrier effect in the quantitative analysis.

1.4.3 Counterfactual Exercises

1.4.3.1 Increasing Competitive Pressure From Foreign Firms

In this section, I assess the impact of increasing competitive pressure from foreign firms on individual firms' behavior, particularly their overall innovation, composition of innovation, and the employment growth rate using the calibrated model. More specifically, I increase the value of $\overline{x}_o$ from 0.045 to 0.054 (20% increase). This is equivalent to an increase in the import penetration ratio in the U.S. manufacturing sector from 37.4% to 43.5% (6.1% increase).

To understand the effects of rising competitive pressure from foreign firms at

Table 1.3: Innovation Intensities Changes

description	variables	before	after	% change
foreign creative destruction arrival rate	$\overline{x}_o$	0.045	0.054	20.00%
creative destruction arrival rate	$\overline{x}$	0.120	0.123	2.98%
prob. of internal innovation ($\Delta^1 = 1$)	z^1	0.224	0.224	-0.03%
prob. of internal innovation ($\Delta^2 = \lambda$)	z^2	0.653	0.656	0.60%
prob. of internal innovation ($\Delta^3 = \eta$)	z^3	0.453	0.456	0.54%
prob. of internal innovation ($\Delta^4 = \frac{\eta}{\lambda}$)	z^4	0.438	0.440	0.44%
prob. of external innovation, incumbents	x	0.097	0.095	-2.20%
prob. of external innovation, potential startups	x_e	0.033	0.032	-3.33%
conditional takeover probability	$\overline{x}_{takeover}$	0.747	0.746	-0.23%
unconditional takeover probability	$x_{takeover}$	0.073	0.071	-2.43%

the firm level, Table 1.3 reports changes in variables related to innovation intensity. An exogenous increase in the foreign creative destruction arrival rate $\overline{x}_o$ increases the aggregate creative destruction arrival rate. As reported in Table 1.5, the expected profits from internal innovation and production ($\{A_\ell\}_{\ell=1}^4$) and external innovation (B) decrease. These have negative Schumpeterian effects on firms' incentives for internal and external innovation. However, the escape-competition effect dominates for product lines with positive technology gaps. Thus, incumbent firms attempt to protect their existing product lines by increasing their internal innovation intensity for product lines with technology gap higher than one, where the relative magnitudes of changes are in alignment with Corollary 2. Due to this increased internal innovation intensity and the heightened overall external innovation intensity—the higher value for the aggregate creative destruction arrival rate—the technology gap distribution changes, as reported in Table 1.4 and shown in Figure 1.3 graphically. Along with increased probabilities of internal innovation, this change in the technology gap distribution towards higher densities of Δ^3 and Δ^4 lowers the value of

Table 1.4: Technology Gap Distribution Change

description	variables	before	after	% change
Technology gap distribution (shares)	$\Delta^1 = 1$	0.613	0.612	-0.10%
	$\Delta^2 = \lambda$	0.303	0.301	-0.55%
	$\Delta^3 = \eta$	0.072	0.074	2.93%
	$\Delta^4 = \frac{\eta}{\lambda}$	0.012	0.013	1.34%

$\overline{x}_{takeover}$, the conditional takeover probability, which is what I call the technological-barrier effect. Both the Schumpeterian effect and the technological-barrier effect affect firms' incentive for external innovation negatively. Therefore, firms optimally lower their investment in external innovation. Recall that the probability of external innovation x is a function of $A_{takeover}$, where

$$A_{takeover} \equiv \frac{1}{2}(1 - z^3)A_1\mu(\Delta^3) + \left(1 - \frac{1}{2}z^4\right) A_2\lambda\mu(\Delta^4) + A_3\eta\mu(\Delta^1) + (1 - z^2)A_4\frac{\eta}{\lambda}\mu(\Delta^2) .$$

Thus we can decompose the changes in x into two parts: one resulting from the Schumpeterian effect and the other from the technological-barrier effect. Holding the expected future profits fixed at their initial levels, I find that 10.3% of the changes in x are due to the technological-barrier effect. Similarly, potential startups' external innovation intensity also drops, and this drives the decrease in the total mass of domestic startups.

The change in the technology gap distribution is affected by the assumption of only one external innovation at a time. Firms can have a product line with technology gap equal to either Δ^3 or Δ^4 only through external innovation. Since

Table 1.5: Firm Value Change

description	variables	before	after	% change
Firm Values	A_1	0.290	0.283	-2.18%
	A_2	0.305	0.299	-2.00%
	A_3	0.313	0.307	-1.95%
	A_4	0.295	0.289	-2.11%
	B	0.393	0.377	-4.03%

incumbent firms are allowed to add only one product line per period, a large share of product lines with technology gap equal to Δ^3 or Δ^4 belong to startups (both domestic and foreign). Thus the share of product lines with technology gap equal to Δ^3 or Δ^4 increases more than that of Δ^2 after an increase in the total mass of potential startups from the foreign country. I conjecture this is the reason why I see a drop in the share of product lines with Δ^2 despite a general increase in internal innovation intensity. This change in the technology gap distribution is one of the reasons for the mild decrease in the conditional takeover probability $\overline{x}_{takeover}$.

Table 1.6 reports changes in some of the model generated moments. Importantly, the R&D to sales ratio drops as a result of increasing competitive pressure from foreign firms. This is because external innovation falls by more than the increase in internal innovation. Consequently, the external R&D intensity, measured as the ratio of total domestic R&D expenses for external innovation to total domestic R&D expenses for all innovation, also drops. The total masses of both domestic firms and domestic startups decrease. However, the total mass of domestic firms decreases by more, so that the domestic firm entry rate increases. The average number of products for each firm decreases after an increase in competitive pressure from foreign firms. This is in alignment with wthe empirical findings ofhat Bernard et

Table 1.6: Domestic Firm Entry, Exit, and Other Moments

description	before	after	% change
R&D to sales ratio (%)	4.124	4.064	-1.46%
external R&D intensity (%)	50.774	49.910	-1.70%
total mass of domestic firms	0.429	0.393	-8.23%
total mass of domestic startups	0.025	0.024	-3.56%
domestic firm entry rate (%)	5.833	6.130	5.09%
avg. number of products	1.461	1.435	-1.78%
avg. sales growth rate (%)	1.048	1.058	0.93%

Table 1.7: Aggregate Growth Decomposition

description	before	after	% change
aggregate growth (1+g)	1.0105	1.0106	0.01%
growth from internal innovation	0.9179	0.9154	-0.27%
growth from domestic external innovation	0.0322	0.0288	-10.45%
growth from domestic startups	0.0258	0.0249	-3.56%
growth from foreign external innovation	0.0346	0.0414	19.72%
growth from domestic firms	0.9759	0.9692	-0.69%

al. (2011). Using the U.S. Linked/Longitudinal Firm Trade Transaction Database and the U.S. Census of Manufactures, they find that firms experiencing higher tariff reductions after the Canada-U.S. Free Trade Agreement reduce the number of products they produce relative to firms experiencing smaller tariff reductions. The average firm sales growth rate, which is equal to the aggregate growth rate g in the model economy, increases after an increase in competitive pressure from foreign firms. This increase, however, is completely driven by foreign exporters. Table 1.7 reports the decomposition of the change in the aggregate growth rate. After subtracting the contribution accounted for by foreign exporters, the aggregate growth accounted for by domestic firms falls by 0.69% after an increase in competitive pressure from foreign firms.

Lastly, Table 1.8 shows the 90th, 50th, and 10th percentiles of the employment-

Table 1.8: Firm Employment Growth Rate Changes

description	before	after
p90 emp. growth rate (%)	22.843	20.997
p50 emp. growth rate (%)	0.254	0.246
p10 emp. growth rate (%)	-12.151	-12.082

weighted distribution of firm employment growth rates before and after the increase in competitive pressure from foreign firms. The growth rate of high-growth firms, measured as the 90th percentile of the distribution, decreases from 22.8% to 21.0% after an increase in competitive pressure from foreign firms. The 50th percentile decreases after the increase in competitive pressure from foreign firms. The 10th percentile, however, increases, because firms are better at protecting their product markets with increased internal innovation.

1.4.3.2 Comparison I: Economy with High External Innovation Cost

To show how the effect of the same-sized shock to competitive pressure changes if we consider an economy with low creativity—a low external innovation intensity due to increased frictions—I run the same exercise of increasing creative destruction arrival rate by outside firms, $\overline{x}_o$, by 20%, in an economy in which $\widetilde{\chi}$, the parameter governing the cost of external R&D is 50 times higher than the baseline calibration of 1.184.

Columns 2 and 3 of Table 1.9 compare this low creativity economy with the economy calibrated to the U.S. (baseline calibration with $\widetilde{\chi} = 1.184$). As we can see, this economy is less dynamic compared to the U.S., with lower R&D, a lower number of startups, lower economic growth, and lower high-growth firm growth than

Table 1.9: Moment Comparison: U.S. vs. Economy with High External Innov. Cost

Moment	Baseline	w/ high ext. innov. cost	after shock	% change
R&D to sales ratio (%)	4.124	1.451	1.480	2.02
avg. number of products	1.461	1.022	1.019	-0.31
total mass of domestic firms	0.429	0.355	0.300	-15.43
total mass of domestic startups	0.025	0.020	0.019	-7.39
avg. sales growth rate (%)	1.011	0.842	0.867	2.96
p90 emp. growth rate (%)	22.843	9.111	9.089	-0.24

the baseline economy.

Columns 3 and 4 of Table 1.9 compare the moments of the low creativity economy before and after an increase in competitive pressure from foreign firms. Compared to the U.S. counterparts, all the moments except for the R&D to sales ratio move in the same direction, but the magnitudes are smaller. Importantly, the domestic R&D to sales ratio increases in this economy, whereas this ratio decreases in the baseline model. In this economy, firms put very little effort into external innovation. Thus, although external innovation decreases after an increase in foreign competitive pressure, the reduction is very small in absolute terms. Therefore it is more than offset by increased investment for internal innovation for defensive reasons. This result highlights the importance of examining changes in the composition of innovation along with changes in the overall amount of innovation.

Table 1.10 shows changes in innovation intensities. Compared to the numbers reported in Table 1.3, we see that innovation intensities are smaller in magnitude in the economy with low creativity. However, the direction of changes in response to increasing competitive pressure from foreign firms are identical in both economies.

Table 1.10: Innov. Intensities Changes in an Economy w/ High Ext. Innov. Cost

description	variables	before	after	% change
foreign creative destruction arrival rate	$\overline{x}_o$	0.045	0.054	20.00%
creative destruction arrival rate	$\overline{x}$	0.070	0.077	10.10%
prob. of internal innovation ($\Delta^1 = 1$)	z^1	0.225	0.224	-0.14%
prob. of internal innovation ($\Delta^2 = \lambda$)	z^2	0.581	0.594	2.24%
prob. of internal innovation ($\Delta^3 = \eta$)	z^3	0.411	0.418	1.76%
prob. of internal innovation ($\Delta^4 = \frac{\eta}{\lambda}$)	z^4	0.403	0.409	1.57%
prob. of external innovation, incumbents	x	0.003	0.003	-6.43%
prob. of external innovation, potential startups	x_e	0.024	0.023	-6.67%
conditional takeover probability	$\overline{x}_{takeover}$	0.831	0.825	-0.78%
unconditional takeover probability	$x_{takeover}$	0.003	0.002	-7.16%

1.4.3.3 Comparison II: Increased Competitive Pressure From Domestic Startups

In this exercise, I lower $\widetilde{\chi}^e$, the parameter governing the cost of external R&D for potential startups, by 11.34%. This increases the aggregate creative destruction arrival rate $\overline{x}$ from 0.120 to 0.123 (a 2.98% increase), which is identical to the increase in the previous exercise due to increasing the foreign creative destruction arrival rate by 20%.

Table 1.11 shows the results. Since the aggregate creative destruction arrival rate is the same, all the moments related to individual incumbent firms are virtually identical to the numbers reported in Tables 1.8, 1.6, and 1.3. However, the total mass of domestic firms, the total mass of domestic startups, and the probability of external innovation by potential startups increase in this case. This is because the increasing competitive pressure is induced by an increase in the mass of domestic startups, rather than by foreign firms. This exercise shows that changes in moments related to the number of domestic firms and startups are keys for identifying whether

Table 1.11: Changes in Moments: Economy with Low Entry Cost

description	before	after	% change
total mass of domestic firms	0.429	0.444	3.54%
total mass of domestic startups	0.025	0.027	8.83%
R&D to sales ratio (%)	4.124	4.065	-1.44%
avg. number of products	1.461	1.435	-1.78%
avg. sales growth rate (%)	1.048	1.058	0.92%
p90 emp. growth rate (%)	22.843	20.997	-8.08%
prob. of external innovation, potential startups	0.033	0.037	9.08%

an increase in competitive pressure is coming from the domestic entry margin or foreign firm entry.

1.5 Conclusion

In this chapter, I investigate the effect of competition on the level and composition of innovation by developing an endogenous growth model with heterogeneous innovation and imperfect technology spillovers. Firms improve their own product quality and production processes through internal innovation and use external innovation to get into new markets and drive incumbent firms out. External innovation, however, is subject to imperfect technology spillovers, in that it takes time to learn others' technology.

This chapter shows that having different types of innovation and imperfect technology spillovers are crucial in analyzing the effect of increasing competition on firm innovation. Increasing competition lowers firms' incentive to invest in external innovation while it encourages investment in internal innovation for product lines with a large technology gap accumulated through recent innovation.

This chapter also shows that the decomposition of innovation into two types is also potentially important in understanding the differential effect of competition on firm innovation across firms in different sectors or countries. The direction of incumbent firms' responses of internal and external innovation to competition is similar regardless of the cost of external innovation. However, overall innovation, which combines internal and external innovation, increases in an economy with high external innovation costs in response to increased competition, while it decreases in an economy with low external innovation costs, such as the U.S. This is because firms do very little external innovation in an economy with high external innovation costs even before an increase in competition, which implies that there is very little room for further downward adjustment. Thus, the decrease in external innovation is completely dominated by an increase in internal innovation.

To the best of my knowledge, this is the first attempt to develop an endogenous growth model incorporating an escape-competition effect where firms are allowed to grow through product scope expansion à la Klette and Kortum (2004) with firm entry and exit. Additionally, my model can explain why the change in overall innovation in response to increasing competition can differ across countries with different fundamentals.

Chapter 2: The Effect of Globalization on Firm Innovation and Firm Growth in the U.S.

2.1 Introduction

The past decades have seen declining business dynamism in the U.S. economy in various measures, such as startup rates, job creation and destruction rates, and activity among high-growth firms, a significant portion of which are young firms (Decker et al., 2014). In the manufacturing sector, for example, the startup rate fell from 8.3% in 1992 to 6.3% in 2007, whereas the employment growth rate of the top decile of firm employment growth declined from 22.5% in 1992 to 17.0% in 2007.[1] Startups and high-growth firms account for 70% of gross job creation in typical years (Decker et al., 2014).[2] Furthermore, high-growth firms also play disproportionately important roles in output growth and productivity growth (Haltiwanger et al., 2016). Thus, the decline in startup rates and the activity of high-growth young firms is a large concern.

[1]The startup rates are Author's calculation from the Business Dynamics Statistics (BDS). The top-decile firm is the 90th percentile firm of the employment-weighted distribution of firm employment growth rates. The two employment growth rates are based on Hodrick-Prescott trend, where data is from Decker et al. (2016). Economy-wide, the two numbers changed from 32.8% in 1992 to 26.3% in 2007.

[2]Here, high-growth firms are defined as firms with employment growth rate more than 25% per year.

Simultaneously, the U.S. economy experienced increasing international trade. Exports of goods and services, for example, rose from 8.0% of GDP in 1992 to 11.5% in 2007, and the import penetration ratio rose from 9.2% in 1992 to 15.5% in 2007.[3] While a significant body of research has examined the link between international trade and macroeconomic outcomes such as output growth and unemployment, less attention has been paid to the impact of international trade on high-growth firm activity.

In this chapter, I argue that the decline in high-growth firm activity and startup rates in the U.S. is a result of multi-product firms' optimal innovation decisions in response to increasing competitive pressure from foreign firms due to globalization. I do this by building on the theoretical predictions developed in Chapter 1. The first prediction of my model is that firms who have innovated intensively in recent periods increase internal innovation more when they are faced with higher competition, compared to their low innovation counterparts. The second prediction is that firms do less external innovation if other firms have innovated more intensively in recent periods. The third prediction is that firms do more internal innovation if they expect to get higher profits from their current product markets in the near future.

Thus, increasing competitive pressure from foreign firms induces innovation-intensive (and thus high-growth) firms to focus their innovative effort on improving

[3] Import penetration ratio is defined as the imports of goods and services divided by the total expenditure on goods and services, measured as the GDP minus the exports of goods and services plus the imports of goods and services. Both exports of goods and services per GDP, and import penetration ratio is the author's calculation from FRED economic data in real terms, then Hodrick-Prescott trends are reported. Exports of goods and services per total expenditure rose from 7.9% in 1992 to 11.0% in 2007

their existing products to defend themselves from competitors rather than expanding their market scope.[4] And because innovative incumbents devote more effort to protecting their markets with heightened technological barriers, all types of firms, including highly innovative firms and potential startups, find it difficult to enter into others' markets through external innovation. Thus, the startup rate falls, and all firms reduce their investment in external innovation. Furthermore, because external innovation makes firms grow faster than internal innovation, this shift of innovation activity causes innovation-intensive firms to grow more slowly.[5]

I first test the three model predictions empirically. To do so, I construct a unique data by combining firm-level datasets from the U.S. Census Bureau with patent data from the United States Patent and Trademark Office (USPTO) from 1976 to 2016. This comprehensive dataset has information for the population of U.S. patenting firms, such as employment, international transactions, and the 6-digit NAICS industries in which each firm operates. I use China's WTO accession in 2001 as an exogenous change in competitive pressure from foreign firms, use the patent self-citation ratio as a measure of the likelihood each patent is used for internal innovation, and provide regression results consistent with the first model prediction.

[4]Graham et al. (2018) show that compared to non-patenting firms, patenting firms on average grow three percentage points faster, and young patenting firms grow even faster. Also, they shed fewer jobs compared to non-patenting counterparts. Acemoglu et al. (2018) show that among innovative firms, young and small firms have higher innovation intensity than mature firms as measured by the ratio of R&D spending to sales.

[5]Akcigit and Kerr (2018) empirically show that external innovation generates more forward citations and is associated with higher employment growth compared to internal innovation. They also show, through the lens of their structural model, that external innovation brings higher product quality improvement, and contributes more to economic growth. They use patenting firms in the Longitudinal Business Database (LBD) from 1982 to 1997 to arrive at these conclusions both theoretically and empirically. Bernard et al. (2010) suggest that product switching contributes to a reallocation of resources within firms toward their most efficient use. Thus, experimentation through external innovation is very important for firm growth.

I also show that the positive association between patenting and employment growth for innovation-intensive firms falls by one-third after an increase in competitive pressure from foreign firms, as more patents are used for internal innovation. Next, I find regression results consistent with the second model prediction by using changes in foreign patent growth (in other words, the recent innovation activity of other firms) as a measure of an exogenous variation in technological barriers. Finally, by using log differences in advanced countries' exports to China (excluding the U.S.) as a proxy for exogenous changes in Chinese demand for U.S. products (an export shock), I find regression results supporting the third model prediction.

To quantify the effect of the rise of China after 2001 on U.S. firms' innovation decisions and growth, I extend the baseline model developed in Chapter 1 into a two-country framework. To my best knowledge, this is the first theoretical model of defensive innovation with two countries that allows individual firms to grow both by improving in their existing markets and by taking over other firms' markets, through two different types of innovation.

This two-country model shares the same features as the baseline model developed in Chapter 1, especially the importance of the technology gap in each product market for the optimal internal innovation-decision rule. In the two-country extension, quality differences between the same products sold in different countries also matter for firms' internal innovation decisions. When international trade is costly, similar quality products are not traded but instead are produced and consumed domestically. This creates a global technology gap—the gap between a U.S. firm's technology and a foreign firm's technology in each product market. Domestic incumbents with lagging technologies invest more in internal innovation if potential foreign competitors in their markets have technology high enough that they could overcome the trade cost and start exporting their products to the domestic market by performing additional internal innovation. On the other hand, global technological leaders also are motivated to perform internal innovation, which could enable them to overcome the trade cost and become exporters. I find supporting empirical evidence for an enhanced internal innovation incentive for global technological leaders.

Using my model, I perform a counterfactual exercise of reducing tariff rates bilaterally by 4.16 percentage points, which is equal to the estimated reduction in expected tariff rates faced by Chinese firms after 2001. I find that model firms, on average, shift their innovation activities toward more internal innovation after they are exposed to higher international trade. This causes high-growth, innovation-intensive firms to grow more slowly. Also, startup rates fall, as the increased technological advantage accumulated by incumbent firms through internal innovation makes it harder for startups to enter the economy through external innovation. I provide industry-level regression results consistent with these predictions.

This chapter contributes to an emerging literature on the decline in business dynamism in the U.S. Decker et al. (2014) and Decker et al. (2016) show that business dynamism in the U.S. has been declining in various measures, and these declines accelerated after 2000. Previous studies, such as Karahan et al. (2019) and Hopenhayn et al. (2018), examine the effect of demographic changes on business dynamism, while Akcigit and Ates (2019a,b) focus on the effect of declines in knowledge diffusion. To my knowledge, I am the first to propose increasing foreign competition and U.S. firms' endogenous changes in the allocation of innovative activity as a channel that explains the decline in high-growth firm activity and startup rates through the lens of a structural model, with supporting empirical evidence.

This chapter also contributes to the literature that investigates the impact of international trade on firm innovation. Existing studies mainly combine internal and external innovation and estimate the effect of trade on firms' overall innovation. The results are mixed. Bloom et al. (2016) show that surviving firms in developed European countries fight Chinese import competition by increasing their overall innovation. Autor et al. (2019), on the other hand, show that publicly traded U.S. firms lower their overall innovation when firms are allowed to exit in the regression sample. Aghion et al. (2017), meanwhile, focus on French exporters' innovation decisions when competition in the export market increases. They show that more productive exporters do more innovation in response to increasing competition in the export market, while less productive firms do less innovation.

Atkeson and Burstein (2010) theoretically investigate the effect of trade liberalization on firm innovation when incumbents do internal innovation while startups are born with new products. In their model, the impact of trade on firm innovation operates through wage changes, and is not heterogeneous across firms. This paper is closely related to Akcigit et al. (2018), who build a two-country endogenous growth model with internal step-by-step innovation. Similar to the model developed in this paper, their model distinguishes the competition effect and the market size effect on firm innovation. The difference is that, in my framework, firms are also allowed to grow through product scope expansion by taking over others' markets. With two types of innovation, firm growth can slow down even if firms increase internal innovation to escape competition.

This chapter contributes to this large strand of literature in three ways. First, I study the differential effect of international trade on two types of innovation, internal and external, that make asymmetric contributions to firm employment growth and economic growth. This provides a potential explanation for the recent decline in business dynamism in the U.S. economy. Second, I study why firms with different initial characteristics can react differentially to the same trade shock, while explaining the underlying mechanisms through a rich theoretical framework that allows us to decompose the firm's incentives for innovation in detail. And lastly, I empirically study the effect of international trade on different types of firm innovation using a population of patenting firms by matching the USPTO patent database to internal Census Bureau datasets. To my own best knowledge, this project is the first to accomplish these three objectives.

The rest of the chapter proceeds as follows. Section 2.2 presents empirical results for the effect of international competition on the composition of firm innovation. Section 2.3 develops a two-country baseline general equilibrium model. Section 2.4 presents results from quantitative analysis. Section 2.5 concludes.

2.2 Empirics

In this section, I empirically examine the relationships among firm innovation, firm growth, and international trade for firms with different characteristics. I identify the causal effect of international trade on the composition of firm innovation (internal vs. external) and test the predictions of the simplee three-period model developed in Chapter 1. The analysis focuses on the early 1990s to mid-2000s, especially the years after 2000, as this period witnessed changes in the trends for many important economic variables, especially the employment growth rate of high-growth firms and the number of patent applications filed by U.S. firms. The rise of China in the U.S. markets after China's WTO accession in 2001, and increased Chinese demand for U.S. products, will be treated as quasi-experiments.

2.2.1 Data and Measurement

To construct a comprehensive firm-level dataset containing measures of innovation and international trade, I combine the following seven datasets: the USPTO PatentsView database, the Longitudinal Business Database (LBD), the Longitudinal Firm Trade Transactions Database (LFTTD), the Census of Manufactures (CMF), the UN Comtrade Database, the NBER-CES database, and the tariff data compiled by Feenstra et al. (2002).

The LBD tracks the universe of establishments and firms in the U.S. non-farm private sector with at least one paid employee annually from 1976 onward.[6] An

[6] Details for the LBD and its construction can be found in Jarmin and Miranda (2002).

establishment corresponds to the physical location where business activity occurs. Establishments that are operated by the same entity, identified through the Economic Census and the Company Organization Survey, are grouped under a common firm identifier. I aggregate establishment-level information into firm-level observations using these firm identifiers. Firm size is measured by either total employment or total payroll. Firm age is based on the age of the oldest establishment of the firm when the firm is first observed in the data. The firm's main industry of operation is based on the six-digit North American Industry Classification System (NAICS) code associated with the highest level of employment. Time-consistent NAICS codes for LBD establishments are constructed by Fort and Klimek (2018), and the 2012 NAICS codes are used throughout the entire analysis.

The LFTTD tracks all U.S. international trade transactions starting from 1992 onward at the firm level.[7] The LFTTD provides the U.S. dollar value of shipments, and the origin and destination country for each transaction, as well as a related-party flag, which indicates whether the U.S. importer and the foreign exporter are related by ownership of at least 6 percent.

The USPTO PatentsView database tracks all patents ultimately granted by the USPTO from 1976 onward.[8] This database contains detailed information for granted patents including application and grant dates, technology class, other patents cited, and the name and address of patent assignees. It also provides the list of inventors responsible for each patent with their locations. In the following analyses, I

[7] Bernard et al. (2009) describe the LFTTD in greater detail.
[8] See http://www.patentsview.org/download/.

use the citation-adjusted number of utility patent applications as the main measure of firm innovation.[9] By using detailed information for each patent, I distinguish domestic innovation from foreign innovation, and measure the extent to which each patent represents internal innovation. The year in which a patent application is filed is used as a proxy for the innovation year. The citation-adjusted average of the internal innovation measure for the flow of patent applications in each firm-year is used as a proxy for the overall extent of internal innovation at each firm in each year. I discuss the measure of internal innovation in detail shortly.

I match the USPTO patent database to the LBD to assign detailed firm-level information and firm-industry-level changes in trade flows to each patent. In the following analyses, I compare firms' patenting behavior across different years. Thus, match quality is important – failing to match a firm in the USPTO patent database in a particular year to its LBD counterpart will result in mismeasuring innovation. This problem arises because the USPTO doesn't track a consistent unique firm ID. The USPTO assigns patent applications to self-reported firm names. Thus, it is vulnerable to misspelling of firm names. To overcome this match quality issue, I adopt the Autor et al. (2019) methodology, which utilizes the machine-learning capacities of the internet search engine. I use all patents granted up to December 26, 2017 during the matching procedure, and use patent applications up to 2007 in the subsequent analyses. Thus, the following analyses are virtually free from the right censoring issue (mismeasuring firms' innovation activities due to patents applied for

[9] See Cohen (2010) for a comprehensive review of the literature on the determination of firms' and industries' innovative activity and performance and how patent-related measures are used.

but not yet granted). Table B.4 in the Appendix reports summary statistics for patenting firms in 1992.

The quinquennial CMF provides detailed information for activities by establishments in the manufacturing sector. It also provides detailed product codes and breaks down the value of shipments for all products each establishment sells. I use five-digit SIC codes for observations up to 1997, and seven-digit NAICS codes for observations from 2002 onward, to measure firms' product choices.

The UN Comtrade Database provides information for world trade flows at the six-digit HS product-level from 1991 to 2016.[10] The six-digit HS codes are concorded to six-digit 2012 NAICS industries using the Pierce and Schott (2009, 2012) crosswalks. I construct an industry-level export shock measure using the UN Comtrade Database. Also, I obtain U.S. tariff schedules from Feenstra et al. (2002) to measure industry-level Trade Policy Uncertainty (TPU), which is used as a measure of shocks to foreign competitive pressure. The construction of these two trade shocks is discussed in detail in the following section.

The NBER CES Manufacturing Industry Database, assembled by Becker et al. (2013), is used to obtain the industry-level deflator for the value of shipments for manufacturing industries from 1976 to 2011.[11] All nominal values are converted to 1997 U.S. dollars using this industry-level deflator for the value of shipments for manufacturing industries, and the BEA's Consumer Price Index for other industries. In the following analyses, I use subsets of a sample of USPTO patents matched to

[10] https://comtrade.un.org/db/default.aspx.
[11] http://www.nber.org/nberces/.

U.S. firms in the LBD and industry-level trade data from 1982 to 2007 for each regression specification.

2.2.1.1 Measure of the likelihood each patent is used for internal innovation

In this study, I use the self-citation ratio as a measure of whether a patent primarily reeflects internal innovation. Each granted patent is required to cite all prior patents on which it builds. When a cited patent belongs to the owner of the citing patent, these citations are called self-citations. Akcigit and Kerr (2018) use the self-citation ratio—defined as the ratio of self-citations to total citations—as a measure of the likelihood each patent is used for internal innovation. The more an idea is based on the firm's internal knowledge stock (self-citation), the more likely the innovation is used for improving the firm's existing products (internal innovation). A higher self-citation ratio means that a patent is more likely to reflect internal innovation.[12]

2.2.1.2 Measures of Trade Shocks

As shown by Handley and Limão (2017), over one-third of the growth of imports from China to the U.S. in the first half of the 2000s can be explained by the U.S. granting permanent normal trade relations (PNTR) to China upon China's 2001 accession to the WTO. Nonmarket economies such as China are subject to rel-

[12]Thus, 100% self-citation means the patent is used for internal innovation with a 100% probability, and 0% self-citation means the patent is used for external innovation with a 100% probability.

atively high tariff rates, originally set under the Smoot-Hawley Tariff Act of 1930, when they export to the U.S. These rates are known as non-Normal Trade Relations (non-NTR) or column 2 tariffs. On the other hand, the U.S. offers WTO member countries NTR or column 1 tariffs, which are substantially lower than non-NTR tariffs. The Trade Act of 1974 allows the President of the United States to grant temporary NTR status to nonmarket countries on an annually renewable basis after approval by Congress. Starting from 1980, U.S. Presidents granted such waivers to China.

While China never lost these waivers and the tariff rates applied to Chinese products were kept low, the process of annual approval by Congress created uncertainty about whether the low tariffs would revert to non-NTR rates. After the Tiananmen Square protests in 1989, Congress voted on a bill to revoke China's temporary NTR status every year from 1990 to 2001. Following the bilateral agreement on China's entry into the WTO between the U.S. and China in 1999, Congress passed a bill granting China PNTR status in October 2000. Upon China's accession to the WTO in December 2001, PNTR became effective and was implemented on January 1, 2002. PNTR removed the uncertainty about U.S. trade policy toward China by permanently setting tariff rates on Chinese products at NTR levels. This lowered the expected U.S. import tariffs on Chinese products, and eliminated any option value of waiting for firms to incur large fixed costs associated with exporting products from China to the U.S. Thus, PNTR reduced trade policy uncertainty (TPU), the more so for industries with a large gap between tariff rates under NTR and non-NTR regimes.

I use the industry-level gap between NTR tariff rates reserved for WTO members and non-NTR tariff rates for non-market economies in the year 1999 as a proxy for the industry-level competitive pressure shock from China occurring in 2001.[13] Thus, for industry j,

$$NTRGap_j = Non\,NTR\,Rate_j \; - \; NTR\,Rate_j \; .$$

Also, following Aghion et al. (2017), I use log differences in advanced countries' exports to China (excluding the U.S.) as a proxy for exogenous changes in Chinese demand for U.S. products (an export shock).[14] Thus,

$$\Delta ExportShock_{j\tau} = \log(EX_{j\tau 1}) - \log(EX_{j\tau 0}) \; ,$$

where EX_{jt} represents total exports by eight advanced countries to China in industry j in year t, $\tau \in \{1992 - 1999\,,\; 2000 - 2007\}$ are the two periods of interest, $\tau 0$ is the start-year for each period, and $\tau 1$ is the end-year for each period. If a firm operates in multiple 6-digit NAICS industries, I use the employment-weighed average $NTRGap_j$ and $\Delta ExportShock_{j\tau}$. I use unweighted average trade shocks and shocks to firms' main industry as robustness checks. Table B.1 and Table B.2 in the Appendix report summary statistics for each trade shock measure.

[13] We can consider the NTR gap as a first-order Taylor approximation of model-based TPU measures, such as Handley and Limão (2017), that is positively related to non-NTR rate and negatively related to NTR rate.

[14] These advanced countries are Australia, Denmark, Finland, Germany, Japan, New Zealand, Spain, and Switzerland. These are the advanced countries for which we can obtain disaggregated bilateral HS trade data back to 1991, as explained in Autor et al. (2019)

2.2.2 Empirical Strategies and Main Results

The theory developed in Chapter 1 provides three empirically testable predictions: i) the escape-competition effect, ii) the technological-barrier effect, and iii) the expected profit effect. I now test these three model predictions.

2.2.2.1 The Escape-Competition Effect

The first prediction of my model is that firms who have innovated intensively in recent periods increase internal innovation more when they are faced with higher competition, compared to their low innovation counterparts. This is because innovation-intensive firms can escape competition more easily through additional internal innovation, by leveraging their higher-than-average production technologies (technological advantages, or technological barriers) that they built in their own markets through recent intensive innovation.

Following Handley and Limão (2017) and Pierce and Schott (2016), I use a Difference-in-Difference (DD) specification to identify the effect of the China competitive pressure shock on U.S. firm innovation for two periods, $p \in \{1992-1999\ ,\ 2000-2007\}$, for firm i in industry j:

$$\begin{aligned}\Delta y_{ijp} \ &= \beta_1 Post_p \times NTRGap_{ijp0} \times InnovIntens_{ijp0} \\ &+\ \beta_2 Post_p \times NTRGap_{ijp0}\ +\ \beta_3 Post_p \times InnovIntens_{ijp0} \\ &+ \beta_4 NTRGap_{ijp0} \times InnovIntens_{ijp0} \\ &+\ \beta_5 NTRGap_{ijp0}\ +\ \beta_6 InnovIntens_{ijp0}\end{aligned} \tag{2.1}$$

$$+ \mathbf{X}_{ijp0}\,\gamma_1 \;+\; \mathbf{X}_{jp0}\,\gamma_2 \;+\; \delta_j \;+\; \delta_p \;+\; \alpha \;+\; \varepsilon_{ijp}\,.$$

In these specifications, firms in low TPU industries are the control group, whereas firms in high TPU industries are the treatment group. I use the 2000 cohort of firms to measure firm innovation before the policy change, which occurred in December 2001. In this way, the composition of firms in terms of their innovation is minimally affected by the policy change.

$Post_p$ is a dummy variable equal to one for the period 2000-2007 and zero otherwise. It captures changes in firm innovation after China's WTO accession. $\mathbf{X}_{ijp0}$ is a vector of firm controls, and $\mathbf{X}_{jp0}$ is a vector of industry controls, both measured at the start-year for each period.[15] δ_j is an industry fixed effect (six-digit NAICS), and δ_p is a period fixed effect. All models are unweighted, and standard errors are clustered on the 6-digit NAICS industries.

Δy_{ijp} is the DHS (Davis et al., 1996) growth rate of either i) the total citation-adjusted number of patents, or ii) the citation-weighted average self-citation ratio between the start-year and end-year for each period $p \in \{1992-1999\,,\ 2000-2007\}$. An increase in the self-citation ratio means that the firm's innovations became more internal. To maximize the sample size, I include firms that applied for at least one patent in the start-year and at least one patent in or before the end-year for each period, and compute the DHS growth rates for the longest span of years available. I

[15] Firm controls include: firm employment, firm age, past 5-year growth of U.S. patents in the CPC technology classes in which the firm operates, and dummy variables for publicly traded firms, exporters, importers, and offshoring firms. Industry control variables include NTR rates measured at the start of each period.

Table 2.1: Escape-competition effect

	ΔPatents (1)	ΔPatents (2)	ΔSelf-cite (3)	ΔSelf-cite (4)
NTR gap × Post × Innov.-inten.	0.077	-0.017	0.732**	0.784***
	(0.231)	(0.233)	(0.299)	(0.268)
Observations	6,500	6,500	6,500	6,500
Fixed effects	j, p	j, p	j, p	j, p
Controls	no	full	no	full

Notes: Full controls include past 5-year U.S. patent growth in firms' own technology fields, log employment, firm age, NTR rate, dummy for publicly traded firms, dummy for firms with total imports > 0, dummy for firms with total exports > 0, and dummy for firms with imports from relative parties > 0. Estimates for industry (j) and the period (p) fixed effects as well as the constant are suppressed. Robust standard errors adjusted for clustering at the level of the firms' major industries are displayed below each coefficient. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * p < 0.1, ** p < 0.05, *** p < 0.01.

also require firms to have at least one patent before the start-year of each period, or to have age > 0, to avoid the effect coming from firm entry. The sample includes all LBD firms matched to the USPTO patent database that meet these three criteria, except for firms in FIRE industries.

$InnovIntens_{ijp0}$ is a continuous variable equal to the past five-year average of the ratio of the number of firm i's patent applications to total employment, measured in the start year for each period $p0$. I control for industry-fixed effects for this measure by dividing it by its time-average at the 2-digit NAICS level. Thus, I am examining the impact of heterogeneity of innovation intensity within industries rather than differences across industries. The escape-competition hypothesis predicts β_1 to be positive when changes in the self-citation ratio are used as Δy_{ijp}.

Table 2.1 shows the estimates of β_1.[16] As indicated in column (4) of Table 2.1, the estimate for β_1 is positive and statistically significant when the growth rate of the

[16]To conserve space, Table 2.1 reports coefficients estimates for triple interaction terms only. Results including coefficients for all the interaction terms are reported in Table B.8 in the Appendix.

self-citation ratio is the dependent variable, consistent with the model predictions. This estimated value for β_1 implies 4.1 percentage points increase in the growth rate of the average self-citation ratio for a firm with average innovation intensity (0.18) in an industry with an average NTR gap (0.291). The average value of the seven-year growth rate of the average self-citation ratio between 2000 and 2007 is 28.2%. Thus, this is about a 14.6% increase.

The estimated effect is economically important as well. Table B.11 in Appendix B.1.4 shows that for an average firm, creating 4 more patents is associated with a 3.4 percentage points increase in employment growth, but the association becomes smaller in magnitude if the average self-citation ratio of the new patents is high. The estimates in Table 2.1, combined with Table B.11, suggest that the association between patenting and employment growth is decreased by 1.13 percentage points for firms with average innovation intensity following the competitive pressure shock from China.

Lastly, Table B.8 in the Appendix shows that Chinese competitive pressure shock has no statistically significant effect on firms' overall innovation. My model predicts that some firms increase their internal innovation while others decrease theirs, and overall, firms lower their external innovation. When these heterogeneous responses are combined, we should see a non-significant effect on average. Thus, the regression results are consistent with the model prediction. And because firms do not change their overall innovation, the increasing self-citation ratio implies that innovative firms (firms with above-average innovation intensity) increase their internal innovation while decreasing their external innovation.

2.2.2.1.1 Discussion: PNTR as a Measure of Competitive Pressure

As discussed extensively in Pierce and Schott (2016) and Facchini et al. (2019), the main channel by which the removal of trade policy uncertainty affects trade between the U.S. and China is by persuading Chinese firms to export their products to the U.S. The two papers verify this channel by estimating the effect of the removal of TPU on changes in Chinese exports to the U.S. using the LFTTD at the product level, and Chinese Custom Data at the firm level. Table B.9 in the Appendix shows OLS estimates of the effect of PNTR on changes in U.S. imports from China from 2000 to 2007 at the 8-digit HS level and the 6-digit NAICS level separately. As indicated in the table, the NTR gap is positively associated with changes in U.S. imports from China regardless of the level of aggregation. However, statistical significance falls from the 1% to the 10% level as we move from the 8-digit HS level to the 6-digit NAICS level, where the latter is the level of aggregation used in this paper.

As is clear from the simple three-period model introduced in Section 1.3, one critical factor firms consider when they decide how much to invest in innovation is competitive pressure—the probability of encountering competitors in a firm's own market in the near future. In the real world, pressure can come from both realized competition (an increase in the number of competitors) and from anticipated competition (an increase in the number of potential entrants). Table B.10 shows OLS results from regressing the two dependent variables of interest on interaction involving the realized changes in U.S. imports from China, to estimate the effect of realized competition on the composition of firm innovation. Here, I simply replace the NTR gap terms in equation 2.1 with the realized changes in U.S. imports from China and use the same two seven-year periods used in the previous analysis, 1992-1999 and 2000-2007. As the table indicates, changes in U.S. imports from China from 1992 to 2007 do not have any statistically significant effect on U.S. firms' composition of innovation after I control for firm characteristics.

This analysis, however, has two concerns: i) changes in U.S. imports from China (a measure for realized competition) are endogenous due to various factors, and importantly, ii) competitive pressure from anticipated future competition is (potentially more) important for firms' innovation decisions, and successful escape competition by U.S. firms can make realized competition low even if competitive pressure is substantial. The first concern can be addressed by using the imposition of PNTR as an instrument for changes in imports. However, as Table B.9 shows, the NTR gap has low statistical power for predicting changes in U.S. imports from China at the 6-digit NAICS level. This indicates that the NTR gap is a weak instrument for realized competition.

My model suggests that the second concern is important, and that measures of realized competition inherently cannot capture the amount of competition escaped. The removal of trade policy uncertainty, however, can be an excellent proxy for increased competitive pressure, as it is associated with an increase in Chinese firms' opportunity to enter the U.S. market. For example, Handley and Limão (2017), through the lens of their structural model, show that a reduction in TPU provides greater incentive for incumbents to incur irreversible investments to enter foreign markets. Erten and Leight (2019) further show that the imposition of PNTR induces Chinese manufacturing firms to increase their investment and their value-added per worker. These findings suggest a tight relationship between the imposition of PNTR and an increase in potential future competition. Thus, finding direct evidence for this relationship, such as a link between PNTR and the number of Chinese startups or the number of Chinese firms with the ability to export their products to the U.S., is a priority for future research.

2.2.2.1.2 Validity of the Identification Strategy and Robustness Tests

Previous studies using PNTR with China as a competitive pressure shock, such as Pierce and Schott (2016) and Handley and Limão (2017), provide rich evidence for the exogeneity of PNTR for U.S. firms' decisions in the 1990s and 2000s. Thus, I focus on testing the parallel pre-trends assumption, the key identifying assumption for the DD model. To test the assumption for the dependent variables of interest, I estimate (2.1) for two seven-year periods before the policy change, 1984-1991 and 1992-1999. Table B.12 in the Appendix shows the results, which support the validity of the parallel pre-trends assumption.

To further confirm the validity of my results, I perform several robustness checks, with results reported in the Appendix. I find that my results are robust to a variety of different specifications. First, I include upstream and downstream competitive pressure shocks as covariates in model (2.1). By using the 1992 BEA input-output table, I construct upstream and downstream competitive pressure shocks as weighted averages of industry-level trade shocks. The upstream effect of trade is the effect of trade shocks propagating upstream from an industry's buyers, and the downstream effect of trade is the effect of trade shocks propagating downstream from its suppliers.[17] Table B.13 in the Appendix shows that including controls for I-O linkages does not change the main results.

The second test uses different weights for constructing firm-level NTR gaps. Because patenting firms are typically multi-industry firms, in my baseline regressions I use employment in the start year of each period as weights and construct a weighted average of industry-level NTR gaps for all industries in which each firm operates as the firm-level NTR gap. As a robustness check, I also use an unweighted average of this measure, and industry-level NTR gaps for firms' main industry (the industry with the most employment) as alternative measures for TPU in model (2.1). Table B.15 in the Appendix shows that using these alternative measures does not change the main results.

The third test addresses possible selection bias resulting from including only firms with a positive number of patents granted in the start year and in any of the last four years of each period in the regression analysis. This selection is inevitable as I need to compute the self-citation ratio for two years for each period. I correct for this bias by re-weighting the regression sample using the inverse of the propensity

[17]Following Pierce and Schott (2016), for each 6-digit NAICS industry, I set the I-O weights to zero for both up and downstream industries belonging to the same 3-digit NAICS broad industries while computing the indirect effects to take into account the findings from Bernard et al. (2010) that U.S. manufacturing establishments often produce clusters of products within the same 3-digit NAICS sector.

scores from a logit model with an indicator for being in the analysis sample as the dependent variable as weights. Table B.16 in the Appendix shows that this reweighting does not change the results. The fourth test adds the cumulative number of patents as a firm-level control variable in the model (2.1). The self-citation ratio can mechanically increase because the firm's patent stock increases as the firm becomes older. Adding the cumulative number of patents as a firm-level covariate addresses this issue, and Table B.17 in the Appendix shows that this does not change the results.

The fifth test clusters standard errors on firms. The second test indicates that most variation in the firm-level NTR gap occurs at the industry-level. Thus, I cluster standard errors at the six-digit NAICS level in the main analysis. As a robustness check, I cluster standard errors on firms, and Table B.18 in the Appendix shows this does not change our inference on the main results. Finally, I test the robustness of my results by using the number of products added—an alternative measure for external innovation (the inverse of internal innovation)—as the dependent variable. Table B.19 in the Appendix shows results that support the model prediction, that higher competitive pressure reduces number of new products added for innovative firms.

2.2.2.2 The Technological-Barrier Effect

Another prediction from my model is that firms do less external innovation if other firms have performed more innovation in the past period. Intensive innovation by other firms raises the technology barrier in other markets on average, which implies that business take over through external innovation becomes more difficult. Thus, firms optimally reduce their R&D spending on external innovation. To test this theoretical prediction, I use the recent increase in the number of foreign patent applications as a proxy for increasing innovation intensity in other markets. Since I don't have product-market information for foreign firms, I use patent technology class (CPC) as a proxy for product in this exercise. Foreign patents are defined as patents filed by foreign firms whose first listed inventor is a foreigner. I use the pre-shock years from the period 1989 to 2000 and construct non-overlapping five-year first differences (DHS growth for 1989-1994 and 1995-2000) to estimate the following fixed-effects model:

$$\Delta Y_{ijt+5} = \beta_1 \overline{\Delta S}^{Own}_{ijt-5} + \beta_2 \overline{\Delta S}^{Outside}_{ijt-5} + \mathbf{X}_{ijt}\,\gamma_1 + \delta_{jt+5} + \varepsilon_{ijt+5}$$

ΔY_{ijt+5} is either the 5-year DHS growth rate of the citation-adjusted number of patents or the average self-citation ratio between t and $t+5$, and $\overline{\Delta S}^{tech}_{ijt-5}$ for $tech \in \{Own,\ Outside\}$ is the lagged average 5-year DHS growth rate of foreign patents inside firm i's own technology space (Own) and outside firm i's technology space ($Outside$).

To be more specific, for each technology class c in CPC, denote the total number of foreign patents filed in year t as $S_{c,t}$. Then the DHS growth rate of foreign patents belonging to c between year $t-5$ and t can be written as

$$\Delta S_{c,t-5} \equiv \frac{S_{c,t} - S_{c,t-5}}{0.5 \times (S_{c,t} + S_{c,t-5})} \, .$$

Denote Q_t as the set of all the patent technology classes available until year t, and Q_{ijt} as the portfolio of patent technology classes firm i accumulated through year t. This defines the technology space in which firm i operates. Furthermore, denote $\omega_{c,i,j,t}$ as the share of patent technology class c in firm i's technology portfolio through year t. Then the lagged growth in innovation intensity in firm i's own technology space, $\overline{\Delta S}^{Own}_{ijt-5}$, is defined as

$$\overline{\Delta S}^{Own}_{ijt-5} \equiv \sum_{c \in Q_{ijt}} \omega_{i,j,c,t} \Delta S_{c,t-5} \, ,$$

while the counterpart firm i's outside of own space, $\overline{\Delta S}^{Outside}_{ijt-5}$, is defined as

$$\overline{\Delta S}^{Outside}_{ijt-5} \equiv \frac{1}{\|Q^c_{ijt}\|} \sum_{c \in Q^c_{ijt}} \Delta S_{c,t-5} \, ,$$

where $Q^c_{ijt} \equiv Q_t \backslash Q_{ijt}$ is the complement of the set Q_{ijt}, and $\|Q^c_{ijt}\|$ is the number of technology classes in Q^c_{ijt}. Table B.3 in the Appendix reports summary statistics for the technology shock measures. The regression is unweighted and standard errors are clustered by firm. I include industry-period fixed effects to control for industry-level shocks. The theory predicts β_2 to be positive when the change in the self-citation ratio is the dependent variable, and insignificant or negative for changes in the total

Table 2.2: Technological-barrier effect

	ΔPatents (1)	ΔSelf-cite (2)
Past 5 year Δforeign patent, outside of firm's own tech. fields	-5.984**	9.076***
	(2.756)	(2.711)
Observation	7,600	7,600
Fixed effects	*jp*	*jp*

Notes: Controls include past 5-year U.S. patent growth in firms' own technology fields, log payroll, firm age, dummy for publicly traded firms, dummy for firms with total imports > 0, dummy for firms with total exports > 0, and dummy for firms with imports from relative parties > 0. Estimates for industry-period (jp) fixed effects as well as the constant are suppressed.Robust standard errors adjusted for clustering at the firm-level are displayed below each coefficient. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

number of patents.

Table 2.2 shows estimates of β_2.[18] As the table indicates, U.S. firms create fewer patent applications when recent outside innovation by foreign firms is high, and firms' innovation is more internal in nature. This suggests that U.S. firms perform less external innovation when the technological barrier is high in product markets outside of their own.

2.2.2.3 The Ex-post Schumpeterian Effect

The final prediction of my model that I test is that firms do more internal innovation if they expect to get higher profits from their current product markets in the near future. To test this prediction, I use the export shock described in Section 2.2.1.2 as a proxy for changes in future profits. Thus, for firm i in industry

[18]Table B.20 in the Appendix shows the estimation results for own technology field shock, as well as the results including the interaction with firms' innovation intensities. I also run the same regression specification using concurrent technology shock, and Table B.21 in the Appendix shows the results. The results are widely consistent with that of the lagged technology shock.

Table 2.3: Effect of export shocks on firm innovation composition

	ΔPatents (1)	ΔPatents (2)	ΔSelf-cite (3)	ΔSelf-cite (4)
Export shock	0.046	0.047	-0.013	-0.014
	(0.032)	(0.032)	(0.035)	(0.035)
X Innovation intensity		0.003		0.014*
		(0.008)		(0.008)
Observations	6,500	6,500	6,500	6,500
Fixed effects	j, p	j, p	j, p	j, p

Notes: Controls include past 5-year U.S. patent growth in firms' own technology fields, log employment, firm age, dummy for publicly traded firms, dummy for firms with total imports > 0, dummy for firms with total exports > 0, and dummy for firms with imports from relative parties > 0. Estimates for industry (j) and the period (p) fixed effects as well as the constant are suppressed. Robust standard errors adjusted for clustering at the level of the firms' major industries are displayed below each coefficient. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * p < 0.1, ** p < 0.05, *** p < 0.01.

j in period $p \in \{1992-1999, 2000-2007\}$, I estimate the following regression model:

$$\Delta y_{ijp} = \beta_2 \Delta ExportShock_{jp} + \mathbf{X}_{ijp0}\,\gamma_1 + \delta_j + \delta_p + \alpha + \varepsilon_{ijp}\,, \tag{2.2}$$

where the descriptions for each variable are the same as described in model (2.1).

Table 2.3 reports the results. As the table indicates, there is no statistically significant effect of the export shock on the average firm's level or compotition of innovation. These weak results might be because few U.S. firms were exporting to China even in 2007, and the share of the total value of shipments accounted for by the value of exports to China is quite small, as shown in Table B.5 and B.6 of the Appendix. The interaction term with firm-level innovation intensity, however, is statistically significant and positive when the change in the self-citation ratio is the dependent variable. Thus, firms with above-average recent innovation intensity increase their internal innovation when they are faced with increased opportunities for exporting their products. We will see in the quantitative analysis in Section 2.4.4 that this result is consistent with the prediction from the baseline two-country model.

2.3 Baseline Two-Country Model

In this section, I extend the baseline model developed in Chapter 1 into a two-country framework. Time is discrete. Two countries, home (H) and foreign (F), are endowed with $\overline{L}_H$ and $\overline{L}_F$ units of labor, which are potentially different. In each country, there is a single final good producer operating in a perfectly competitive market, and a continuum of differentiated good producers operating in monopolistically competitive markets. The mass of differentiated good producers is determined through endogenous entry and exit. In each period, there is a fixed mass of potential startups in the differentiated good sector in each country, and those which successfully take over existing good markets through external innovation enter the economy. Differentiated goods are tradable but subject to variable trade costs, and producers from the two countries compete for technological leadership in a continuum of measure one goods markets through internal and external innovation. External innovation requires learning another firm's technology, but learning takes time. Thus, there is an imperfect technology spillover in the form of lagged learning, as firms can only learn other firms' past-period technologies. Below I describe the economy mainly for the home country H, and super/subscript H is omitted whenever there is no confusion. Time subscript t is also omitted whenever there is no confusion.[19]

[19]I use the term technology and product quality interchangeably.

2.3.1 Representative Household

The representative household has a logarithmic utility function and is populated by a continuum of individuals with total measure $\overline{L}$. Each individual supplies one unit of labor each period inelastically and consumes a portion C_t of a unique final good (consumption bundle) in the economy produced by the final good producer. The household's lifetime utility is

$$U = \sum_{t=0}^{\infty} \beta^t \log(C_t) \, . \tag{2.3}$$

Homogeneous workers are employed in the final goods (L) and differentiated goods ($\widetilde{L}$) sectors. Thus, in each period, the labor market satisfies

$$L + \widetilde{L} = \overline{L} \, . \tag{2.4}$$

The household maximizes its lifetime utility (2.3) subject to the period-by-period budget constraint

$$C_t \leqslant w_t \overline{L} + \Pi_t + \widetilde{\Pi}_t + G_t \, , \tag{2.5}$$

where w_t is the wage, Π_t is the final good producer's profits, $\widetilde{\Pi}_t$ is differentiated good producers' total profits net of R&D expenses, and G_t is government transfers including tariff revenues.

2.3.2 Final Good Producer

Both countries produce an identical final good. The final good is used for consumption and R&D expenditure for differentiated goods. The final good producer uses labor (L_H) and a continuum of differentiated goods indexed by $j \in [0,1]$ to produce a final good, where some of the differentiated goods can be produced by foreign exporters. Denote $\mathcal{J}_{cH}$ as an index set for differentiated goods sold in the home country that are produced by firms in country $c \in \{H, F\}$, and y_j^{cH} as the quantity of good j sold from c to H. Then a constant returns to scale production technology w.r.t. labor and differentiated goods can be written as

$$Y_H = \frac{(L_H)^\theta}{1-\theta} \Bigg[\underbrace{\int_0^1 \left(q_j^H\right)^\theta \left(y_j^{HH}\right)^{1-\theta} \mathcal{I}_{\{j \in \mathcal{J}_{HH}\}} \, dj}_{\text{domestic absorbtion}} + \underbrace{\int_0^1 \left(q_j^F\right)^\theta \left(y_j^{FH}\right)^{1-\theta} \mathcal{I}_{\{j \in \mathcal{J}_{FH}\}} \, dj}_{\text{imports}} \Bigg], \tag{2.6}$$

where q_j^H is the quality of good j in country H, possibly different from that in country F, and $\mathcal{I}_{\{\cdot\}}$ are indicator functions. The final good is produced competitively, and input prices—w_H for labor and p_j^H for good j sold in H—as well as product quality q_j^c are taken as given. When there are multiple potential suppliers for the same good from the home and/or foreign countries, the final good producer chooses the supplier with the combination of product quality and marginal cost of production (adjusted by trade costs for imported goods) that gives the final good supplier the highest profits.

To simplify the model and allow trade imbalances in the differentiated goods sector, I make the following assumption.

Assumption 2. *The final good is traded without friction and absorbs possible imbalances in differentiated goods trade.*

Free trade in the final good sector, along with identical final good production functions in both countries, imply that the final good price in both countries, P_H and P_F, will be the same. I normalize that price to one in each period for both countries without loss of generality.

2.3.3 Differentiated Goods Producers

There is a set of measure $\mathcal{F}_H$ of home firms and a set of measure $\mathcal{F}_{FH}$ of foreign exporters with $\mathcal{F}_H + \mathcal{F}_{FH} \in (0, 1)$, which are determined endogenously in equilibrium. These firms produce differentiated goods each period and sell their products in the home market. Some of the home firms ($\mathcal{F}_{HF} \leqslant \mathcal{F}_H$) export a portion of their products as well, which is also determined endogenously based on their product quality and marginal cost of production. Each good is produced in the producer's own country using local labor. Each operating firm owns at least one product line, and a single firm owns each product line in each country. Thus, a firm f can be characterized as its collection of product lines $\mathcal{J}^f = \{j : j \text{ is owned by firm } f\}$, where $n_f \equiv \| \mathcal{J}^f \|$ is the number of products firm f produces.

Because international trade is costly and the marginal cost of production can be different across countries, some domestic firms may have zero demand from the foreign final good producer, and foreign demand is absorbed by foreign differentiated good firms. In such cases, the quality of product j in the home country (q_j^H), along with the ownership of the good j, can be different from that in a foreign country (q_j^F). Then a global technology gap can be defined as $\Delta_{j,t}^G \equiv \frac{q_{j,t}^H}{q_{j,t}^F}$. If a firm in country H exports good j, then $\Delta_{j,t}^G$ is not defined as there is no firm in country F producing good j. In this case, I simply define $\Delta_{j,t}^G = \infty$. In the case where a country F firm exports good j, I define $\Delta_{j,t}^G = -\infty$.

Also, because there are imperfect technology spillovers, the current period frontier technology can be different from the last period technology that another firm can learn. The local technology gap for each market j in each country $c \in \{H, F\}$ is defined as $\Delta_{j,t}^c \equiv \frac{q_{j,t}^c}{q_{j,t-1}^c}$. Thus, each product line can be characterized by its quality and technology gaps—local technology gaps in home and foreign markets, and the global technology gap—$\left(q_j^c, \Delta_j^H, \Delta_j^F, \Delta_j^G\right)$.

Denote y_j^{HH} as the quantity of good j produced by a home firm and supplied to home market j. Each good $j \in [0,1]$ is produced using domestic labor ℓ_j^{HH} with a linear technology;

$$y_j^{HH} = \overline{q}_H \, \ell_j^{HH} \,, \tag{2.7}$$

where $\overline{q}_H \equiv \int_{\mathcal{J}_{HH}} q_j^H dj + \int_{\mathcal{J}_{FH}} q_j^F dj$ is the average product quality (average production technology) of differentiated goods traded in home markets. If good j is exported by a home firm to foreign market j, then it is produced using the same technology;

$$y_j^{HF} = \overline{q}_H \, \ell_j^{HF} \,,$$

but it is subject to an iceberg cost $d_{HF} > 1$ and an ad-valorem tariff $\widetilde{\tau}_{HF} \geqslant 1$. Thus, in order to sell y_j^{HF} units of good j in the foreign market, the home firm needs to ship $\tau_{HF} \times y_j^{HF}$ units, where $\tau_{HF} \equiv \widetilde{\tau}_{HF} \times d_{HF}$.

2.3.4 Innovation by Differentiated Good Producers

The differentiated good producers engage in two types of R&D—internal and external—to increase their profits from the products they currently produce, to protect their product markets from competitors, and to expand their businesses, where the R&D output takes the form of improvements in product quality (equivalently, production technology). Innovation outcomes are realized at the beginning of the next period. To allow incumbent firms to protect their own product markets from competitors (the escape-competition effect) and to make it more difficult to take over other firms' product markets when overall innovation intensity in the economy is high (the technological-barrier effect), I introduce imperfect technological spillovers, which are captured by lagged learning: firms that don't own product line j can only learn the incumbent's last period technology, $q_{j,t-1}$. Thus, external innovation builds on the past-period technology used in the domestic market. A home firm can learn a foreign firm's technology if and only if that foreign firm sells its products in the home country.

In this setup, learning another firm's technology is costly in the sense that i) outside firms can only learn last period's technology, and ii) learning involves R&D—only firms with strictly positive R&D expenditure can learn another firm's past technology through undirected learning.[20] Product line-specific current period technology $q_{j,t}$, and thus the local technology gap $\Delta_{j,t} \equiv \frac{q_{j,t}}{q_{j,t-1}}$, are observable only to the firms operating in product line j in that period. However, aggregate variables and the distribution of local technology gap (the share of product lines with a certain level of local technology gap) are publicly observable. Thus, a stationary equilibrium can be well defined. When two firms' technologies are neck and neck in one product line, a coin-toss tiebreaker rule applies as in Acemoglu et al. (2016) to make sure each product is produced by only one firm. An unused technology (idea) is assumed to depreciate by an amount sufficient to ensure that it becomes unprofitable to innovate on top of it next period.[21]

With the last two assumptions, only the winning firm from the coin toss keeps the product line until it is taken over by another firm through creative destruction (external innovation), while the losing firm never tries to enter the same market through internal innovation in the neck and neck case. Thus, the undirected nature of external innovation is ensured, and only a firm producing a product in a current period is allowed to do internal innovation on that product. Finally, to maintain tractability I assume that each firm can do only one external innovation in each period regardless of the total number of product lines the firm owns.

[20]Firms do not know which product line technology they will learn prior to their learning. This assumption helps the model tractable.

[21]If you don't recall your skill or idea frequently, you gradually forget about it. This is in some sense consistent with the literature discussing displaced workers' human capital depreciation.

2.3.4.1 Internal Innovation

Successful internal innovation improves the current quality $q_{j,t}$ for differentiated good j by $\lambda > 1$. The probability of internal innovation, $z_{j,t}$, is determined by the level of R&D expenditure $R^{in}_{j,t}$ in units of the final good:

$$z_{j,t} = \left(\frac{R^{in}_{j,t}}{\widehat{\chi} q_{j,t}} \right)^{\frac{1}{\widehat{\psi}}} ,$$

where $\widehat{\chi} > 0$ and $\widehat{\psi} > 1$. Thus incumbent firm's good j quality realized at the beginning of $t+1$, assuming the firm is not displaced by creative destruction, is:[22]

$$\left\{ q^{in}_{j,t+1} \right\} = \begin{cases} \left\{ \lambda q_{j,t} \right\} & \text{with probability } z_{j,t} \\ \left\{ q_{j,t} \right\} & \text{with probability } 1 - z_{j,t} \,. \end{cases}$$

As time is discrete and firms are multi-product firms, internal innovation outcomes follow a binomial process as in Ates and Saffie (2016).

2.3.4.2 External Innovation

Incumbents and potential startups attempt to take over other incumbents' markets through external innovation. Successful external innovation generates an improvement in product quality by a factor of $\eta > 1$ relative to the incumbent's lagged technology, where R&D results are realized at the beginning of next period.

[22] Hereafter, I write the quality of good j as a point set. This makes it easy to write the case when external innovation fails and firm does not acquire any product lines, which will be written as product quality set to be an empty set.

I assume $\lambda^2 > \eta > \lambda$. This assumption ensures that firms can protect their own product lines from outside firms through internal innovation, while $\eta > \lambda$ reflects the idea that external innovation introduces a new way of producing an existing product more efficiently. Thus, external innovation contributes more to both firm employment and aggregate growth than internal innovation, as found empirically in Akcigit and Kerr (2018). Both potential startups' and incumbent firms' external innovations are undirected in the sense that they are realized in any other product line with equal probability.

Existing firms with at least one product line ($n_f > 0$) decide the probability of external innovation x_t by choosing R&D expenditures R_t^{ex} in units of the final good:

$$x_t = \left(\frac{R_t^{ex}}{\widetilde{\chi}\, \bar{q}_t} \right)^{\frac{1}{\widetilde{\psi}}} ,$$

where $\widetilde{\chi} > 0$, $\widetilde{\psi} > 1$, and $\bar{q}_t$ is the average quality in the country where the firm is located. Thus, for prospective external innovators whose takeover is not pre-empted by the incumbent's successful defensive innovation, the distribution of quality at the start of the next period is:

$$\left\{ q_{j,t+1}^{ex} \right\} = \begin{cases} \left\{ \eta q_{j,t-1} \right\} & \text{with probability } x_t \\ \\ \varnothing & \text{with probability } 1 - x_t \, . \end{cases}$$

With probability $1 - x_t$, the external innovation fails, which implies there is zero probability that the firm will take over product line j. In this case, product quality for product line j for the potential entrant does not exist.

As a rival firm can only learn last period's technology, the local technology gap is an important factor determining an incumbent firm's success/failure at protecting its product line through internal innovation. With the above setup for innovation there are four possible technology gaps in this model economy, $\Delta^1 = 1$, $\Delta^2 = \lambda$, $\Delta^3 = \eta$, and $\Delta^4 = \frac{\eta}{\lambda}$. Detail is provided in Lemma 1 in Chapter 1.

2.3.4.3 Entry and Exit in the Differentiated Good Sector

At the beginning of each period, there is an exogenously determined $\mathcal{E}_H$ mass of new potential domestic startups trying to start businesses in the differentiated good sector. To start a business, a potential startup needs to invest in external R&D and take over one of the product lines from an incumbent firm. The potential startups, who have no existing product lines, decide the probability of external innovation $x_{e,t}$ by choosing R&D expenditure R_t^e in units of the final good:

$$x_{e,t} = \left(\frac{R_t^e}{\widetilde{\chi}_e \, \bar{q}_t} \right)^{\frac{1}{\widetilde{\psi}_e}},$$

where $\widetilde{\chi}_e > 0$, and $\widetilde{\psi}_e > 1$. For potential startups whose takeover attempt is not thwarted by defensive innovation by the incumbent, the distribution of quality at $t+1$ is

$$\left\{q^e_{j,t+1}\right\} = \begin{cases} \left\{\eta q_{j,t-1}\right\} & \text{with probability } x_{e,t} \\ \\ \varnothing & \text{with probability } 1 - x_{e,t} \ . \end{cases}$$

Incumbent firms in the differentiated good sector are engaged in internal and external innovation in each period. Thus, not only do they expand by developing improved versions of their existing products, they also expand by adding new product lines to their portfolio. However, as there are other firms engaged in external innovation as well, an individual incumbent firm is always faced with a positive probability of losing some of its own product markets to competitors. As there is a continuum of measure one product lines and a continuum of differentiated good producers, each product line faces the same probability of encountering a competitor. This probability is called the aggregate endogenous creative destruction arrival rate and it is equal to the average probability of external innovation in the economy:

$$\overline{x} = \underbrace{\mathcal{F}_H x^H + \mathcal{E}_H x_e^H}_{\equiv \overline{x}^H} + \underbrace{\mathcal{F}_F x^F + \mathcal{E}_F x_e^F}_{\equiv \overline{x}^F} , \tag{2.8}$$

where $\mathcal{F}_c$ is the mass of incumbents, $\mathcal{E}_c$ is the mass of potential startups , x^c is the probability of external innovation by incumbents, and x_e^c is the probability of external innovation by potential startups in country c. Here, I write the probability of external innovation for each group of firms as equal across all the firms in the same group. I verify this holds in equilibrium in the later section. Thus, $\overline{x}^c$ is the portion of the aggregate creative destruction arrival rate due to external innovation by firms in country c. An incumbent firm losing all of its product lines to competitors exits the economy, and it receives the value equal to the sum of discounted expected profits from a successful external innovation when it exits. This compensation for its accumulated knowledge stock ensures that incumbents with no product lines optimally do not to attempt to perform external innovation to re-enter the economy.

2.3.5 Equilibrium

2.3.5.1 Production

The standard profit maximization problem of the final good producer in country $c \in \{H, F\}$ gives us their inverse demand curve for differentiated good j produced by a firm in country $\tilde{c} \in \{H, F\}$:

$$p_j^c = P_c^{-1} L_c^{\theta} \left(q_j^{\tilde{c}}\right)^{\theta} \left(y_j^{\tilde{c}c}\right)^{-\theta} \ , \tag{2.9}$$

and demand for labor:

$$L_c = \frac{\theta}{w_c} P_c Y_c \ , \tag{2.10}$$

where p_j^c is the price for differentiated good j sold in country c, and P_c is the final good price in country c, which is equal to one. In deriving demand for good j I assume that each good is supplied by a single firm in a particular country. However, past incumbent firms in domestic markets that lost technological leadership to the current leader could in principle try to produce and sell their products through limit pricing, as the marginal cost of production is equal across all domestic firms. To avoid such cases and to simplify the model, I adopt the following two-stage price-bidding game assumption.

Assumption 3. *In a given product line j in a given country, the current incumbents and any former incumbents in the same line enter a two-stage price-bidding game. In the first stage, each firm pays a fee of $\varepsilon > 0$. In the second stage, all firms that paid the fee announce their prices.*

This assumption ensures that only the technological leader (adjusted for marginal costs and trade frictions) in a given country enters the first stage and announces its price in equilibrium. By using (2.9), the profit maximization problem of the differentiated good producer in country c owning product line $j \in [0, 1]$ is then

$$\pi^c(q_j^c) = \begin{cases} \max\limits_{y_j^{cc} \geqslant 0} \left\{ P_c^{-1} L_c^{\theta} \left(q_j^c\right)^{\theta} \left(y_j^{cc}\right)^{1-\theta} - \dfrac{w_c}{\overline{q}_c}\, y_j^{cc} \right\} & \text{if not exporter} \\ \max\limits_{y_j^{cc} \geqslant 0} \left\{ \sum\limits_{\hat{c}=c}^{\tilde{c}} \left[P_{\hat{c}}^{-1} L_{\hat{c}}^{\theta} \left(q_j^c\right)^{\theta} \left(y_j^{c\hat{c}}\right)^{1-\theta} - \tau_{c\hat{c}} \dfrac{w_c}{\overline{q}_c}\, y_j^{c\hat{c}} \right] \right\} & \text{if exporter,} \end{cases}$$

where $\tau_{cc} = 1$. The first order conditions of the above problem (and its foreign firm counterpart) yield the optimal price for differentiated good j in country c:

$$p_j^c = \begin{cases} \dfrac{1}{1-\theta}\,\dfrac{w_c}{\overline{q}_c} & \text{for domestic suppliers} \\ \\ \dfrac{1}{1-\theta}\,\tau_{\widetilde{c}c}\,\dfrac{w_{\widetilde{c}}}{\overline{q}_{\widetilde{c}}} & \text{for imports} \end{cases} \tag{2.11}$$

which is an unconstrained monopoly price given that the seller is the technological leader (taking into account trade frictions and the marginal costs) in each country c. The optimal price is independent of the individual product quality. Optimal quantities supplied by firms in country c are equal to

$$y_j^{cc}(q_j^c) = (1-\theta)^{\frac{1}{\theta}}\,(P_c)^{\frac{1}{\theta}}\,L_c\left(\frac{w_c}{\overline{q}_c}\right)^{-\frac{1}{\theta}}\,q_j^c \tag{2.12}$$

$$y_j^{c\widetilde{c}}(q_j^c) = (1-\theta)^{\frac{1}{\theta}}\,(P_{\widetilde{c}})^{\frac{1}{\theta}}\,L_{\widetilde{c}}\left(\tau_{c\widetilde{c}}\frac{w_c}{\overline{q}_c}\right)^{-\frac{1}{\theta}}\,q_j^c\,. \tag{2.13}$$

Then, profits for a firm in country c with technology q_j^c selling to market j in its own country are equal to

$$\pi^{cc}(q_j^c) = \underbrace{\theta(1-\theta)^{\frac{1-\theta}{\theta}}\,L_c\left(\frac{w_c}{\overline{q}_c}\right)^{-\frac{1-\theta}{\theta}}\,(P_c)^{\frac{1}{\theta}}}_{\equiv\pi^{cc}}\,q_j^c\,,$$

and profits for the same firm from selling to market j in country $\widetilde{c}$ are equal to

$$\pi^{c\widetilde{c}}(q_j^c) = \underbrace{\theta(1-\theta)^{\frac{1-\theta}{\theta}}\,L_{\widetilde{c}}\left(\tau_{c\widetilde{c}}\,\frac{w_c}{\overline{q}_c}\right)^{-\frac{1-\theta}{\theta}}\,(P_{\widetilde{c}})^{\frac{1}{\theta}}}_{\equiv\pi^{c\widetilde{c}}}\,q_j^c\,.$$

Importantly, both expressions are linear in q_j^c. Notice that product quality of good j sold in country $\widetilde{c}$ by a firm in country c is denoted as q_j^c. This is because product quality is firm-specific, in that if a firm produces good j in its own country c with quality q_j^c, then the quality of good j the firm can sell in country $\widetilde{c}$ is also equal to q_j^c.

Denote total product quality of goods produced by firms in country $\widetilde{c} \in \{H, F\}$ that are sold in country $c \in \{H, F\}$ as

$$\mathcal{Q}_{\widetilde{c}c} \equiv \int_0^1 q_j^c \, \mathcal{I}_{\{j \in \mathcal{J}_{\widetilde{c}c}\}} dj \; .$$

Then, the wage expressed in units of total quality in country c satisfies the following equation:

$$\frac{w_c}{\overline{q}_c} = \theta(1-\theta)^{\frac{1-2\theta}{\theta}} \left[\left(\frac{w_c}{\overline{q}_c} \right)^{-\frac{1-\theta}{\theta}} \frac{\mathcal{Q}_{cc}}{\overline{q}_c} + \left(\tau_{\widetilde{c}c} \frac{w_{\widetilde{c}}}{\overline{q}_{\widetilde{c}}} \right)^{-\frac{1-\theta}{\theta}} \frac{\mathcal{Q}_{\widetilde{c}c}}{\overline{q}_c} \right] (P_c)^{\frac{1}{\theta}} \; . \tag{2.14}$$

Total labor hired by differentiated good producers in country c is equal to

$$\widetilde{L}_c = (1-\theta)^{\frac{1}{\theta}} \left(\frac{w_c}{\overline{q}_c} \right)^{-\frac{1}{\theta}} \left[L_c \, (P_c)^{\frac{1}{\theta}} \, \frac{\mathcal{Q}_{cc}}{\overline{q}_c} + L_{\widetilde{c}} \, (P_{\widetilde{c}})^{\frac{1}{\theta}} \, (\tau_{c\widetilde{c}})^{-\frac{1}{\theta}} \, \frac{\mathcal{Q}_{c\widetilde{c}}}{\overline{q}_c} \right] , \tag{2.15}$$

and total labor hired by the final good producer is equal to $L_c = \overline{L}_c - \widetilde{L}_c$. The last three equations for the two countries can be solved for w_c, $\widetilde{L}_c$, and L_c as functions of aggregate qualities, price indices, and trade costs.

Total final good output expressed in units of total quality in country c is

$$\frac{Y_c}{\overline{q}_c} = (1-\theta)^{\frac{1-2\theta}{\theta}} (P_c)^{\frac{1-\theta}{\theta}} L_c \left[\left(\frac{w_c}{\overline{q}_c} \right)^{-\frac{1-\theta}{\theta}} \frac{\mathcal{Q}_{cc}}{\overline{q}_c} + \left(\tau_{\tilde{c}c} \frac{w_{\tilde{c}}}{\overline{q}_{\tilde{c}}} \right)^{-\frac{1-\theta}{\theta}} \frac{\mathcal{Q}_{\tilde{c}c}}{\overline{q}_c} \right] . \qquad (2.16)$$

Other equations are described in Technical Appendix B.3.5.

2.3.5.2 International Trade of Differentiated goods

Denote $MC^H \equiv \frac{w_H}{\overline{q}_H}$ as the marginal cost of production for domestic differentiated good firms, and $MC^F \equiv \frac{w_F}{\overline{q}_F}$ as the foreign counterpart. Recall that τ_{FH} is the trade cost to foreign firms exporting to domestic markets, and τ_{HF} is the trade cost to domestic firms exporting to the foreign country. Proposition 6 shows how these values define ranges for the global technology gap Δ^G corresponding to the direction of trade between home and foreign countries, which come from a profit maximizing final good producer that values product quality.

Proposition 6. *Denote threshold ratios of marginal cost for home firms to foreign firms in home and foreign markets as*

$$\underline{\Omega} \equiv \left(\frac{1}{\tau_{FH}} \right)^{\frac{1-\theta}{\theta}} \left(\frac{MC^H}{MC^F} \right)^{\frac{1-\theta}{\theta}} , \quad \overline{\Omega} \equiv (\tau_{HF})^{\frac{1-\theta}{\theta}} \left(\frac{MC^H}{MC^F} \right)^{\frac{1-\theta}{\theta}} , \qquad (2.17)$$

and the global technology gap for product j as

$$\Delta_j^G \equiv \frac{q_j}{q_j^F} .$$

Then, the home firm exports good j to the foreign country iff

$$\Delta_j^G > \overline{\Omega}\,,$$

while the home final good firm imports j from the foreign country iff

$$\Delta_j^G < \underline{\Omega}\,.$$

There is no trade of good j between the two countries iff $\Delta_j^G \in \left[\underline{\Omega}, \overline{\Omega}\right]$.

Proof: See Technical Appendix B.3.3.1.

Proposition 6 shows that depending on the relative size of (trade cost-adjusted) marginal costs, which are equivalent to quality-adjusted wages, foreign products with low quality (technology) can be sold in domestic markets. The global technology gap can also be defined by using 2-tuple integers. Denote m as the number of internal innovations, and n as the number of external innovations, in which home firms advance compared to foreign firms. Assuming that initial quality of good j in both countries is the same, $\Delta_j^G = \frac{q_j^H}{q_j^F} = \lambda^m \times \eta^n$ and this can be written as $\widetilde{\Delta^G}_j = (m, n)$.

As briefly explained earlier, there is free trade in the final good sector, in which all of the trade imbalance in the differentiated good sector is absorbed. Thus,

$$P_c X_c = \int_{j \in \mathcal{J}_{\tilde{c}c}} p_j^c y_j^{\tilde{c}c} dj \;-\; \frac{P_c}{P_{\tilde{c}}} \int_{j \in \mathcal{J}_{c\tilde{c}}} p_j^{\tilde{c}} y_j^{c\tilde{c}} dj\,,$$

which implies

$$X_c = (1-\theta)^{\frac{1-\theta}{\theta}} \left[(P_c)^{\frac{1-\theta}{\theta}} L_c \left(\tau_{\widetilde{c}c} \frac{w_{\widetilde{c}}}{\overline{q}_{\widetilde{c}}} \right)^{1-\frac{1}{\theta}} \mathcal{Q}_{\widetilde{c}c} - (P_{\widetilde{c}})^{\frac{1-\theta}{\theta}} L_{\widetilde{c}} \left(\tau_{c\widetilde{c}} \frac{w_c}{\overline{q}_c} \right)^{1-\frac{1}{\theta}} \mathcal{Q}_{c\widetilde{c}} \right] \tag{2.18}$$

where X_c is the net quantity of final goods exported by country c. $X_c > 0$ means country c exports final goods to country $\widetilde{c}$, and $X_c < 0$ means country c imports final goods from country $\widetilde{c}$. The first term in the RHS is the total value of differentiated goods imported from country $\widetilde{c}$, and the second term is the total value of differentiated goods exported to $\widetilde{c}$.

2.3.5.3 Firm Values and Optimal Innovation Decision

The value of a firm in country c with a production technology portfolio

$$\Phi^f \equiv \left\{ \left(q_j^c \, , \, \Delta_j^H \, , \, \Delta_j^F \, , \, \Delta_j^G \right) \right\}_{j \in \mathcal{J}^f}$$

is equal to

$$\begin{aligned} V^c\left(\Phi^f\right) = \max_{\left\{z_j^c\right\}_{j\in\mathcal{J}^f}, \, x^c} & \left\{ \sum_{j\in\mathcal{J}^f} \pi^c(q_j^c) - \left(\sum_{j\in\mathcal{J}^f} \widehat{\chi}\left(z_j^c\right)^{\widehat{\psi}} q_j^c + \widetilde{\chi}\left(x^c\right)^{\widetilde{\psi}} \overline{q}_c \right) \right. \\ & \left. + \widetilde{\beta}\, \mathbb{E}\Big[V^c\left(\Phi^{f\prime} \,\middle|\, \Phi^f \, , \, \left\{z_j^c\right\}_{j\in\mathcal{J}^f} \, , \, x^c \right) \Big] \right\}, \end{aligned}$$

where

$$\pi^c(q_j^c) = \begin{cases} \left(\pi^{cc}\right)\ q_j^c & \text{if firm is not an exporter} \\ \left(\pi^{cc} + \pi^{c\tilde{c}}\right)\ q_j^c & \text{if firm is an exporter} \end{cases}$$

are profits from production net of labor costs and tariffs, as defined in the previous section, and where the second and third terms in parentheses are R&D expenses for internal and external innovation. Since all firms are owned by the household, they discount their future profits using households' stochastic discount factor, $\widetilde{\beta} \equiv \frac{P_H' C_H'}{\beta P_H C_H}$. The last conditional expectation term for future values, $\mathbb{E}\Big[V^c\left(\Phi^{f\prime} \,\middle|\, \Phi^f\ ,\ \left\{z_j^c\right\}_{j\in\mathcal{J}^f}\ ,\ x^c\right)\Big]$ is defined in Appendix B.2.1.1.

Proposition 7. *For a given joint distribution over local technology gaps for home and foreign markets and global technology gaps, the value function of a firm in country c with product quality and technology gap portfolio* $\Phi^f \equiv \left\{\left(\ q_j^c\ ,\ \Delta_j^H\ ,\ \Delta_j^F\ ,\ \Delta_j^G\ \right)\right\}_{j\in\mathcal{J}^f}$ *is of the form:*

$$V^c\left(\Phi^f\right) = \sum_{j\in\mathcal{J}^f} A^c\left(\ \Delta_j^H\ ,\ \Delta_j^F\ ,\ \Delta_j^G\ \right) q_j^c\ +\ B^c\overline{q}_c\ ,$$

where the coefficients for values from existing products, $A^c\left(\ \Delta_j^H\ ,\ \Delta_j^F\ ,\ \Delta_j^G\ \right)$*, are independent of product quality* q_j^c*. The value from external innovation is equal to* $B^c\overline{q}_c$*, which is also equal to the exit value of an incumbent firm,* $V^c(\varnothing) = B^c\overline{q}_c$*. Furthermore, optimal internal innovation intensity* z_j^c *also depends only on the technology gap* $\left(\ \Delta_j^H\ ,\ \Delta_j^F\ ,\ \Delta_j^G\ \right)$*. Finally, optimal external innovation intensity* x^c *is independent of firm characteristics and equal across all incumbent firms.*

Proof: See Technical Appendix B.3.4.2.

Analytic expressions for $A^c\left(\Delta_j^H, \Delta_j^F, \Delta_j^G\right)$, $z^c\left(\Delta_j^H, \Delta_j^F, \Delta_j^G\right)$, B^c, and x^c are provided in Technical Appendix B.3.4.2.

2.3.5.4 Potential Startups

Let $V^c\left(\left\{q_j\, \Delta_j^H\, \Delta_j^F\ \ \Delta_j^G\right\}\right)$ denote the value of a firm in country c that has only one product line j, with product quality q_j^c, local technology gaps in home and foreign markets Δ_j^H and Δ_j^F, and global technology gap Δ_j^G. Then a potential startup's expected profits from entering through R&D are

$$\Pi_c^e = x_e^c\, \widetilde{\beta}_c\, \mathbb{E}\Big[\, V^c\left(\left\{q_j^{c\,\prime}\, \Delta_j^{H\prime}\, \Delta_j^{F\prime}\ \ \Delta_j^{G\prime}\right\}\right)\ \Big] - \ \widetilde{\chi}^e\left(x_e^c\right)^{\widetilde{\psi}^e}\ \overline{q}_c\,.$$

An analytic expression for optimal external innovation for potential startups is derived in Technical Appendix B.3.4.3.

2.3.5.5 Evolution of the Technology-Gap Distribution and Aggregate Growth

As shown in the previous section, product j can be completely described by its technology gaps $\left(\Delta_j^H\, \Delta_j^F\ \ \Delta_j^G\right)$ and its quality q_j. Thus, the index for each product, j, is redundant. Furthermore, what matters for firms' optimal decisions are the technology gaps, and firms need to know the distribution of technology gaps across markets—local technology gaps in home and foreign markets, Δ^H and

Δ^F, and global technology gaps, Δ^G. Denote the technology gap distribution as $\mu\left(\Delta^H, \Delta^F, \Delta^G\right)$. Appendix B.3.1 shows how technology gaps evolve over time according to firms' innovation decisions for each possible set of local and global technology gaps, $\left(\Delta^H, \Delta^F, \Delta^G\right)$. In a stationary equilibrium, inflows should be equal to outflows for each technology gap state $\mu\left(\Delta^H, \Delta^F, \Delta^G\right)$, where inflows and outflows for each technology gap state are described in Technical Appendix B.3.1.4.

2.3.6 Aggregate Quality Evolution

Proposition 8. *Define* $\boldsymbol{\Delta} \equiv \left(\Delta^H, \Delta^F, \Delta^G\right)$ *and* $\boldsymbol{\Delta}' \equiv \left(\Delta^{H\prime}, \Delta^{F\prime}, \Delta^{G\prime}\right)$. *Then for* $c, \widetilde{c} \in \{H, F\}$ *with* $c \neq \widetilde{c}$, *and for* $\widehat{c} \in \{c, \widetilde{c}\}$, *aggregate quality along a balanced growth path evolves according to*

$$\begin{aligned}\mathcal{Q}'_{\widehat{c}c} = &\left[\sum_{\boldsymbol{\Delta}}\sum_{\boldsymbol{\Delta}'} \Delta^{H\prime}\ \mathcal{I}'_{\widehat{c}c}\left(\Delta^{G\prime}\right)\ \mathcal{I}_{cc}\left(\Delta^{G}\right)\ \mathcal{P}\left(\boldsymbol{\Delta}'|\boldsymbol{\Delta}\right)\ \mu\left(\boldsymbol{\Delta}\right)\right]\mathcal{Q}_{cc} \\ &+\left[\sum_{\boldsymbol{\Delta}}\sum_{\boldsymbol{\Delta}'} \Delta^{H\prime}\ \mathcal{I}'_{\widehat{c}c}\left(\Delta^{G\prime}\right)\ \mathcal{I}_{\widetilde{c}c}\left(\Delta^{G}\right)\ \mathcal{P}\left(\boldsymbol{\Delta}'|\boldsymbol{\Delta}\right)\ \mu\left(\boldsymbol{\Delta}\right)\right]\mathcal{Q}_{\widetilde{c}c}\,,\end{aligned} \tag{2.19}$$

where $\mathcal{P}\left(\boldsymbol{\Delta}'|\boldsymbol{\Delta}\right)$ *is the probability of* $\boldsymbol{\Delta}$ *becoming* $\boldsymbol{\Delta}'$, *which is described in Technical Appendix B.3.1.* $\mathcal{I}_{\breve{c}c}\left(\Delta^G\right)$ *is an index function equal to one if* Δ^G *falls into the range for which a firm from country* $\breve{c}$ *produces and sells its product in country* c *for* $\breve{c} \in \{c, \widetilde{c}\}$. $\mathcal{I}'_{\breve{c}c}\left(\Delta^{G\prime}\right)$ *is the next period counterpart.*

Proof: See Appendix B.2.1.2.1

A complete description of $\mathcal{P}\left(\boldsymbol{\Delta}'|\boldsymbol{\Delta}\right)$ is provided in Technical Appendix B.3.1. and a complete description for the evolution of $\mathcal{Q}_{c\tilde{c}}$ is provided in Technical Appendix B.3.3.

Equation (2.16) shows that aggregate output growth is determined by the growth rate of average quality $\overline{q}_c$. The following lemma characterizes the aggregate growth rate.

Lemma 3. *The aggregate growth rate g_c along the balanced growth path is determined by*

$$g_c = \sum_{\boldsymbol{\Delta}} \sum_{\boldsymbol{\Delta}'} \Delta^{c\,\prime}\, \mathcal{P}\left(\boldsymbol{\Delta}'|\boldsymbol{\Delta}\right)\, \mu(\boldsymbol{\Delta}) - 1\,. \tag{2.20}$$

Proof: This follows from the proof of Proposition 8.

2.3.6.1 Aggregate Variables and Balanced Growth Path (BGP) Equilibrium

Total R&D expenses in country c, R_c, are

$$\begin{aligned} R_c \;=& \sum_{(\,\Delta^H\,,\,\Delta^F\,,\,\Delta^G\,)} \widehat{\chi}\left(z_j^c\left(\,\Delta^H\,,\,\Delta^F\,,\,\Delta^G\,\right)\right)^{\widehat{\psi}} \mu\left(\,\Delta^H\,,\,\Delta^F\,,\,\Delta^G\,\right)\; \mathcal{Q}_{cc} \\ & + \widetilde{\chi}\left(x^c\right)^{\widetilde{\psi}}\; \mathcal{F}_c\, \overline{q}_c + \widetilde{\chi}^e\left(x_e^c\right)^{\widetilde{\psi}^e}\; \mathcal{E}_c\, \overline{q}_c\,, \end{aligned} \tag{2.21}$$

where the first term is the sum of all internal R&D expenses by incumbent firms, the second term is the sum of all external R&D expenses by incumbent firms, and the last term is the sum of all external R&D expenses by potential startups. Note that $z_j^H\left(\,\Delta^H\,,\,\Delta^F\,,\,-\infty\,\right) = 0$ and $z_j^F\left(\,\Delta^H\,,\,\Delta^F\,,\,\infty\,\right) = 0$, $\forall\, \Delta^H,\, \Delta^F\, \in \left\{\,1,\, \frac{\eta}{\lambda},\, \lambda,\, \eta\,\right\}$.

Total profits by incumbent firms net of R&D expenses are then

$$\widetilde{\Pi}_c \;=\; \pi^{cc}\,\mathcal{Q}_{cc} \;+\; \pi^{c\widetilde{c}}\,\mathcal{Q}_{c\widetilde{c}} - P_c R_c\,. \tag{2.22}$$

Since the final good producer is perfectly competitive, its profit is zero, $\Pi_c = 0$.

The government transfer, G_c, for $c \neq \widetilde{c}$ is equal to total tariff revenue:

$$G_c = \widetilde{\tau}_{\widetilde{c}c}\,(1-\theta)^{\frac{1-\theta}{\theta}}\,(P_c)^{\frac{1}{\theta}}\,L_c\left(\tau_{\widetilde{c}c}\frac{w_{\widetilde{c}}}{\overline{q}_{\widetilde{c}}}\right)^{1-\frac{1}{\theta}}\mathcal{Q}_{\widetilde{c}c}\,.$$

Finally, consumption is determined by the resource constraint

$$P_c C_c = P_c Y_c - P_c X_c - R_c + G_c\,, \tag{2.23}$$

which is equal to the households' total income defined by the households' budget constraint (2.5) with equality. I now close this section by defining the equilibrium.

Definition 2 (Balanced Growth Path Equilibrium)**.** *Let the world economy consist of two countries $c \in \{H, F\}$. A balanced growth path equilibrium of this economy consists of the following tuple for every t, $c, \widetilde{c} \in \{H, F\}$, $j \in [0,1]$, q_j^c and $\overline{q}_c$:*

$$\left\{ y_j^{c\widetilde{c}*},\, p_j^{c*},\, w_c^*,\, L_c^*,\, \widetilde{L}_c^*,\, x^{c*},\, \{z_j^{c*}(\mathbf{\Delta})\}_{\mathbf{\Delta}},\, x_e^{c*},\, \overline{x}^{c*},\, \overline{x}^*,\, \mathcal{F}_c^*,\, R_c^*,\, X_c^*,\, Y_c^*,\, C_c^*,\, g_c^*,\, \mathcal{Q}_{c\widetilde{c}},\, \underline{\Omega},\, \overline{\Omega},\, \{\mu^*(\mathbf{\Delta})\}_{\mathbf{\Delta}} \right\}$$

such that (i) $y_j^{c\widetilde{c}*}$ and p_j^{c*} satisfy (2.11)-(2.13); (ii) w_c^*, L_c^*, and $\widetilde{L}_c^*$ satisfy (2.14), (2.15), and $L_c = \overline{L}_c - \widetilde{L}_c$; (iii) x^{c*} is equal to (B.68); (iv) $\left\{z_j^{c*}(\mathbf{\Delta})\right\}_{\mathbf{\Delta}}$ is equal to (B.75)-(B.80) and (B.69)-(B.74) according to the value of $\mathbf{\Delta}$; (v) x_e^{c*} is equal to (B.81); (vi) $\overline{x}^{c*}$ is as defined in (2.8); (vii) $\overline{x}^*$ is equal to (2.8); (viii) $\mathcal{F}_c^*$ is consistent *with optimal innovation decisions; (ix) R_c^* satisfies* (2.21)*; (x) X_c^* satisfies* (2.18)*; (xi) Y_c^* satisfies* (2.16)*; (xii) C_c^* satisfies* (2.23)*; (xiii) g_c^* is given by* (2.20)*; (xiv) $\mathcal{Q}_{c\tilde{c}}$ evolves according to the evolution of the technology gaps* (2.19)*; (xv) $\underline{\Omega}$ and $\overline{\Omega}$ satisfy* (2.17)*; and (xvi) $\{\mu^* (\mathbf{\Delta})\}_{\mathbf{\Delta}}$ evolves according to the laws of motion* (B.20)-(B.43) *according to the value of $\mathbf{\Delta}$.*

2.4 Quantitative Analysis

2.4.1 Calibration

There are nineteen structural parameters (assuming symmetry across the two countries for innovation and production) that I need to calibrate, seven of which I calibrate internally. Table 2.4 shows the list of parameters and their values used for the counterfactual exercise. I map my two-country model to the U.S. and China. As all the products in my model economy are tradable, I calibrate the model to the U.S. manufacturing sector in 2000.

One complication with this setup is that tariff rates imposed by the U.S. government on Chinese products in 2000 were virtually unchanged after China's WTO accession. However, because there was a possibility of tariff rate increases, the effective tariff rates Chinese firms perceived before 2001 were higher than the actual values. To capture this and to run a counterfactual exercise to analyze the effect of trade liberalization on the composition of firm innovation that mimics what happened after China's WTO accession in the U.S., I estimate the effective tariff rate facing Chinese firms in 2000. Specifically, I assume a 13% probability of the

Table 2.4: Structural Parameters

	Parameter	Description	Value	Identification
1.	β	Time discount factor	0.9615	Annual interest rate of 4%
2.	$\widetilde{\tau}_{HF}$	Tariff rates for exports from H to F	1.0816	External calibration
3.	$\widetilde{\tau}_{FH}$	Tariff rates for exports from F to H	1.0816	External calibration
4.	$\mu_0(\Delta^G)$	Initial global technology gap distribution	Matrix	External calibration
5.	$\widehat{\psi}$	Curvature of internal R&D	2	(Akcigit and Kerr, 2018)
6.	$\widetilde{\psi}$	Curvature of external R&D	2	(Akcigit and Kerr, 2018)
7.	$\widetilde{\psi}^e$	Curvature of external R&D, startup	2	(Akcigit and Kerr, 2018)
8.	θ	Quality share in final good production	0.109	(Akcigit and Kerr, 2018)
9.	$\overline{L}_H$	Mass of labor in country H	1	External calibration
10.	$\overline{L}_F$	Mass of labor in country F	1	External calibration
11.	$\mathcal{E}_H$	Mass of potential startups in H	0.5	External calibration
12.	$\mathcal{E}_F$	Mass of potential startups in F	0.5	External calibration
13.	λ	Quality multiplier of internal innovation	1.044	Indirect inference
14.	η	Quality multiplier of external innovation	1.067	Indirect inference
15.	$\widehat{\chi}$	Scale of internal R&D	0.119	Indirect inference
16.	$\widetilde{\chi}$	Scale of external R&D	0.714	Indirect inference
17.	$\widetilde{\chi}^e$	Scale of external R&D, startup	11.696	Indirect inference
18.	d_{HF}	Iceberg trade cost for exports from H to F	1.01	Indirect inference
19.	d_{FH}	Iceberg trade cost for exports from F to H	1.01	Indirect inference

tariff rate increasing to the non-NTR rate, as estimated by Handley and Limão (2017), an average non-NTR rate of 36%, and an average NTR rate of 4%, to get an effective tariff rate of 8.16%.

As one period in my model is one year, I set the time discount factor to 0.9615, implying a real interest rate of 4%. I set the mass of labor to 1 and the mass of potential startups to 0.5 in both countries, as the counterfactual exercise will compare the two balanced growth path equilibria before and after China's WTO accession, and this requires the two countries to be symmetric. I set the initial global technology gap distribution to be a symmetric random matrix. This is innocuous as the effect from the initial values of this matrix will be washed away during the simulation. I set the curvature of the R&D cost functions to 2, which is a standard value in the firm innovation literature. I set the quality share in final good production to 0.109, the value estimated by Akcigit and Kerr (2018).

Table 2.5: Model Fit

	Moment	Data	Model	Source
1.	p90 emp. growth, emp. weighted (%)	19.86	18.46	Decker et al. (2016)
2.	Startup rates (%)	6.68	5.64	BDS
3.	Agg. domestic sales growth (%)	2.14	1.70	NBER-CES
4.	Avg. # of products firms produce	2.27	1.88	CMF
5.	Success prob. of adding a product (%)	29.20	22.68	CMF
6.	Share of US firms exporting to CN (%)	2.30	1.12	LFTTD

2.4.2 Indirect Inference

There are seven remaining parameters to be estimated: λ, η, $\widehat{\chi}$, $\widetilde{\chi}$, $\widetilde{\chi}^e$, d_{HF}, and d_{FH}. However, as the two countries are symmetric, $d_{HF} = d_{FH}$. Thus, I have six remaining parameters and these are estimated using an indirect inference approach: for each set of six parameter values, I compute six model-generated moments, compare them to the data moments, and find a set of parameter values that minimizes the objective function

$$\min \sum_{i=1}^{6} \frac{\mid \text{model moments}_i - \text{data moments}_i \mid}{\frac{1}{2} \mid \text{model moments}_i \mid + \frac{1}{2} \mid \text{data moments}_i \mid}$$

where the six moments are listed in Table 2.5.

The six moments are chosen in consideration of both their importance in answering the central question of this paper, and the relationships among the moments and the parameters coming from the choice of functional forms in the model. Although all the parameter values contribute substantially in determining the value for each model-generated moment, the tight relationship between specific sub-groups of parameters and moments can be noted.

Firms perform internal and external R&D to adjust the number of product lines they operate. Since R&D cost is one of the crucial factors determining the level of R&D intensity, and hence the number of product lines the firm owns, I discipline the scale parameters of internal R&D ($\widehat{\chi}$) and external R&D ($\widetilde{\chi}$) using the average number of products owned by firms. How quickly firms add new products to their portfolio depends on the level of external R&D investment. Thus, I discipline the scale of external R&D ($\widetilde{\chi}$) using the success probability of adding a product (average number of products added by firms). Potential startups learn and improve existing technologies to enter the market, and the success probability of entry is tightly related to their level of R&D expenditure. Thus I discipline the scale of external R&D for startups ($\widetilde{\chi}^e$) using the startup rate.

Firms grow in terms of both sales and the number of employees by improving the qualities of their existing products and/or adding new product lines to their product portfolios. How quickly they grow depends on how much product quality improvement they can achieve. Thus I discipline the quality multipliers of internal innovation (λ) and external innovation (η) using the average sales growth rate and employment growth rate at the 90th percentile of the employment-weighted firm employment growth distribution.

Finally, I discipline the iceberg trade costs $d_{cc'}$ using the share of U.S. firms exporting to China. Table 2.5 reports the model generated moments and their empirical counterparts.

2.4.3 Solution Algorithm

Since I don't have an analytic expression for the firm distribution, I pin down values for the masses of firms, $\mathcal{F}_H$ and $\mathcal{F}_F$, through simulation during the numerical solution algorithm. I simulate 200,000 products over 600 years, then take an average across outcomes from the last 200 years to capture the model-implied moments. I solve the model as a fixed point over a vector of growth rates $(g^H,\ g^F)$. Below I describe the solution algorithm in detail.

Solution Algorithm

1. Guess the stationary distribution $\mu\left(\Delta^H\ \Delta^F\ \ \Delta^G\right)$, BGP growth rates g^c, total external innovation rates $\overline{x}^c$, and total quality ratios $\frac{\mathcal{Q}_{cc}}{\overline{q}_c}$ for $c \in \{H, F\}$ with $\frac{\mathcal{Q}_{\tilde{c}c}}{\overline{q}_c} = 1 - \frac{\mathcal{Q}_{cc}}{\overline{q}_c}$.

2. Using $\frac{\mathcal{Q}_{cc}}{\overline{q}_c}$ and $\frac{\mathcal{Q}_{\tilde{c}c}}{\overline{q}_c}$ for $c \in \{H, F\}$,

    (a) Compute $\frac{w_c}{\overline{q}_c}$, L_c, and $\widetilde{L}_c$.

    (b) Compute $\pi^{c\tilde{c}}$ for $c, \tilde{c} \in \{H, F\}$.

    (c) Compute the two thresholds $\underline{\Omega}$, and $\overline{\Omega}$, and identify the range of $\Delta^G \in \left[\underline{\Omega}, \overline{\Omega}\right]$.

3. Using $\overline{x}^c$ and g^c,

    (a) Compute $A^c\left(\Delta^H\ \Delta^F\ \ \Delta^G\right)$, $z^c\left(\Delta^H\ \Delta^F\ \ \Delta^G\right)$, x^c and x_e^c for $c \in \{H, F\}$.

    (b) Compute $\mathcal{F}_c = \frac{\overline{x}^c - x_e^c \mathcal{E}_c}{x^c}$. If $\mathcal{F}_c \notin (0, 1)$, adjust $\overline{x}^c$ and redo 3a.

4. Simulate to get updates for g^c, $\frac{\mathcal{Q}_{cc}}{\overline{q}_c}$, and $\mathcal{F}_c$ for $c \in \{H, F\}$:

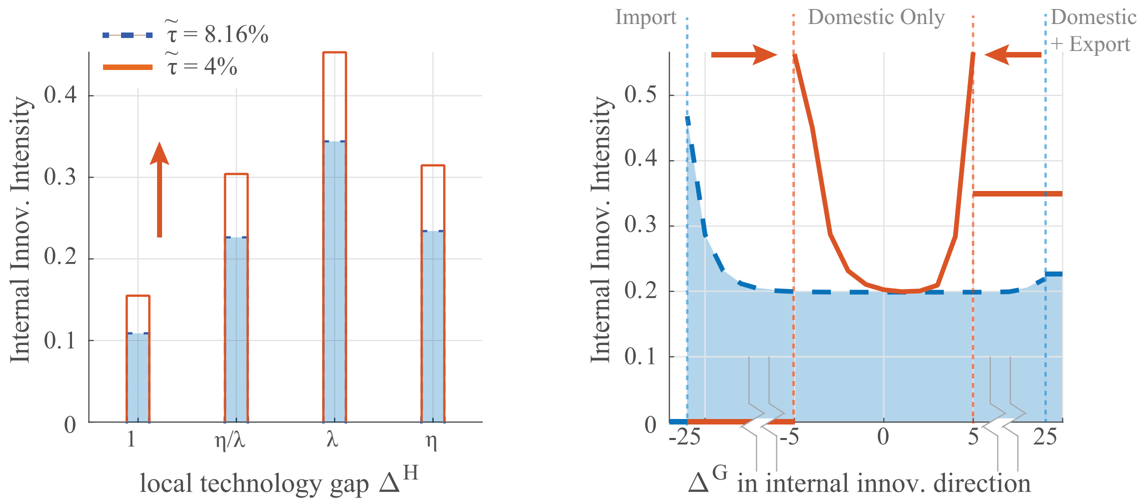

Figure 2.1: Internal Innovation Decision Rule

(a) Draw a sample of N_p product lines from the posited stationary distribution $\mu\left(\Delta^H\ \Delta^F\ \ \Delta^G\right)$.

(b) Assign N_f^c number of firms implied by $\mathcal{F}_c$ computed in 3b to the sample product lines randomly.

(c) Simulate the model while allowing for firm entry and exit until $\| g_{t+1}^c - g_t^c \| < \varepsilon$

(d) Compute $\frac{\mathcal{Q}_{cc}}{\overline{q}_c}$, $\mathcal{F}_c = \frac{total\ nb\ of\ firms_c}{total\ nb\ of\ products_c}$, and $\overline{x}^c = x^c\mathcal{F}_c + x_e^c\mathcal{E}_c$.

5. Compute a stationary distribution $\mu_\infty\left(\Delta^H\ \Delta^F\ \ \Delta^G\right)$ by using the law of motion and innovation rates (use z^c and x_e^c from 3a, and $\overline{x}^c$ from 4d).

6. Compare the initial growth rates in 1 with the values from 4. If the values are sufficiently different, update 1 with 5 and 4d, and redo the process 2 through 4. Iterate until the two growth rates converge.

2.4.4 Characteristics of Optimal Innovation Decision Rules

The blue lines in Figure 2.1 show two cross-sections from the internal innovation decision rule for the baseline parameter values, which is four-dimensional. The left panel shows the average internal innovation decision rule as a function of the local-technology-gap in the home county (Δ^H). As we can see, innovation intensity (the success probability of innovation) increases with Δ^H at first, then drops when $\Delta^H = \eta$. In the latter case, incumbents have such a high technological advantage that competitors are unlikely to take over their businesses even when incumbents fail at internal innovation. The right panel shows the internal innovation decision rule as a function of the global-technology-gap (Δ^G), which is similar to Akcigit et al. (2018). Internal innovation intensity peaks near two thresholds. Firms have higher incentives to do internal innovation near the export-threshold (right), as additional internal innovation makes firms exporters, which leads to higher profits. Firms also have higher incentives to do internal innovation near the import-threshold (left), as the failure of internal innovation leads to losing the product market to foreign firms.

2.4.5 Counterfactual Exercise

I run a counterfactual exercise using a 4.16 percentage point drop in the bilateral tariff rate (from 8.16% to 4%) as a trade shock, which is equivalent to the average drop in the effective tariff rates faced by Chinese firms after 2001, and compare the two BGP equilibria. Figure 2.1 shows changes in the optimal internal innovation decision rule, and Table 2.6 shows changes in firm and aggregate-level

Table 2.6: Reduction in bilateral tariff rates from 8.16% to 4%

Description	Before	After	% change
Avg. internal innov. intensity z^H (%)	19.28	20.99	8.83
Firm level external innov. intensity x^H (%)	26.63	24.81	-6.84
Success prob. of adding a product (%)	22.68	20.63	-9.04
Technological barrier (%)	14.82	16.83	13.56
p90 emp. growth (%)	18.46	15.15	-17.94
p10 emp. growth (%)	-41.77	-37.89	-9.28
Startup rate (%)	5.64	5.06	-10.26
Aggregate domestic sales growth (%)	1.70	1.69	-0.55
R&D to sales ratio (%)	4.54	4.16	-8.51
Internal R&D expense share	21.84	31.96	46.35
Share of firms exporting (%)	1.12	3.44	206.45
Share of export sales in total sales (%)	0.46	1.64	256.37

moments. Increasing international competition leads firms to shift their innovation from external to internal, which leads to lower employment growth rates for high-growth firms (firms at the 90th percentile of the firm employment growth distribution). Employment growth of low-growth firms (firms at the 10th percentile of the firm employment growth distribution), however, increases, and this leads to a decline in the skewness of the firm employment growth distribution measured as the p90-p10 differential. Firms become better at protecting their own product market through defensive internal innovation but lose their power of creative destruction. The economy becomes a place where incumbent firms have a high technological advantage in their own market on average. This is reflected as an increased technological barrier (measured as one minus the ratio of the success probability of adding a product divided by the firm-level external innovation intensity). Thus, the startup rate also declines as external innovation becomes harder. These results are consistent with industry-level regression results using the imposition of PNTR as a foreign competitive pressure shock, as shown in Table B.22 in the Appendix.

2.5 Conclusion

In this chapter, I investigate how competitive pressure from foreign firms affects firm innovation, high-growth firm activity and firm entry by testing predictions of the model developed in Chapter 1, and developing a two-country endogenous growth model with two types of innovation and imperfect technology spillovers, which is an extension of the baseline model developed in Chapter 1. An increase in competitive pressure from foreign firms lowers firms' incentive to invest in external innovation while it encourages investment in internal innovation for products with a high technological advantage. Therefore, innovation-intensive (and thus high-growth) firms don't grow as quickly, and the domestic startup rate falls, after an increase in competitive pressure from foreign firms.

To test the three model predictions developed in Chapter 1 empirically, I first construct a comprehensive dataset for a population of patenting firms from 1976 to 2016 using firm-level data from the U.S. Census Bureau integrated with firm-level patent data from the USPTO. Then I run reduced-form regressions, where I use the patent data to measure the level and composition of firm innovation. I find regression results consistent with the model predictions.

Quantitative analysis using my theoretical framework confirms the proposed mechanism. A 4.16 percentage point reduction in bilateral tariff rates in my model causes firms to shift their innovation activities toward more internal innovation due to higher competitive pressure from foreign firms. Consequently, high-growth firms grow more slowly, as they become less willing to experiment and add new products.

Also, the startup rate falls, as the heightened technological advantage accumulated by incumbent firms through internal innovation makes it harder to enter the economy through external innovation.

To the best of my knowledge, this is the first attempt to develop a two-country endogenous growth model with an escape-competition effect, in which firms are allowed to grow both through product scope expansion à la Klette and Kortum (2004) and own product quality improvement as in Aghion et al. (2001), and in which there is firm entry and exit. Also, this is the first attempt to identify increasing competitive pressure from foreign firms as a reason for declining business dynamism in the U.S. economy, and I provide supporting empirical evidence.

Appendix A: Chapter 1 Appendix

A.1 Baseline Model

A.1.1 Optimal Production and Employment

Final goods producer's production function is of the form:

$$Y = \frac{L^\theta}{1-\theta}\left[\int_0^1 q_j^\theta\, y_j^{1-\theta}\, \mathcal{I}_{\{j\in\mathcal{D}\}}\, dj \;+\; \int_0^1 q_j^\theta\, y_j^{1-\theta}\, \mathcal{I}_{\{j\notin\mathcal{D}\}}\, dj\right],$$

where $\mathcal{D}$ is the index set for differentiated products produced by domestic firms, and final good price is normalized to one $P = 1$. Thus profits are

$$\Pi^{\text{FG}} = Y = \frac{L^\theta}{1-\theta}\left[\int_0^1 q_j^\theta\, y_j^{1-\theta}\, \mathcal{I}_{\{j\in\mathcal{D}\}}\, dj \;+\; \int_0^1 q_j^\theta\, y_j^{1-\theta}\, \mathcal{I}_{\{j\notin\mathcal{D}\}}\, dj\right] - wL - \int_0^1 p_j\, y_j\, dj\,.$$

FONCs of final good producer's profit maximization problem w.r.t. k_j and L are

$$\frac{\partial}{\partial y_j}: \quad p_j = q_j^\theta\, L^\theta\, y_j^{-\theta} \tag{A.1}$$

$$\frac{\partial}{\partial L}: \quad w = \frac{\theta}{1-\theta}\, L^{\theta-1}\left[\int_0^1 q_j^\theta\, y_j^{1-\theta}\, \mathcal{I}_{\{j\in\mathcal{D}\}}\, dj \;+\; \int_0^1 q_j^\theta\, y_j^{1-\theta}\, \mathcal{I}_{\{j\notin\mathcal{D}\}}\, dj\right]. \tag{A.2}$$

Intermediate good producers, both domestic firms and foreign exporters, take differentiated product demand (A.1) as given and solve for the profit maximization problem:

$$\pi(q_j) = \max_{y_j \geqslant 0} \left\{ L^{\theta}\, q_j^{\theta}\, y_j^{1-\theta} - y_j \right\} \ .$$

The FOC of this problem gives us:

$$\frac{\partial}{\partial y_j} : \quad (1-\theta) L^{\theta}\, q_j^{\theta}\, y_j^{-\theta} = 1 \quad \Rightarrow \quad y_j = (1-\theta)^{\frac{1}{\theta}} L\, q_j \ , \text{ and } p_j = \frac{1}{1-\theta} \ .$$

By plugging in the two optimal choices, differentiated product producer's profits from a product line j become

$$\pi(q_j) = \underbrace{\theta (1-\theta)^{\frac{1-\theta}{\theta}}\, L}_{\equiv \pi}\ q_j \ .$$

By plugging in optimal differentiated product production rule to (A.2), we get the wage rule that depends only on average product qualities

$$\begin{aligned} w &= \frac{\theta}{1-\theta}\, L^{\theta-1} \left[\int_0^1 q_j^{\theta}\, (1-\theta)^{\frac{1-\theta}{\theta}}\, L^{1-\theta}\, q_j^{1-\theta}\, \mathcal{I}_{\{j \in \mathcal{D}\}}\, dj + \int_0^1 q_j^{\theta}\, (1-\theta)^{\frac{1-\theta}{\theta}}\, L^{1-\theta}\, q_j^{1-\theta}\, \mathcal{I}_{\{j \notin \mathcal{D}\}}\, dj \right] \\ &= \frac{\theta}{1-\theta}\, L^{\theta-1}\, (1-\theta)^{\frac{1-\theta}{\theta}}\, L^{1-\theta} \int_0^1 q_j\, dj \\ &\Rightarrow w = \theta (1-\theta)^{1-2\theta} \overline{q} \end{aligned} \tag{A.3}$$

Finally, using the labor market clearing condition

$$L = 1\,, \tag{A.4}$$

we get the equilibrium conditions:

$$Y = (1-\theta)^{\frac{1-2\theta}{\theta}}\overline{q} \tag{A.5}$$

$$y_j = (1-\theta)^{\frac{1}{\theta}}q_j \tag{A.6}$$

$$p_j = \frac{1}{1-\theta} \tag{A.7}$$

$$\pi = \theta(1-\theta)^{\frac{1-\theta}{\theta}}\,. \tag{A.8}$$

A.1.2 Product Quality Determination

In this section, I will consider all possible cases where firm keeps or loses its product lines next period and compute the probabilities as functions of internal innovation intensities and creative destruction arrival rate. Clearly, past period technology gap $\Delta_t = \frac{q_t}{q_{t-1}}$ is the only information needed to compute these probabilities, as incumbent firm and outside firm trying to take over incumbent firm's product line compete with the level of next period product qualities they come up with, where product quality in period $t+1$ the incumbent firm will have after internal innovation improves or fail to improve the product quality by $\Delta_{j,t+1}$ is $q^{in}_{j,t+1} = \Delta_{j,t+1}\Delta_{j,t}q_{j,t-1}$, and product quality the outside firm will have after successful external innovation is $q^{en}_{j,t+1} = \eta q_{j,t-1}$. I will first show Δ_t can assume only four values, $\Delta^1 = 1$, $\Delta^2 = \lambda$,

$\Delta^3 = \eta$, and $\Delta^4 = \frac{\eta}{\lambda}$.

A.1.2.1 Proof of Lemma 1

Proof. To make argument clearer, let's consider the cases where 1) there is no ownership change between $t-1$ and t, and 2) there is ownership change between $t-1$ and t.

1) No ownership change between $t-1$ and t: In this case, $q_{j,t} = \Delta_{j,t} q_{j,t-1}$ should hold, where only $\Delta_{j,t} \in \{\Delta^1 = 1, \Delta^2 = \lambda\}$ are possible due to the fact that $\Delta_{j,t}$ is an outcome of internal innovation.

2) Ownership change between $t-1$ and t: In this case, $q_{j,t} = \eta q_{j,t-2}$ should hold. Let's consider all potentially possible cases where i. $\Delta_{j,t} = 1$, ii. $\Delta_{j,t} = \lambda$, iii. $\Delta_{j,t} = \eta$, iv. $\Delta_{j,t} = \frac{\eta}{\lambda}$, v. $\Delta_{j,t} = \frac{\eta^n}{\lambda^m}$ with $n \geqslant m > 0$, and vi. $\Delta_{j,t} = \frac{\lambda^n}{\eta^m}$ with $n > m > 0$. These are the only potentially possible values Δ can assume, as there are only three step sizes (1, λ, and η) product quality can change between two periods and there cannot be a technology regression ($q_t < q_{t-1}$). In the end, we will see that only the first four cases are possible.

case 2)-i. $\Delta_{j,t} = 1$

For this to be true, $q_{j,t} = q_{j,t-1}$ should hold. Since $q_{j,t} = \eta q_{j,t-2}$, this implies $q_{j,t-1} = \eta q_{j,t-2}$. This is possible if there was external innovation between $t-2$ and $t-1$, and no internal innovation between $t-3$ and $t-1$, thus $q_{j,t-2} = q_{j,t-3}$. Thus $\Delta_{j,t} = 1$ is possible with ownership change between $t-1$ and t.

case 2)-ii. $\Delta_{j,t} = \lambda$

For this to be true, $\Delta_{j,t-1} = \frac{\eta}{\lambda}$ should hold, as $\Delta_{j,t} = \frac{q_{j,t}}{q_{j,t-1}} = \frac{\eta q_{j,t-2}}{\Delta_{j,t-1} q_{j,t-2}}$. This can be possible if there is internal innovation between $t-3$ and $t-2$, and external innovation between $t-2$ and $t-1$, but no internal innovation between $t-2$ and $t-1$. In this case, $q_{j,t-2} = \lambda q_{j,t-3}$, and $q_{j,t-1} = \eta q_{j,t-3}$. Thus $\Delta_{j,t-1} = \frac{q_{j,t-1}}{q_{j,t-2}} = \frac{\eta q_{j,t-3}}{\lambda q_{j,t-3}} = \frac{\eta}{\lambda}$. So I proved both $\Delta_{j,t} = \lambda$ and $\Delta_{j,t} = \frac{\eta}{\lambda}$ are possible and $\Delta_{j,t} = \frac{\eta}{\lambda}$ can be realized only through external innovation between $t-1$ and t.

case 2)-iii. $\Delta_{j,t} = \eta$

For this to be true, $q_{j,t-1} = q_{j,t-2}$ should hold. This is possible if there is no ownership change and no internal innovation between $t-1$ and $t-2$. Thus $\Delta_{j,t} = \eta$ is possible.

case 2)-iv. $\Delta_{j,t} = \frac{\eta}{\lambda}$

The possibility of this case is shown in case 2)-ii.

case 2)-v. $\Delta_{j,t} = \frac{\eta^n}{\lambda^m}$ with $n \geqslant m > 0$

Let's suppose this is the case. Since $\Delta_{j,t} \notin \{\Delta^1 = 1, \Delta^2 = \lambda\}$ there should be an ownership change between $t-1$ and t. Thus $q_{j,t} = \eta q_{j,t-2}$ should hold, and this implies $q_{j,t-1} = \frac{\lambda^m}{\eta^{n-1}} q_{j,t-2}$. $m \leqslant n-1$ is not possible as this implies technology regression. Let's suppose $m > n-1$. Since $n \geqslant m > 0$, this implies $m = n$ should hold.

Suppose this is the case, thus $g_{j,t-2} = \frac{\lambda^m}{\eta^{m-1}} q_{j.t-1}$. If the values for λ, η, and m are such that $\frac{\lambda^m}{\eta^{m-1}} < 1$, then this means technology regression, which is not possible. Let's suppose $\frac{\lambda^m}{\eta^{m-1}} > 1$ is true. If $m = 1$, we are back in the case 2)-ii and case 2)-iv. Let's suppose $m > 1$. Since $\frac{\lambda^m}{\eta^{m-1}} \neq 1$ or λ, there should be an ownership change between $t-2$ and $t-1$. Thus $q_{j,t-1} = \eta q_{j,t-3}$, and this implies $q_{j,t-2} = \frac{\eta^m}{\lambda^m} q_{j,t-3}$.

Thus if $\Delta_{j,t} = \frac{\eta^n}{\lambda^m}$ is possible, then

$$q_{j,t-s} = \begin{cases} \frac{\eta^m}{\lambda^m} q_{j,t-s-1} & , s\text{: even number} \\ \\ \frac{\lambda^m}{\eta^{m-1}} q_{j,t-s-1} & , s\text{: odd number .} \end{cases}$$

Thus in this case, either $q_{j,1} = \frac{\eta^m}{\lambda^m} q_{j,0}$ or $q_{j,1} = \frac{\lambda^m}{\eta^{m-1}} q_{j,0}$ should hold, which is not possible (or I assume this case out). Thus $\Delta_{j,t} = \frac{\eta^n}{\lambda^m}$ with $n \geqslant m > 0$ is not possible.

case 2)-vi. $\Delta_{j,t} = \frac{\lambda^n}{\eta^m}$ with $n > m > 0$

With a similar argument, this case is not possible.

Therefore $\Delta_{j,t}$ can assume only four values, $\left\{1, \lambda, \eta, \frac{\eta}{\lambda}\right\}$. ■

A.1.2.2 Product Quality Evolution for Outsider Firms

Let's denote z_j^ℓ as an internal innovation intensity for product line j when it's technology gap is $\frac{q_{j,t}}{q_{j,t-1}} = \Delta^\ell$, such that $\Delta^1 = 1$, $\Delta^2 = \lambda$, $\Delta^3 = \eta$, and $\Delta^4 = \frac{\eta}{\lambda}$.

Then product quality in period $t+1$ evolves probabilistically as:

$$q_{j,t+1}(\Delta_t = 1) = \begin{cases} \lambda q_{j,t-1}\ , & \text{with prob. } \ (1-\overline{x})\, z_j^1 \\ q_{j,t-1}\ , & \text{with prob. of } \ (1-\overline{x})\,(1-z_j^1) \\ \eta q_{j,t-1}\ , & \text{with prob. } \ \overline{x}\ , \end{cases}$$

where $q_{j,t-1} = q_{j,t}$,

$$q_{j,t+1}(\Delta_t = \lambda) = \begin{cases} \lambda^2 q_{j,t-1}\ , & \text{with prob. } \ z_j^2 \\ \lambda q_{j,t-1}\ , & \text{with prob. } \ (1-\overline{x})\,(1-z_j^2) \\ \eta\, q_{j,t-1}\ , & \text{with prob. } \ \overline{x}(1-z_j^2)\ , \end{cases}$$

where $q_{j,t-1} = \frac{1}{\lambda} q_{j,t}$,

$$q_{j,t+1}(\Delta_t = 1+\eta) = \begin{cases} \lambda\eta\, q_{j,t-1}\ , & \text{with prob. } \ z_j^3 \\ \eta\, q_{j,t-1}\ , & \text{with prob. } \ (1-\overline{x})(1-z_j^3) + \frac{1}{2}\,\overline{x}\,(1-z_j^3) \\ \eta\, q_{j,t-1}\ , & \text{with prob. } \ \frac{1}{2}\,\overline{x}\,(1-z_j^3)\ , \end{cases}$$

where $q_{j,t-1} = \frac{1}{\eta} q_{j.t}$, and

$$q_{j,t+1}\left(\Delta_t = \frac{\eta}{\lambda}\right) = \begin{cases} \lambda\, \frac{\eta}{\lambda}\, q_{j,t-1}\ , & \text{with prob. } \ (1-\overline{x})\, z_j^4 + \frac{1}{2}\,\overline{x}\, z_j^4 \\ \frac{\eta}{\lambda}\, q_{j,t-1}\ , & \text{with prob. } \ (1-\overline{x})\,(1-z_j^4) \\ \eta q_{j,t-1}\ , & \text{with prob. of } \ \overline{x}\,(1-z_j^4) + \frac{1}{2}\,\overline{x}\, z_j^4\ , \end{cases}$$

where $q_{j,t-1} = \frac{\lambda}{1+\eta} q_{j,t}$.

A.1.2.3 Product Quality Evolution for an Incumbent Firm

For each Δ^{ℓ}, transition dynamics for product quality and technology gap for product line j_i can be represented using two indicator functions I_i^z and $I_i^{\overline{x}}$, where $\Delta'_{j_i} = 0$ (or equivalently $\{q'_{j_i}\} = \phi$) implies firm loses product line j_i in the next period. Here, I write down the expressions as if incumbent firm is doing coin-tossing at all times.

A.1.2.3.1 i) $\Delta_{j_i} = \Delta^1 = 1$

$I_i^{\overline{x}}$	I_i^z	prob. $\frac{1}{2}$ (win)	prob. $\frac{1}{2}$ (lose)
1	0	$\Delta'_{j_i} = 0$	$\Delta'_{j_i} = 0$
1	1	$\Delta'_{j_i} = 0$	$\Delta'_{j_i} = 0$
0	0	$\Delta'_{j_i} = 1$	$\Delta'_{j_i} = 1$
0	1	$\Delta'_{j_i} = \lambda$	$\Delta'_{j_i} = \lambda$

$$\Rightarrow \begin{cases} \Delta'_{j_i} = (1 - I_j^{\overline{x}})(\lambda I_i^z) & \text{prob. } \frac{1}{2} \text{ (win)} \\ \Delta'_{j_i} = (1 - I_j^{\overline{x}})(\lambda I_i^z) & \text{prob. } \frac{1}{2} \text{ (lose)} \\ \{q'_{j_i}\} = \left\{(1 - I_j^{\overline{x}})(\lambda I_i^z)\, q_{j_i}\right\} \backslash \{0\} & \text{prob. } \frac{1}{2} \text{ (win)} \\ \{q'_{j_i}\} = \left\{(1 - I_j^{\overline{x}})(\lambda I_i^z)\, q_{j_i}\right\} \backslash \{0\} & \text{prob. } \frac{1}{2} \text{ (lose)} \end{cases}$$

A.1.2.3.2 ii) $\Delta_{j_i} = \Delta^2 = \lambda$

		prob. $\frac{1}{2}$ (win)	prob. $\frac{1}{2}$ (lose)
$I_i^{\overline{x}}$	I_i^z		
1	0	$\Delta'_{j_i} = 0$	$\Delta'_{j_i} = 0$
1	1	$\Delta'_{j_i} = \lambda$	$\Delta'_{j_i} = \lambda$
0	0	$\Delta'_{j_i} = 1$	$\Delta'_{j_i} = 1$
0	1	$\Delta'_{j_i} = \lambda$	$\Delta'_{j_i} = \lambda$

$$\Rightarrow \begin{cases} \Delta'_{j_i} = \left[1 - (1 - I_i^z) I_j^{\overline{x}}\right] (\lambda I_i^z) & \text{prob. } \frac{1}{2} \text{ (win)} \\ \Delta'_{j_i} = \left[1 - (1 - I_i^z) I_j^{\overline{x}}\right] (\lambda I_i^z) & \text{prob. } \frac{1}{2} \text{ (lose)} \\ \{q'_{j_i}\} = \left\{ \left[1 - (1 - I_i^z) I_j^{\overline{x}}\right] (\lambda I_i^z)\, q_{j_i} \right\} \backslash \{0\} & \text{prob. } \frac{1}{2} \text{ (win)} \\ \{q'_{j_i}\} = \left\{ \left[1 - (1 - I_i^z) I_j^{\overline{x}}\right] (\lambda I_i^z)\, q_{j_i} \right\} \backslash \{0\} & \text{prob. } \frac{1}{2} \text{ (lose)} \end{cases}$$

A.1.2.3.3 iii) $\Delta_{j_i} = \Delta^3 = \eta$

		prob. $\frac{1}{2}$ (win)	prob. $\frac{1}{2}$ (lose)
$I_i^{\overline{x}}$	I_i^z		
1	0	$\Delta'_{j_i} = 1$	$\Delta'_{j_i} = 0$
1	1	$\Delta'_{j_i} = \lambda$	$\Delta'_{j_i} = \lambda$
0	0	$\Delta'_{j_i} = 1$	$\Delta'_{j_i} = 1$
0	1	$\Delta'_{j_i} = \lambda$	$\Delta'_{j_i} = \lambda$

$$\Rightarrow \begin{cases} \Delta'_{j_i} = \lambda I_i^z & \text{prob. } \frac{1}{2} \text{ (win)} \\ \Delta'_{j_i} = \left[1 - (1 - I_i^z) I_j^{\overline{x}}\right] (\lambda I_i^z) & \text{prob. } \frac{1}{2} \text{ (lose)} \\ \{q'_{j_i}\} = \left\{ (\lambda I_i^z)\, q_{j_i} \right\} \backslash \{0\} & \text{prob. } \frac{1}{2} \text{ (win)} \\ \{q'_{j_i}\} = \left\{ \left[1 - (1 - I_i^z) I_j^{\overline{x}}\right] (\lambda I_i^z)\, q_{j_i} \right\} \backslash \{0\} & \text{prob. } \frac{1}{2} \text{ (lose)} \end{cases}$$

A.1.2.3.4 iv) $\Delta_{j_i} = \Delta^4 = \frac{\eta}{\lambda}$

		prob. $\frac{1}{2}$ (win)	prob. $\frac{1}{2}$ (lose)
$I_i^{\overline{x}}$	I_i^z		
1	0	$\Delta'_{j_i} = 0$	$\Delta'_{j_i} = 0$
1	1	$\Delta'_{j_i} = \lambda$	$\Delta'_{j_i} = 0$
0	0	$\Delta'_{j_i} = 1$	$\Delta'_{j_i} = 1$
0	1	$\Delta'_{j_i} = \lambda$	$\Delta'_{j_i} = \lambda$

$$\Rightarrow \begin{cases} \Delta'_{j_i} = \left[1 - (1 - I_i^z) I_j^{\overline{x}}\right] (\lambda I_i^z) & \text{prob. } \frac{1}{2} \text{ (win)} \\ \Delta'_{j_i} = (1 - I_i^{\overline{x}})(\lambda I_i^z) & \text{prob. } \frac{1}{2} \text{ (lose)} \\ \{q'_{j_i}\} = \left\{ \left[1 - (1 - I_i^z) I_j^{\overline{x}}\right] (\lambda I_i^z)\, q_{j_i} \right\} \backslash \{0\} & \text{prob. } \frac{1}{2} \text{ (win)} \\ \{q'_{j_i}\} = \left\{ (1 - I_i^{\overline{x}})(\lambda I_i^z)\, q_{j_i} \right\} \backslash \{0\} & \text{prob. } \frac{1}{2} \text{ (lose)} \end{cases}$$

A.1.3 Value Function and Optimal Innovation Decisions

Conditional expectation inside of the expression for the value function is over the success/failure of internal and external innovation, creative destruction shock arrival, winning/losing from coin-tosses (c-t), the current period product quality q distribution, and the current period technology gap Δ^{ℓ} distribution. Thus $\mathbb{E}\left[V\left(\Phi^{f\prime} \mid \Phi^{f}\right) \middle| \{z_j\}_{j\in\mathcal{J}^f}, x\right]$ is equal to

$$
\begin{aligned}
&\sum_{I_1^{\overline{x}}, I_2^{\overline{x}}, \ldots, I_{n_f}^{\overline{x}}=0}^{1} \sum_{I_1^{z}, \ldots, I_{n_f}^{z}=0}^{1} \sum_{\text{c-t}_1, \ldots, \text{c-t}_{n_f} = \text{win}}^{\text{lose}} \sum_{I^x=0}^{1} \left[\prod_{i=1}^{n_f} \overline{x}^{I_i^{\overline{x}}} (1-\overline{x})^{1-I_i^{\overline{x}}} z_i^{I_i^z} (1-z_i)^{1-I_i^z}\right] \\
&\times \left[x^{I^x}(1-x)^{1-I^x}\right] \left(\frac{1}{2}\right)^{n_f} \\
&\times \mathbb{E}_{q,\Delta}\, V\left(\left[\bigcup_{i=1}^{n_f}\left[\left\{\left(\Delta'_{j_i} q_{j_i}, \Delta'_{j_i}\right) \mid \left(q_{j_i}, \Delta_{j_i}\right), I_i^{\overline{x}}, I_i^{z}, \text{c-t}_i\right\} \backslash \{\mathbf{0}\}\right]\right]\right. \\
&\qquad\qquad \left.\bigcup \left[\left\{\left(\frac{\eta}{\Delta_{-j}}\, I^x\, q_{-j},\ \frac{\eta}{\Delta_{-j}} I^x\right)\right\} \backslash \{\mathbf{0}\}\right]\right).
\end{aligned}
$$

The first term inside of the value function, $\bigcup_{i=1}^{n_f}\left[\left\{\left(\Delta'_{j_i} q_{j_i}, \Delta'_{j_i}\right) \mid \left(q_{j_i}, \Delta_{j_i}\right), I_i^{\overline{x}}, I_i^{z}, \text{c-t}_i\right\} \backslash \{\mathbf{0}\}\right]$, depicts subsets of possible realizations for $\Phi^{f\prime}$ from internal innovation, creative destruction, and coin-toss, and the second term, $\left\{\left(\frac{\eta}{\Delta_{-j}}\, I^x\, q_{-j},\ \frac{\eta}{\Delta_{-j}} I^x\right)\right\} \backslash \{\mathbf{0}\}$, depicts subsets of possible realizations for $\Phi^{f\prime}$ from external innovation, where $\{q'_{j_i}\} = \{\Delta'_{j_i} q_{j_i}\} \backslash \{0\}$, and $\{q'_{-j}\} = \{\frac{\eta}{\Delta_{-j}}\, I^x\, q_{-j}\} \backslash \{0\}$. If $\Delta'_{j_i} = 0$, then firm f loses product line j_i and $\{(q'_{j_i}, \Delta'_{j_i})\} \backslash \{\mathbf{0}\} = \{\mathbf{0}\} \backslash \{\mathbf{0}\} = \varnothing$.

A.1.3.1 Proof of Proposition 1

Proof. Due to the linearity of expectation, $\sum_{\ell=1}^{4} A_\ell \sum_{j\in\mathcal{J}^f|\Delta_j=\Delta^\ell} q_j$ portion of conjectured value function from $\mathbb{E}\left[V\left(\Phi^{f\prime} \mid \Phi^f\right) \middle| \{z_j\}_{j\in\mathcal{J}^f}, x\right]$ can be written as

$$\mathbb{E}\left[\sum_{\ell=1}^{4} A_\ell \sum_{j\in\mathcal{J}^{f\prime}|\Delta'_j=\Delta^\ell} q'_j\right] = \mathbb{E}\left[\sum_{\ell=1}^{2} A_\ell \sum_{j\in\mathcal{J}^f|(\Delta'_j|\Delta_j)=\Delta^\ell} \Delta^\ell q_j\right] + \mathbb{E}\left[\sum_{\ell=1}^{4} A_\ell\, I_{\left\{\frac{\eta}{\Delta_j}=\Delta^\ell\right\}} \frac{\eta}{\Delta_j}\, q_j\right],$$

where the first term is expected value from existing product lines and the second term is expected value from a new product line added through external innovation.

Since realization of internal innovation success/failure and creative destruction shock are independent from realization of external innovation success/failure, expected value from a new product line is

$$\begin{aligned}
\mathbb{E}\left[\sum_{\ell=1}^{4} A_\ell\, I_{\left\{\frac{\eta}{\Delta_j}=\Delta^\ell\right\}} \frac{\eta}{\Delta_j}\, q_j\right] &= \sum_{I^x=0}^{1} x^{I^x}(1-x)^{1-I^x}\mathbb{E}_{q_j,\Delta_j}\left[\sum_{\ell=1}^{4} A_\ell\, I_{\left\{\frac{\eta}{\Delta_j}=\Delta^\ell\right\}}\, I^x \frac{\eta}{\Delta_j}\, q_j\right] \\
&= x\,\mathbb{E}_{q_j}\Bigg[\frac{1}{2}(1-z^3)A_1\mu(\Delta^3) + \left(1-\frac{1}{2}z^4\right)A_2\lambda\mu(\Delta^4) \\
&\qquad + A_3\eta\mu(\Delta^1) + (1-z^2)A_4\frac{\eta}{\lambda}\mu(\Delta^2)\Bigg]\, q_j \\
&= x\Bigg[\frac{1}{2}(1-z^3)A_1\mu(\Delta^3) + \left(1-\frac{1}{2}z^4\right)A_2\lambda\mu(\Delta^4) + A_3\eta\mu(\Delta^1) \\
&\qquad + (1-z^2)A_4\frac{\eta}{\lambda}\mu(\Delta^2)\Bigg]\,\overline{q}\,.
\end{aligned}$$

The second equality follows from the fact that randomly chosen product line with a quality q_j can have technology gap Δ^ℓ with the probability $\mu(\Delta^\ell)$ and probability of taking over this product line depends on its technology gap. The third equality follows by integrating product quality over all product line indices.[1]

First expectation can further divided into four cases, depending on current period technology gap Δ:

$$\mathbb{E}\left[\sum_{\ell=1}^{2} A_\ell \sum_{j\in\mathcal{J}^f|(\Delta'_j|\Delta_j=\Delta^\ell)} \Delta^\ell q_j\right] = \sum_{\tilde{\ell}=1}^{4} \mathbb{E}\left[\sum_{\ell=1}^{2} A_\ell \sum_{j\in\mathcal{J}^f|(\Delta'_j|\Delta_j=\Delta^{\tilde{\ell}})=\Delta^\ell} \Delta^\ell q_j\right] .$$

To make formulas easy to write, let's re-order the product quality portfolio q_j according to technology gap Δ^ℓ and renumber them according to:

$$q^f = \Big\{ \underbrace{q_{j_1},\, q_{j_2},\, \ldots,\, q_{j_{n_f^1}}}_{\Delta^1},\, \underbrace{q_{j_{n_f^1+1}},\, \ldots,\, q_{j_{n_f^1+n_f^2}}}_{\Delta^2},\, \underbrace{q_{j_{n_f^1+n_f^2+1}},\, \ldots,\, q_{j_{n_f^1+n_f^2+n_f^3}}}_{\Delta^3},$$
$$\underbrace{q_{j_{n_f^1+n_f^2+n_f^3+1}},\, \ldots,\, q_{j_{n_f^1+n_f^2+n_f^3+n_f^4}}}_{\Delta^4} \Big\} .$$

Then for $i = 1, 2, \ldots, n_f^1$ ($\Delta_{j_i} = \Delta^1 = 1$),

$$\mathbb{E}\left[\sum_{\ell=1}^{2} A_\ell \sum_{j_i\in\mathcal{J}^f|(\Delta'_{j_i}|\Delta_{j_i}=\Delta^1)=\Delta^\ell} \Delta^\ell q_{j_i}\right] = \sum_{i=1}^{n_f^1} \Big[A_1(1-\overline{x})(1-z_i^1) + \lambda A_2(1-\overline{x})z_i^1\Big] q_{j_i} ,$$

for $i = n_f^1 + 1, \ldots, n_f^1 + n_f^2$ ($\Delta_{j_i} = \Delta^2 = \lambda$),

$$\mathbb{E}\left[\sum_{\ell=1}^{2} A_\ell \sum_{j_i\in\mathcal{J}^f|(\Delta'_{j_i}|\Delta_{j_i}=\Delta^2)=\Delta^\ell} \Delta^\ell q_{j_i}\right] = \sum_{i=n_f^1+1}^{n_f^1+n_f^2} \Big[A_1(1-\overline{x})(1-z_i^2) + \lambda A_2 z_i^2\Big] q_{j_i} ,$$

[1] Only the share of technology gap $\{\mu(\Delta^\ell)\}_{\ell=1}^4$ and average quality $\overline{q}$ are contained in individual firm's information set in terms of firm distribution. That is, for an individual firm, technology gap and product quality are independent.

for $i = n_f^1 + n_f^2 + 1, \dots, n_f - n_f^4$ $(\Delta_{j_i} = \Delta^3 = \eta)$,

$$\mathbb{E}\left[\sum_{\ell=1}^{2} A_\ell \sum_{j_i \in \mathcal{J}^f | (\Delta'_{j_i} | \Delta_{j_i} = \Delta^3) = \Delta^\ell} \Delta^\ell q_{j_i}\right] = \sum_{i=n_f^1+n_f^2+1}^{n_f - n_f^4} \left[A_1 \left(1 - \frac{1}{2}\overline{x}\right)(1 - z_i^3) + \lambda A_2 z_i^3\right] q_{j_i} \,,$$

and for $i = n_f - n_f^4 + 1, \dots, n_f$ $(\Delta_{j_i} = \Delta^4 = \frac{\eta}{\lambda})$,

$$\mathbb{E}\left[\sum_{\ell=1}^{2} A_\ell \sum_{j_i \in \mathcal{J}^f | (\Delta'_{j_i} | \Delta_{j_i} = \Delta^4) = \Delta^\ell} \Delta^\ell q_{j_i}\right] = \sum_{i=n_f - n_f^4}^{n_f} \left[A_1 (1 - \overline{x})(1 - z_i^4) + \lambda A_2 \left(1 - \frac{1}{2}\overline{x}\right) z_i^4\right] q_{j_i} \,.$$

$B\overline{q}$ portion of conjectured value function from $\mathbb{E}\left[V\left(\Phi^{f\prime} \mid \Phi^f\right) \Big| \{z_j\}_{j \in \mathcal{J}^f}, x\right]$ can be written as

$$\mathbb{E}B\overline{q}' = B(1+g)\overline{q} \,,$$

where g is a growth rate of product qualities in balanced growth path (BGP). Thus by plugging in the conjectured value function, the original value function can be written as

$$\sum_{i=1}^{n_f^1} A_1 q_{j_i} + \sum_{i=n_f^1+1}^{n_f^1+n_f^2} A_2 q_{j_i} + \sum_{i=n_f^1+n_f^2+1}^{n_f - n_f^4} A_3 q_{j_i} + \sum_{i=n_f - n_f^4+1}^{n_f} A_4 q_{j_i} + B\overline{q} =$$

$$\max_{\substack{x\in[0,\bar{x}],\\ \{z_i\in[0,\bar{z}]\}_{i=1}^{n_f}}} \left\{ \begin{array}{l} \sum_{i=1}^{n_f}\left[\pi q_{j_i} - \hat{\chi} z_i^{\hat{\psi}} q_{j_i}\right] - \bar{q}\widetilde{\chi} x^{\widetilde{\psi}} \\ \\ +\widetilde{\beta}\sum_{i=1}^{n_f^1}\left[A_1(1-\overline{x})(1-z_i^1) + \lambda A_2(1-\overline{x}) z_i^1\right] q_{j_i} \\ \\ +\widetilde{\beta}\sum_{i=n_f^1+1}^{n_f^1+n_f^2}\left[A_1(1-\overline{x})(1-z_i^2) + \lambda A_2 z_i^2\right] q_{j_i} \\ \\ +\widetilde{\beta}\sum_{i=n_f^1+n_f^2+1}^{n_f-n_f^4}\left[A_1\left(1-\frac{1}{2}\overline{x}\right)(1-z_i^3) + \lambda A_2 z_i^3\right] q_{j_i} \\ \\ +\widetilde{\beta}\sum_{i=n_f-n_f^4}^{n_f}\left[A_1(1-\overline{x})(1-z_i^4) + \lambda A_2\left(1-\frac{1}{2}\overline{x}\right) z_i^4\right] q_{j_i} \\ \\ +\widetilde{\beta} x \left[\frac{1}{2}(1-z^3)A_1\mu(\Delta^3) + \left(1-\frac{1}{2}z^4\right)A_2\lambda\mu(\Delta^4)\right. \\ \\ \qquad\qquad \left. +A_3\eta\mu(\Delta^1) + (1-z^2)A_4\frac{\eta}{\lambda}\mu(\Delta^2)\right]\overline{q} \\ \\ +\widetilde{\beta}B(1+g)\overline{q} \end{array} \right\}$$

Optimal innovation intensities from FONCs are

$$\frac{\partial}{\partial z_i^1} : -\widehat{\psi}\widehat{\chi}(z_i^1)^{\widehat{\psi}-1} q_{j_i} + \widetilde{\beta}(1-\overline{x})\left[\lambda A_2 - A_1\right] q_{j_i} = 0$$

$$\Rightarrow z^1 = \left[\frac{\widetilde{\beta}(1-\overline{x})\left[\lambda A_2 - A_1\right]}{\widehat{\psi}\widehat{\chi}}\right]^{\frac{1}{\widehat{\psi}-1}}$$

$$\frac{\partial}{\partial z_i^2} : -\widehat{\psi}\widehat{\chi}(z_i^2)^{\widehat{\psi}-1} q_{j_i} + \widetilde{\beta}\left[\lambda A_2 - (1-\overline{x})A_1\right] q_{j_i} = 0$$

$$\Rightarrow z^2 = \left[\frac{\widetilde{\beta}\left[\lambda A_2 - (1-\overline{x})A_1\right]}{\widehat{\psi}\widehat{\chi}}\right]^{\frac{1}{\widehat{\psi}-1}}$$

$$\frac{\partial}{\partial z_i^3} : -\widehat{\psi}\widehat{\chi}(z_i^3)^{\widehat{\psi}-1} q_{j_i} + \widetilde{\beta}\left[\lambda A_2 - \left(1-\frac{1}{2}\overline{x}\right)A_1\right] q_{j_i} = 0$$

$$\Rightarrow z^3 = \left[\frac{\widetilde{\beta}\left[\lambda A_2 - \left(1-\frac{1}{2}\overline{x}\right)A_1\right]}{\widehat{\psi}\widehat{\chi}}\right]^{\frac{1}{\widehat{\psi}-1}}$$

$$\frac{\partial}{\partial z_i^4}: - \widehat{\psi}\widehat{\chi}(z_i^4)^{\widehat{\psi}-1} q_{j_i} + \widetilde{\beta}\left[\lambda\left(1-\frac{1}{2}\overline{x}\right)A_2 - (1-\overline{x})A_1\right] q_{j_i} = 0$$

$$\Rightarrow z^4 = \left[\frac{\widetilde{\beta}\left[\lambda\left(1-\frac{1}{2}\overline{x}\right)A_2 - (1-\overline{x})A_1\right]}{\widehat{\psi}\widehat{\chi}}\right]^{\frac{1}{\widehat{\psi}-1}}$$

$$\frac{\partial}{\partial x}: - \widetilde{\psi}\widetilde{\chi}\overline{q}x^{\widetilde{\psi}-1}$$
$$+ \widetilde{\beta}\Bigg[\frac{1}{2}(1-z^3)A_1\mu(\Delta^3) + \left(1-\frac{1}{2}z^4\right)A_2\lambda\mu(\Delta^4) + A_3\eta\mu(\Delta^1) + (1-z^2)A_4\frac{\eta}{\lambda}\mu(\Delta^2)\Bigg]\overline{q}$$
$$= 0$$

$$\Rightarrow x = \left[\frac{\widetilde{\beta}\left[\frac{(1-z^3)A_1\mu(\Delta^3)}{2} + \left(1-\frac{z^4}{2}\right)A_2\lambda\mu(\Delta^4) + A_3\eta\mu(\Delta^1) + (1-z^2)A_4\frac{\eta}{\lambda}\mu(\Delta^2)\right]}{\widetilde{\psi}\widetilde{\chi}}\right]^{\frac{1}{\widetilde{\psi}-1}}$$

By plugging in optimal innovation intensities and equating the LHS to the RHS, we get the five coefficients of the conjectured value function of the form

$$A_1 = \pi - \widehat{\chi}(z^1)^{\widehat{\psi}} + \widetilde{\beta}\Big[A_1(1-\overline{x})(1-z^1) + \lambda A_2(1-\overline{x})z^1\Big]$$

$$A_2 = \pi - \widehat{\chi}(z^2)^{\widehat{\psi}} + \widetilde{\beta}\Big[A_1(1-\overline{x})(1-z^2) + \lambda A_2 z^2\Big]$$

$$A_3 = \pi - \widehat{\chi}(z^3)^{\widehat{\psi}} + \widetilde{\beta}\left[A_1\left(1-\frac{1}{2}\overline{x}\right)(1-z^3) + \lambda A_2 z^3\right]$$

$$A_4 = \pi - \widehat{\chi}(z^4)^{\widehat{\psi}} + \widetilde{\beta}\left[A_1(1-\overline{x})(1-z^4) + \lambda A_2\left(1-\frac{1}{2}\overline{x}\right)z^4\right]$$

$$B = \frac{1}{1-\widetilde{\beta}(1+g)}\Bigg[\widetilde{\beta}x\left[\frac{1}{2}(1-z^3)A_1\mu(\Delta^3) + \left(1-\frac{1}{2}z^4\right)A_2\lambda\mu(\Delta^4) + A_3\eta\mu(\Delta^1)\right.$$
$$\left. + (1-z^2)A_4\frac{\eta}{\lambda}\mu(\Delta^2)\right] - \widetilde{\chi}(x)^{\widetilde{\psi}}\Bigg]$$
$$= \frac{1}{1-\widetilde{\beta}(1+g)}\left(\widetilde{\psi}\widetilde{\chi}\right)^{-\frac{1}{\widetilde{\psi}-1}}\left(1-\frac{1}{\widetilde{\psi}}\right)\Bigg[\widetilde{\beta}\Bigg[\frac{1}{2}(1-z^3)A_1\mu(\Delta^3) + \left(1-\frac{1}{2}z^4\right)A_2\lambda\mu(\Delta^4)$$
$$+ A_3\eta\mu(\Delta^1) + (1-z^2)A_4\frac{\eta}{\lambda}\mu(\Delta^2)\Bigg]\Bigg]^{\frac{\widetilde{\psi}}{\widetilde{\psi}-1}}.$$

■

A.1.3.2 Proof of Corollary 1

Proof. Define $\widetilde{z^\ell} = \frac{\hat{\psi}\hat{\chi}}{\widetilde{\beta}} \left(z^\ell\right)^{(\hat{\psi}-1)}$. Then $z^\ell > z^{\ell'} \Leftrightarrow \widetilde{z^\ell} > \widetilde{z^{\ell'}}$ for $\ell, \ell' \in [1,4] \cap \mathbb{Z}$ with $\widehat{\psi} > 1$. Since $\widetilde{z^2} - \widetilde{z^3} = \frac{1}{2}\overline{x}A_1 > 0$, $\widetilde{z^2} - \widetilde{z^1} = \overline{x}\lambda A_2 > 0$, $\widetilde{z^2} - \widetilde{z^4} = \frac{1}{2}\overline{x}\lambda A_2 > 0$, and $\widetilde{z^4} - \widetilde{z^1} = \frac{1}{2}\overline{x}\lambda A_2 > 0$, we have $z^2 > z^3$, $z^2 > z^1$, $z^2 > z^4$, and $z^4 > z^1$. Now, if we know the sign for $\widetilde{z^3} - \widetilde{z^4} = \frac{1}{2}\overline{x}\left[\lambda A_2 - A_1\right]$ then we know the entire relationships among $\{z^\ell\}_{\ell=1}^4$. But in an equilibrium, $\widetilde{z^1} = (1-\overline{x})\left[\lambda A_2 - A_1\right] > 0$ should hold, which implies $\lambda A_2 - A_1 > 0$. Thus $\widetilde{z^3} > \widetilde{z^4} \Leftrightarrow z^3 > z^4$. Therefore, $z^2 > z^3 > z^4 > z^1$.

■

A.1.3.3 Proof of Corollary 2

Proof. The partial derivatives of $\{z^\ell\}_{\ell=1}^4$ w.r.t. $\overline{x}$, holding A_1 and A_2 fixed are

$$
\begin{aligned}
\left.\frac{\partial z^1}{\partial \overline{x}}\right|_{A_1,A_2} &: \quad -\frac{\widetilde{\beta}}{\widehat{\psi}\widehat{\chi}}\,(z^1)^{2-\hat{\psi}}\left[\,\lambda A_2 - A_1\,\right] < 0 \\
\left.\frac{\partial z^2}{\partial \overline{x}}\right|_{A_1,A_2} &: \quad \frac{\widetilde{\beta}}{\widehat{\psi}\widehat{\chi}}\,(z^2)^{2-\hat{\psi}}\,A_1 > 0 \\
\left.\frac{\partial z^3}{\partial \overline{x}}\right|_{A_1,A_2} &: \quad \frac{\widetilde{\beta}}{\widehat{\psi}\widehat{\chi}}\,(z^3)^{2-\hat{\psi}}\,\frac{1}{2}A_1 > 0 \\
\left.\frac{\partial z^4}{\partial \overline{x}}\right|_{A_1,A_2} &: \quad -\frac{\widetilde{\beta}}{\widehat{\psi}\widehat{\chi}}\,(z^4)^{2-\hat{\psi}}\left[\,\frac{1}{2}\lambda A_2 - A_1\,\right] \gtrless 0\,.
\end{aligned}
$$

Since we know $\lambda A_2 - A_1 > 0$, $\left.\frac{\partial z^1}{\partial \overline{x}}\right|_{A_1,A_2}$ should be negative. Also, since $z^2 > z^3$, $\left.\frac{\partial z^2}{\partial \overline{x}}\right|_{A_1,A_2} > \left.\frac{\partial z^3}{\partial \overline{x}}\right|_{A_1,A_2}$. Since $z^3 > z^4$ and $A_1 > A_1 - \frac{1}{2}\lambda A_2$, $\left.\frac{\partial z^3}{\partial \overline{x}}\right|_{A_1,A_2} > \left.\frac{\partial z^4}{\partial \overline{x}}\right|_{A_1,A_2}$ but the sign for $\frac{1}{2}\lambda A_2 - A_1$ is ambiguous. ■

A.1.4 Potential Startups

By plugging in the value function defined in the previous section, the expected term becomes

$$\begin{aligned}
\mathbb{E}V(\{(q_j', \Delta_j')\}) = \mathbb{E}_{q_j}\Bigg[& \frac{1}{2}x_e(1-z^3)\Big[A_1 q_j + B\overline{q}'\Big]\mu(\Delta^3) + x_e\left(1-\frac{1}{2}z^4\right)\Big[A_2\lambda g_j + B\overline{q}'\Big]\mu(\Delta^4) \\
& + x_e\Big[A_3\eta q_j + B\overline{q}'\Big]\mu(\Delta^1) + x_e(1-z^2)\Big[A_4\frac{\eta}{\lambda}q_j + B\overline{q}'\Big]\mu(\Delta^2)\Bigg] \\
= x_e\Bigg[& \frac{1}{2}(1-z^3)A_1\mu(\Delta^3) + \left(1-\frac{1}{2}z^4\right)A_2\lambda\mu(\Delta^4) + A_3\eta\mu(\Delta^1) \\
& + (1-z^2)A_4\frac{\eta}{\lambda}\mu(\Delta^2)\Bigg]\overline{q} + x_e\Bigg[\frac{1}{2}(1-z^3)\mu(\Delta^3) + \left(1-\frac{1}{2}z^4\right)\mu(\Delta^4) \\
& + \mu(\Delta^1) + (1-z^2)\mu(\Delta^2)\Bigg]B(1+g)\overline{q}\,.
\end{aligned}$$

Thus from FOSC, optimal external innovation intensity for potential startups x_e is

$$\begin{aligned}
x_e = \Bigg[\Bigg[& \left(\frac{1}{2}(1-z^3)A_1\mu(\Delta^3) + \left(1-\frac{1}{2}z^4\right)A_2\lambda\mu(\Delta^4) + A_3\eta\mu(\Delta^1) + (1-z^2)A_4\frac{\eta}{\lambda}\mu(\Delta^2)\right) \\
& + \left(\frac{1}{2}(1-z^3)\mu(\Delta^3) + \left(1-\frac{1}{2}z^4\right)\mu(\Delta^4) + \mu(\Delta^1) + (1-z^2)\mu(\Delta^2)\right)B(1+g)\Bigg] \\
& \times \frac{\widetilde{\beta}}{\widetilde{\psi}_e\,\widetilde{\chi}_e}\Bigg]^{\frac{1}{\widetilde{\psi}_e - 1}}.
\end{aligned}$$

A.1.5 Growth rate

A.1.5.1 Proof of Proposition 2

Proof. In this model economy, output growth rate is equal to product quality growth rate. Pick any q_j. Then it's technology gap is equal to $\Delta_j = \Delta^\ell$ with the probability $\mu(\Delta^\ell)$ and the probability of Δ'_j becoming a certain technology gap depends on this.

$$
\begin{aligned}
\text{If } \Delta_j = \Delta^1, \quad & q'_j = \Delta^1 q_j \quad \text{w/ prob.} \quad (1-\overline{x})(1-z^1) \\
& q'_j = \Delta^2 q_j \quad \text{w/ prob.} \quad (1-\overline{x})z^1 \\
& q'_j = \Delta^3 q_j \quad \text{w/ prob.} \quad \overline{x} \\
& q'_j = \Delta^4 q_j \quad \text{w/ prob.} \quad 0 \\
\text{If } \Delta_j = \Delta^2, \quad & q'_j = \Delta^1 q_j \quad \text{w/ prob.} \quad (1-\overline{x})(1-z^2) \\
& q'_j = \Delta^2 q_j \quad \text{w/ prob.} \quad z^2 \\
& q'_j = \Delta^3 q_j \quad \text{w/ prob.} \quad 0 \\
& q'_j = \Delta^4 q_j \quad \text{w/ prob.} \quad \overline{x}(1-z^2) \\
\text{If } \Delta_j = \Delta^3, \quad & q'_j = \Delta^1 q_j \quad \text{w/ prob.} \quad 1-z^3 \\
& q'_j = \Delta^2 q_j \quad \text{w/ prob.} \quad z^3 \\
& q'_j = \Delta^3 q_j \quad \text{w/ prob.} \quad 0 \\
& q'_j = \Delta^4 q_j \quad \text{w/ prob.} \quad 0
\end{aligned}
$$

$$
\begin{aligned}
\text{If } \Delta_j = \Delta^4, \quad q_j' &= \Delta^1 q_j \quad \text{w/ prob.} \quad (1-\overline{x})(1-z^4) \\
q_j' &= \Delta^2 q_j \quad \text{w/ prob.} \quad z^4 + \overline{x}(1-z^4) \\
q_j' &= \Delta^3 q_j \quad \text{w/ prob.} \quad 0 \\
q_j' &= \Delta^4 q_j \quad \text{w/ prob.} \quad 0
\end{aligned}
$$

Thus

$$
\begin{aligned}
\mathbb{E}\big[q_j' \mid q_j\big] =& \Bigg[\Big[(1-\overline{x})(1-z^1) + \lambda(1-\overline{x})z^1 + \eta\overline{x}\Big]\mu(\Delta^1) \\
&+ \Big[(1-\overline{x})(1-z^2) + \lambda z^2 + \frac{\eta}{\lambda}\overline{x}(1-z^2)\Big]\mu(\Delta^2) + \Big[1 - z^3 + \lambda z^3\Big]\mu(\Delta^3) \\
&+ \Big[(1-\overline{x})(1-z^4) + \lambda(z^4 + \overline{x}(1-z^4))\Big]\mu(\Delta^4)\Bigg] q_j \ ,
\end{aligned}
$$

and

$$
\begin{aligned}
g =& \Bigg[\Big[(1-\overline{x})(1-z^1) + \lambda(1-\overline{x})z^1 + \eta\overline{x}\Big]\mu(\Delta^1) \\
&+ \Big[(1-\overline{x})(1-z^2) + \lambda z^2 + \frac{\eta}{\lambda}\overline{x}(1-z^2)\Big]\mu(\Delta^2) + \Big[1 - z^3 + \lambda z^3\Big]\mu(\Delta^3) \\
&+ \Big[(1-\overline{x})(1-z^4) + \lambda(z^4 + \overline{x}(1-z^4))\Big]\mu(\Delta^4)\Bigg] - 1 \ .
\end{aligned}
$$

The decomposition follows from the straightforward application of the definition of $\overline{x}$ and product quality evolution. ■

A.1.6 Technology Gap Portfolio Composition Distribution Transition

Let's define technology gap portfolio composition with $n_f - k$ number of $\Delta = \Delta^1$, k number of $\Delta = \Delta^2$, zero number of $\Delta = \Delta^3$ and zero number of $\Delta = \Delta^4$ as $\widetilde{\mathcal{N}}(n_f, k) \equiv (n_f, n_f - k, k, 0, 0)$, for $k \in [0, n_f] \cap \mathbb{Z}$, $n_f > 0$. Then without considering external innovation, probability of $\mathcal{N} = \widetilde{\mathcal{N}}(n_f, k)$ becoming $\mathcal{N}' = \widetilde{\mathcal{N}}(n_f, \widetilde{k})$ can be computed as

$$\widetilde{\mathbb{P}}\Big(n_f, \widetilde{k} \mid n_f, k\Big) =$$

$$\begin{cases} \sum_{\widetilde{k}^1 = \max\{0,\, \widetilde{k}-k\}}^{\min\{n_f - k,\, \widetilde{k}\}} \begin{pmatrix} n_f - k \\ \widetilde{k}^1 \end{pmatrix} \begin{pmatrix} k \\ \widetilde{k} - \widetilde{k}^1 \end{pmatrix} \\ \qquad \times \begin{bmatrix} (1-\overline{x})^{n_f - (\widetilde{k} - \widetilde{k}^1)} \, (1 - z^1)^{n_f - k - \widetilde{k}^1} \, (z^1)^{\widetilde{k}^1} \\ \times (1 - z^2)^{k - (\widetilde{k} - \widetilde{k}^1)} \, (z^2)^{\widetilde{k} - \widetilde{k}^1} \end{bmatrix} & \text{for } n_f \geqslant 1, \text{ and } 0 \leqslant \widetilde{k}, k \leqslant n_f \\ 0 & \text{otherwise} \end{cases}$$

where

$$\begin{pmatrix} n \\ k \end{pmatrix} = \frac{n!}{k!(n-k)!}$$

is a combination of selecting k elements from n elements without repetition, where the order of selection does not matter. Range for $\widetilde{k}^1$ is of the form described as above due to the fact that

i. For $0 \leqslant \widetilde{k} \leqslant \min\{n_f - k, k\}$ case, the two combinations are well defined for any $\widetilde{k}^1 \in [0, \widetilde{k}] \cap \mathbb{Z}$ and describes all the possible cases.

ii. For $n_f - k \geqslant k$ case, $\widetilde{k} > k$, $0 \leqslant \widetilde{k} - \widetilde{k}^1$, and $0 \leqslant \widetilde{k}^1 \leqslant n_f - k$ should be satisfied. Thus $\widetilde{k} - k \leqslant \widetilde{k}^1 \leqslant \widetilde{k}$.

iii. For $k \geqslant n_f - k$ case, $\widetilde{k} > n_f - k$, $0 \leqslant \widetilde{k} - \widetilde{k}^1$, and $0 \leqslant \widetilde{k}^1 \leqslant n_f - k$ should be satisfied. Thus $\max\{0,\ \widetilde{k} - k\} \leqslant \widetilde{k}^1 \leqslant n_f - k$.

By using $\widetilde{\mathbb{P}}(n_f, \widetilde{k} \,|\, n_f, k)$, probability of $\mathcal{N} = \widetilde{\mathcal{N}}(n_f,\ k)$ becoming $\mathcal{N}' = \widetilde{\mathcal{N}}(n_f - h,\ \widetilde{k})$ for any $h \geqslant 0$ without considering external innovation can be defined as follows. Take out h^1 number of product lines with $\Delta = \Delta^1$, and $h - h^1$ number of product lines with $\Delta = \Delta^2$ from $\widetilde{\mathcal{N}}(n_f,\ k)$, then compute the probability of $\widetilde{\mathcal{N}}(n_f - h,\ k - (h - h^1))$ becoming $\widetilde{\mathcal{N}}(n_f - h,\ \widetilde{k})$ by using $\widetilde{\mathbb{P}}\Big(n_f - h,\ \widetilde{k} \,|\, n_f - h,\ k - (h - h^1)\Big)$ for all feasible h^1:

$$\widetilde{\mathbb{P}}\Big(n_f - h,\ \widetilde{k} \,|\, n_f,\ k\Big) =$$

$$\begin{cases} \sum_{h^1=\max\{0,\, h-k\}}^{\min\{h,\, n_f-k\}} \Bigg[\begin{pmatrix} n_f - k \\ h^1 \end{pmatrix} \begin{pmatrix} k \\ h - h^1 \end{pmatrix} \overline{x}^h (1-z^2)^{h-h^1} & \\ \qquad \times \widetilde{\mathbb{P}}\Big(n_f - h,\ \widetilde{k} \,|\, n_f - h,\ k - (h - h^1)\Big) \Bigg] & \text{for } 0 \leqslant h < n_f, \\ & n_f \geqslant 1, \\ & 0 \leqslant \widetilde{k} \leqslant n_f - h, \\ & \text{and } 0 \leqslant k \leqslant n_f \\ \overline{x}^{n_f}\,(1-z^2)^k & \text{for } h = n_f \geqslant 1, \\ & \widetilde{k} = 0, \\ & \text{and } 0 \leqslant k \leqslant n_f \\ 0 & \text{otherwise.} \end{cases}$$

Range for h^1 is defined as above, due to the fact that for any h^1, $0 \leqslant h - h^1 \leqslant k$ and $0 \leqslant h^1 \leqslant n_f - k$ should be satisfied.

By using $\widetilde{\mathbb{P}}\Big(n_f - h,\ \widetilde{k} \,|\, n_f,\ k\Big)$, other possible technology gap portfolio composition transition probabilities can be described conveniently.

1-i. Probability of $\mathcal{N} = (n_f, n_f - k, k, 0, 0)$ becoming $\mathcal{N}' = (n_f - h, n_f - h - \widetilde{k}, \widetilde{k}, 0, 0)$ for $h \geqslant -1$ is defined as

$$\mathbb{P}\Big(n_f - h, n_f - h - \widetilde{k}, \widetilde{k}, 0, 0 \ \Big|\, n_f, n_f - k, k, 0, 0\Big) =$$

$$\widetilde{\mathbb{P}}\Big(n_f - h, \widetilde{k} \,\Big|\, n_f, k\Big)\,(1 - x\overline{x}_{takeover})$$
$$+\, \widetilde{\mathbb{P}}\Big(n_f - h - 1, \widetilde{k} \,\Big|\, n_f, k\Big)\, \mu(\Delta^3)\, \frac{1}{2}\, x\,(1 - z^3)$$

$$+ \widetilde{\mathbb{P}}\Big(n_f - h - 1, \widetilde{k} - 1 \,\Big|\, n_f, k\Big)\, \mu(\Delta^4)\, x\, \left(1 - \frac{1}{2} z^4\right) .$$

The first term is the probability of $\mathcal{N}$ becoming $\mathcal{N}'$ directly via firm's existing technology gap portfolio composition change, while external innovation fails. The second term is the probability of $\mathcal{N}$ becoming $\widetilde{\mathcal{N}}(n_f - h - 1, \widetilde{k})$, then successful external innovation adds one product line with $\Delta' = \Delta^1$. Since next period technology gap of product line j from successful external innovation is equal to $\Delta'_j = \frac{q'_j}{q_j} = \frac{\eta q_{j,-1}}{\Delta_j q_{j,-1}} = \frac{\eta}{\Delta_j}$, firm needs to take over product line with technology gap $\Delta = \Delta^3 = 1 + \eta$ to have a product line with technology gap Δ^1 next period. The third term is the probability of $\mathcal{N}$ becoming $\widetilde{\mathcal{N}}(n_f - h - 1, \widetilde{k} - 1)$, then successful external innovation adds one product line with $\Delta' = \Delta^2$ by taking over a product line with technology gap $\Delta = \Delta^4$. For $h = -1$, the first term becomes zero by the definition of $\widetilde{\mathbb{P}}(\,\cdot\,|\,\cdot\,)$. Thus this probability is well defined for any $h \geqslant -1$.

1-ii. Probability of $\mathcal{N} = (n_f, n_f - k, k, 0, 0)$ becoming $\mathcal{N}' = (n_f - h, n_f - h - 1 - \widetilde{k}, \widetilde{k}, 1, 0)$ for $h \geqslant -1$ is defined as

$$\mathbb{P}\Big(n_f - h, n_f - h - 1 - \widetilde{k}, \widetilde{k}, 1, 0 \,\Big|\, n_f, n_f - k, k, 0, 0\Big) = \widetilde{\mathbb{P}}\Big(n_f - h - 1, \widetilde{k} \,\Big|\, n_f, k\Big)\, \mu(\Delta^1)\, x\, .$$

Firm's existing technology gap changes from $\widetilde{\mathcal{N}}(n_f, k)$ to $\widetilde{\mathcal{N}}(n_f - h - 1, \widetilde{k})$, then successful external innovation adds $\Delta' = \Delta^3 = 1 + \eta$.

1-iii. Probability of $\mathcal{N} = (n_f, n_f - k, k, 0, 0)$ becoming $\mathcal{N}' = (n_f - h, n_f - h - 1 -$

$\widetilde{k}, \widetilde{k}, 0, 1)$ for $h \geqslant -1$ is defined as

$$\mathbb{P}\Big(n_f - h, n_f - h - 1 - \widetilde{k}, \widetilde{k}, 0, 1 \,\Big|\, n_f, n_f - k, k, 0, 0\Big) =$$

$$\widetilde{\mathbb{P}}\Big(n_f - h - 1, \widetilde{k} \,\Big|\, n_f, k\Big)\, \mu(\Delta^2)\, x\, (1 - z^2)\,.$$

2-i. For $n_f \geqslant 2$, probability of $\mathcal{N} = (n_f, n_f - 1 - k, k, 1, 0)$ becoming $\mathcal{N}' = (n_f - h, n_f - h - \widetilde{k}, \widetilde{k}, 0, 0)$ for $h \geqslant -1$ is defined as

$$\mathbb{P}\Big(n_f - h, n_f - h - \widetilde{k}, \widetilde{k}, 0, 0 \,\Big|\, n_f, n_f - 1 - k, k, 1, 0\Big) =$$

$$\begin{aligned}
&\left[\begin{array}{l}
\quad\widetilde{\mathbb{P}}\Big(n_f - h, \widetilde{k} \,\Big|\, n_f - 1, k\Big)\, \tfrac{1}{2}\, \overline{x}\, (1 - z^3) \\
+\, \widetilde{\mathbb{P}}\Big(n_f - h - 1, \widetilde{k} \,\Big|\, n_f - 1, k\Big)\, \left(1 - \tfrac{1}{2}\overline{x}\right)\, (1 - z^3) \\
+\, \widetilde{\mathbb{P}}\Big(n_f - h - 1, \widetilde{k} - 1 \,\Big|\, n_f - 1, k\Big)\, z^3
\end{array}\right] \times (1 - x\, \overline{x}_{takeover}) \\
+ &\left[\begin{array}{l}
\quad\widetilde{\mathbb{P}}\Big(n_f - h - 1, \widetilde{k} \,\Big|\, n_f - 1, k\Big)\, \tfrac{1}{2}\, \overline{x}\, (1 - z^3) \\
+\, \widetilde{\mathbb{P}}\Big(n_f - h - 2, \widetilde{k} \,\Big|\, n_f - 1, k\Big)\, \left(1 - \tfrac{1}{2}\overline{x}\right)\, (1 - z^3) \\
+\, \widetilde{\mathbb{P}}\Big(n_f - h - 2, \widetilde{k} - 1 \,\Big|\, n_f - 1, k\Big)\, z^3
\end{array}\right] \times \mu(\Delta^3)\, \frac{1}{2}\, x\, (1 - z^3) \\
+ &\left[\begin{array}{l}
\quad\widetilde{\mathbb{P}}\Big(n_f - h - 1, \widetilde{k} - 1 \,\Big|\, n_f - 1, k\Big)\, \tfrac{1}{2}\, \overline{x}\, (1 - z^3) \\
+\, \widetilde{\mathbb{P}}\Big(n_f - h - 2, \widetilde{k} - 1 \,\Big|\, n_f - 1, k\Big)\, \left(1 - \tfrac{1}{2}\overline{x}\right)\, (1 - z^3) \\
+\, \widetilde{\mathbb{P}}\Big(n_f - h - 2, \widetilde{k} - 2 \,\Big|\, n_f - 1, k\Big)\, z^3
\end{array}\right] \times \mu(\Delta^4)\, x\, \left(1 - \frac{1}{2} z^4\right)\,.
\end{aligned}$$

Three probabilities in the brackets are the probabilities when the existing product line with $\Delta = \Delta^3$ is taken over by other firm, internal innovation fails but firm keeps it, and internal innovation succeeds and firm keeps it. The first bracket is the probability of $\mathcal{N}$ becoming $\mathcal{N}'$ when external innovation fails, the second bracket is the probability of $\mathcal{N}$ becoming $\mathcal{N}'$ when successful external innovation adds a product line with technology gap $\Delta' = \Delta^1$, and the third bracket is the probability of $\mathcal{N}$ becoming $\mathcal{N}'$ when successful external innovation adds a product line with $\Delta' = \Delta^2$. Similarly, for $n_f = 1$,

$$\begin{aligned}\mathbb{P}\Big(1,1,0,0,0 \,\Big|\, 1,0,0,1,0\Big) &= \left(1 - \frac{1}{2}\overline{x}\right)(1-z^3)(1 - x\,\overline{x}_{takeover}) \\ &+ \frac{1}{2}\,\overline{x}\,(1-z^3)\,\mu(\Delta^3)\,\frac{1}{2}\,x\,(1-z^3)\,,\end{aligned}$$

and

$$\mathbb{P}\Big(1,0,1,0,0 \,\Big|\, 1,0,0,1,0\Big) = z^3\,(1 - x\,\overline{x}_{takeover}) + \frac{1}{2}\,\overline{x}\,(1-z^3)\,\mu(\Delta^4)\,x\,\left(1 - \frac{1}{2}z^4\right).$$

2-ii. For $n_f \geqslant 2$, probability of $\mathcal{N} = (n_f, n_f - 1 - k, k, 1, 0)$ becoming $\mathcal{N}' = (n_f - h, n_f - h - 1 - \widetilde{k}, \widetilde{k}, 1, 0)$ for $h \geqslant -1$ is defined as

$$\begin{aligned}&\mathbb{P}\Big(n_f - h, n_f - h - 1 - \widetilde{k}, \widetilde{k}, 1, 0 \,\Big|\, n_f, n_f - 1 - k, k, 1, 0\Big) = \\ &\left[\begin{array}{c} \widetilde{\mathbb{P}}\Big(n_f - h - 1, \widetilde{k} \,\Big|\, n_f - 1, k\Big)\,\tfrac{1}{2}\,\overline{x}\,(1-z^3) \\ + \widetilde{\mathbb{P}}\Big(n_f - h - 2, \widetilde{k} \,\Big|\, n_f - 1, k\Big)\,\left(1 - \tfrac{1}{2}\overline{x}\right)\,(1-z^3) \\ + \widetilde{\mathbb{P}}\Big(n_f - h - 2, \widetilde{k} - 1 \,\Big|\, n_f - 1, k\Big)\,z^3 \end{array}\right] \times \mu(\Delta^1)\,x\end{aligned}$$

$\mathcal{N}$ becomes $\widetilde{\mathcal{N}}(n_f - h - 1, \widetilde{k})$ through internal innovations, then successful external innovation adds a product line with $\Delta' = \Delta^3$ by taking over a product line with $\Delta = \Delta^1$. Similarly, for $n_f = 1$,

$$\mathbb{P}\Big(1,0,0,1,0 \,\Big|\, 1,0,0,1,0\Big) = \frac{1}{2}\,\overline{x}\,(1-z^3)\,\mu(\Delta^1)\,x\,.$$

2-iii. For $n_f \geqslant 2$, probability of $\mathcal{N} = (n_f, n_f - 1 - k, k, 1, 0)$ becoming $\mathcal{N}' = (n_f - h, n_f - h - 1 - \widetilde{k}, \widetilde{k}, 0, 1)$ for $h \geqslant -1$ is defined as

$$\mathbb{P}\Big(n_f - h, n_f - h - 1 - \widetilde{k}, \widetilde{k}, 0, 1 \,\Big|\, n_f, n_f - 1 - k, k, 1, 0\Big) =$$
$$\left[\begin{array}{l} \quad\widetilde{\mathbb{P}}\Big(n_f - h - 1, \widetilde{k} \,\Big|\, n_f - 1, k\Big)\,\tfrac{1}{2}\,\overline{x}\,(1-z^3) \\ +\,\widetilde{\mathbb{P}}\Big(n_f - h - 2, \widetilde{k} \,\Big|\, n_f - 1, k\Big)\,\left(1 - \tfrac{1}{2}\overline{x}\right)\,(1-z^3) \\ +\,\widetilde{\mathbb{P}}\Big(n_f - h - 2, \widetilde{k} - 1 \,\Big|\, n_f - 1, k\Big)\,z^3 \end{array}\right] \times \mu(\Delta^2)\,x\,(1-z^2)\,.$$

Similarly, for $n_f = 1$,

$$\mathbb{P}\Big(1,0,0,0,1 \,\Big|\, 1,0,0,1,0\Big) = \frac{1}{2}\,\overline{x}\,(1-z^3)\,\mu(\Delta^2)\,x\,(1-z^2)\,.$$

3-i. For $n_f \geqslant 2$, probability of $\mathcal{N} = (n_f, n_f - 1 - k, k, 0, 1)$ becoming $\mathcal{N}' = (n_f - h, n_f - h - \widetilde{k}, \widetilde{k}, 0, 0)$ for $h \geqslant -1$ is defined as

$$\mathbb{P}\Big(n_f - h, n_f - h - \widetilde{k}, \widetilde{k}, 0, 0 \,\Big|\, n_f, n_f - 1 - k, k, 0, 1\Big) =$$
$$\left[\begin{array}{l} \quad\widetilde{\mathbb{P}}\Big(n_f - h, \widetilde{k} \,\Big|\, n_f - 1, k\Big)\,\overline{x}\,\left(1 - \tfrac{1}{2}z^4\right) \\ +\,\widetilde{\mathbb{P}}\Big(n_f - h - 1, \widetilde{k} \,\Big|\, n_f - 1, k\Big)\,(1-\overline{x})\,(1-z^4) \\ +\,\widetilde{\mathbb{P}}\Big(n_f - h - 1, \widetilde{k} - 1 \,\Big|\, n_f - 1, k\Big)\,\left(1 - \tfrac{1}{2}\overline{x}\right)\,z^4 \end{array}\right] \times (1 - x\,\overline{x}_{takeover})$$

$$
+\left[\begin{array}{l}
\quad \widetilde{\mathbb{P}}\Big(n_f-h-1,\widetilde{k}\,\Big|\,n_f-1,k\Big)\,\overline{x}\ \left(1-\tfrac{1}{2}z^4\right) \\
+\,\widetilde{\mathbb{P}}\Big(n_f-h-2,\widetilde{k}\,\Big|\,n_f-1,k\Big)\,(1-\overline{x})\,(1-z^4) \\
+\,\widetilde{\mathbb{P}}\Big(n_f-h-2,\widetilde{k}-1\,\Big|\,n_f-1,k\Big)\ \left(1-\tfrac{1}{2}\overline{x}\right)\ z^4
\end{array}\right]\times\mu(\Delta^3)\,\frac{1}{2}\,x\,(1-z^3)
$$

$$
+\left[\begin{array}{l}
\quad \widetilde{\mathbb{P}}\Big(n_f-h-1,\widetilde{k}-1\,\Big|\,n_f-1,k\Big)\,\overline{x}\ \left(1-\tfrac{1}{2}z^4\right) \\
+\,\widetilde{\mathbb{P}}\Big(n_f-h-2,\widetilde{k}-1\,\Big|\,n_f-1,k\Big)\,(1-\overline{x})\,(1-z^4) \\
+\,\widetilde{\mathbb{P}}\Big(n_f-h-2,\widetilde{k}-2\,\Big|\,n_f-1,k\Big)\ \left(1-\tfrac{1}{2}\overline{x}\right)\ z^4
\end{array}\right]\times\mu(\Delta^4)\,x\,\left(1-\frac{1}{2}z^4\right)\,.
$$

Similarly, for $n_f=1$,

$$
\begin{aligned}
\mathbb{P}\Big(1,1,0,0,0\,\Big|\,1,0,0,0,1\Big) =& (1-\overline{x})(1-z^4)(1-x\,\overline{x}_{takeover}) \\
& +\overline{x}\left(1-\frac{1}{2}z^4\right)\mu(\Delta^3)\frac{1}{2}\,x\,(1-z^3)
\end{aligned}
$$

and

$$
\begin{aligned}
\mathbb{P}\Big(1,0,1,0,0\,\Big|\,1,0,0,0,1\Big) =& \left(1-\frac{1}{2}\overline{x}\right)\ z^4(1-x\,\overline{x}_{takeover}) \\
& +\overline{x}\ \left(1-\frac{1}{2}z^4\right)\ \mu(\Delta^4)\,x\ \left(1-\frac{1}{2}z^4\right)\,.
\end{aligned}
$$

3-ii. For $n_f\geqslant 2$, probability of $\mathcal{N}=(n_f,n_f-1-k,k,0,1)$ becoming $\mathcal{N}'=(n_f-h,n_f-h-1-\widetilde{k},\widetilde{k},1,0)$ for $h\geqslant -1$ is defined as

$$
\mathbb{P}\Big(n_f-h,n_f-h-1-\widetilde{k},\widetilde{k},1,0\,\Big|\,n_f,n_f-1-k,k,0,1\Big)=
$$

$$\begin{bmatrix} \widetilde{\mathbb{P}}\Big(n_f-h-1,\widetilde{k}\,\big|\,n_f-1,k\Big)\,\overline{x}\,\left(1-\tfrac{1}{2}z^4\right) \\ +\,\widetilde{\mathbb{P}}\Big(n_f-h-2,\widetilde{k}\,\big|\,n_f-1,k\Big)\,(1-\overline{x})\,(1-z^4) \\ +\,\widetilde{\mathbb{P}}\Big(n_f-h-2,\widetilde{k}-1\,\big|\,n_f-1,k\Big)\,\left(1-\tfrac{1}{2}\overline{x}\right)\,z^4 \end{bmatrix} \times \mu(\Delta^1)\,x\,.$$

Similarly, for $n_f = 1$,

$$\mathbb{P}\Big(1,0,0,1,0\,\big|\,1,0,0,0,1\Big) = \overline{x}\left(1-\frac{1}{2}z^4\right)\,\mu(\Delta^1)\,x\,.$$

3-iii. For $n_f \geqslant 2$, probability of $\mathcal{N} = (n_f, n_f-1-k, k, 0, 1)$ becoming $\mathcal{N}' = (n_f - h, n_f - h - 1 - \widetilde{k}, \widetilde{k}, 0, 1)$ for $h \geqslant -1$ is defined as

$$\mathbb{P}\Big(n_f-h, n_f-h-1-\widetilde{k},\widetilde{k},0,1\,\big|\,n_f,n_f-1-k,k,0,1\Big) =$$
$$\begin{bmatrix} \widetilde{\mathbb{P}}\Big(n_f-h-1,\widetilde{k}\,\big|\,n_f-1,k\Big)\,\overline{x}\,\left(1-\tfrac{1}{2}z^4\right) \\ +\,\widetilde{\mathbb{P}}\Big(n_f-h-2,\widetilde{k}\,\big|\,n_f-1,k\Big)\,(1-\overline{x})\,(1-z^4) \\ +\,\widetilde{\mathbb{P}}\Big(n_f-h-2,\widetilde{k}-1\,\big|\,n_f-1,k\Big)\,\left(1-\tfrac{1}{2}\overline{x}\right)\,z^4 \end{bmatrix} \times \mu(\Delta^2)\,x\,(1-z^2)\,.$$

Similarly, for $n_f = 1$,

$$\mathbb{P}\Big(1,0,0,0,1\,\big|\,1,0,0,0,1\Big) = \overline{x}\left(1-\frac{1}{2}z^4\right)\,\mu(\Delta^2)\,x\,(1-z^2)\,.$$

Now that the probabilities of any particular technology gap portfolio composition becoming other particular technology gap portfolio composition is computed, I can specify the inflows and outflows of a particular technology gap portfolio. Let $\mathcal{F}$ be a total mass of firms in the economy and let $\mu(\mathcal{N})$ be a share of firms with technology gap portfolio $\mathcal{N}$.

i) For $\mathcal{N} = (n_f, n_f - k, k, 0, 0)$ with $n_f \geqslant 2$, any firms with technology gap portfolio next period not equal to $\mathcal{N}$ accounts for outflows. Thus

$$\begin{aligned}\text{outflow}\big(n_f, n_f - k, k, 0, 0\big) =& \Big[\, 1 - \mathbb{P}\big(n_f, n_f - k, k, 0, 0 \,\big|\, n_f, n_f - k, k, 0, 0\big) \,\Big] \\ & \times \mathcal{F}\, \mu\big(n_f, n_f - k, k, 0, 0\big)\,.\end{aligned}$$

Any firms with total number of product line $n \geqslant n_f - 1$ can have technology gap portfolio composition equal to $\mathcal{N}$ through combinations of internal and external innovations. Thus for the maximum number of product lines $\bar{n}_f$,

$$\begin{aligned}&\text{inflow}\big(n_f, n_f - k, k, 0, 0\big) = \\ &\mathcal{F} \sum_{n=n_f-1}^{\bar{n}_f} \sum_{\widetilde{k}=0}^{n} \Bigg[\mu\big(n, n-\widetilde{k}, \widetilde{k}, 0, 0\big)\, \mathbb{P}\big(n_f, n_f - k, k, 0, 0 \,\big|\, n, n - \widetilde{k}, \widetilde{k}, 0, 0\big) \\ &\quad + \mu\big(n, n - 1 - \widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 1, 0\big) \\ &\qquad \times \mathbb{P}\big(n_f, n_f - k, k, 0, 0 \,\big|\, n, n - 1 - \widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 1, 0\big) \\ &\quad + \mu\big(n, n - 1 - \widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 0, 1\big) \\ &\qquad \times \mathbb{P}\big(n_f, n_f - k, k, 0, 0 \,\big|\, n, n - 1 - \widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 0, 1\big) \Bigg] \\ &\quad - \mathcal{F}\, \mu\big(n_f, n_f - k, k, 0, 0\big)\, \mathbb{P}\big(n_f, n_f - k, k, 0, 0 \,\big|\, n_f, n_f - k, k, 0, 0\big)\,.\end{aligned}$$

ii) $\mathcal{N} = (n_f, n_f - 1 - k, k, 1, 0)$ with $n_f \geqslant 2$

$$\text{outflow}\big(n_f, n_f - 1 - k, k, 1, 0\big)$$

$$
\begin{aligned}
&= \Big[\, 1 - \mathbb{P}\big(n_f, n_f - 1 - k, k, 1, 0 \,\big|\, n_f, n_f - 1 - k, k, 1, 0\big) \,\Big] \\
&\quad \times \mathcal{F}\, \mu\big(n_f, n_f - 1 - k, k, 1, 0\big)\,.
\end{aligned}
$$

$$
\begin{aligned}
&\operatorname{inflow}\big(n_f, n_f - 1 - k, k, 1, 0\big) = \\
&\mathcal{F} \sum_{n=n_f-1}^{\overline{n}_f} \sum_{\widetilde{k}=0}^{n} \Big[\, \mu\big(n, n-\widetilde{k}, \widetilde{k}, 0, 0\big)\, \mathbb{P}\big(n_f, n_f - 1 - k, k, 1, 0 \,\big|\, n, n-\widetilde{k}, \widetilde{k}, 0, 0\big) \\
&\qquad + \mu\big(n, n-1-\widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 1, 0\big) \\
&\qquad\quad \times \mathbb{P}\big(n_f, n_f - 1 - k, k, 1, 0 \,\big|\, n, n-1-\widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 1, 0\big) \\
&\qquad + \mu\big(n, n-1-\widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 0, 1\big) \\
&\qquad\quad \times \mathbb{P}\big(n_f, n_f - 1 - k, k, 1, 0 \,\big|\, n, n-1-\widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 0, 1\big) \,\Big] \\
&\qquad - \mathcal{F}\, \mu\big(n_f, n_f - 1 - k, k, 1, 0\big)\, \mathbb{P}\big(n_f, n_f - 1 - k, k, 1, 0 \,\big|\, n_f, n_f - 1 - k, k, 1, 0\big)\,.
\end{aligned}
$$

iii) $\mathcal{N} = (n_f, n_f - 1 - k, k, 0, 1)$ with $n_f \geqslant 2$

$$
\begin{aligned}
&\operatorname{outflow}\big(n_f, n_f - 1 - k, k, 0, 1\big) \\
&\qquad = \Big[\, 1 - \mathbb{P}\big(n_f, n_f - 1 - k, k, 0, 1 \,\big|\, n_f, n_f - 1 - k, k, 0, 1\big) \,\Big] \\
&\qquad\quad \times \mathcal{F}\, \mu\big(n_f, n_f - 1 - k, k, 0, 1\big)\,.
\end{aligned}
$$

$$
\begin{aligned}
&\operatorname{inflow}\big(n_f, n_f - 1 - k, k, 0, 1\big) = \\
&\mathcal{F} \sum_{n=n_f-1}^{\overline{n}_f} \sum_{\widetilde{k}=0}^{n} \Big[\, \mu\big(n, n-\widetilde{k}, \widetilde{k}, 0, 0\big)\, \mathbb{P}\big(n_f, n_f - 1 - k, k, 0, 1 \,\big|\, n, n-\widetilde{k}, \widetilde{k}, 0, 0\big)
\end{aligned}
$$

$$+\,\mu\big(n, n-1-\widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 1, 0\big)$$

$$\times\, \mathbb{P}\big(n_f, n_f-1-k, k, 0, 1 \,\big|\, n, n-1-\widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 1, 0\big)$$

$$+\,\mu\big(n, n-1-\widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 0, 1\big)$$

$$\times\, \mathbb{P}\big(n_f, n_f-1-k, k, 0, 1 \,\big|\, n, n-1-\widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 0, 1\big) \Bigg]$$

$$-\,\mathcal{F}\, \mu\big(n_f, n_f-1-k, k, 0, 1\big)\, \mathbb{P}\big(n_f, n_f-1-k, k, 0, 1 \,\big|\, n_f, n_f-1-k, k, 0, 1\big)\,.$$

iv) $\mathcal{N} = (1, 1, 0, 0, 0)$

$$\text{outflow}\big(1,1,0,0,0\big) = \Big[\, 1 - \mathbb{P}\big(1,1,0,0,0 \,\big|\, 1,1,0,0,0\big) \Big]\, \mathcal{F}\, \mu\big(1,1,0,0,0\big)\,.$$

$$\text{inflow}(1,1,0,0,0) = \mathcal{E}\, x_e\, \mu(\Delta^3)\frac{1}{2}(1-z^3)$$

$$+\,\mathcal{F} \sum_{n=1}^{\overline{n}_f} \sum_{\widetilde{k}=0}^{n} \Bigg[\, \mu\big(n, n-\widetilde{k}, \widetilde{k}, 0, 0\big)\, \mathbb{P}\big(1,1,0,0,0 \,\big|\, n, n-\widetilde{k}, \widetilde{k}, 0, 0\big)$$

$$+\,\mu\big(n, n-1-\widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 1, 0\big)$$

$$\times\, \mathbb{P}\big(1,1,0,0,0 \,\big|\, n, n-1-\widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 1, 0\big)$$

$$+\,\mu\big(n, n-1-\widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 0, 1\big)$$

$$\times\, \mathbb{P}\big(1,1,0,0,0 \,\big|\, n, n-1-\widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 0, 1\big) \Bigg]$$

$$-\,\mathcal{F}\, \mu\big(1,1,0,0,0\big)\, \mathbb{P}\big(1,1,0,0,0 \,\big|\, 1,1,0,0,0\big)\,.$$

v) $\mathcal{N} = (1,0,1,0,0)$

$$\text{outflow}\big(1,0,1,0,0\big) = \Big[\, 1 - \mathbb{P}\big(1,0,1,0,0 \,\big|\, 1,0,1,0,0\big) \,\Big]\, \mathcal{F}\,\mu\big(1,0,1,0,0\big)\,.$$

$$\begin{aligned}
\text{inflow}(1,0,1,0,0) = {}& \mathcal{E}\,x_e\,\mu(\Delta^4)\,\left(1 - \frac{1}{2}z^4\right) \\
&+ \mathcal{F}\sum_{n=1}^{\overline{n}_f}\sum_{\widetilde{k}=0}^{n}\Bigg[\,\mu\big(n, n-\widetilde{k}, \widetilde{k}, 0, 0\big)\,\mathbb{P}\big(1,0,1,0,0 \,\big|\, n, n-\widetilde{k}, \widetilde{k}, 0, 0\big) \\
&\qquad + \mu\big(n, n-1-\widetilde{k}\,I_{\{n>1\}}, \widetilde{k}\,I_{\{n>1\}}, 1, 0\big) \\
&\qquad\quad \times \mathbb{P}\big(1,0,1,0,0 \,\big|\, n, n-1-\widetilde{k}\,I_{\{n>1\}}, \widetilde{k}\,I_{\{n>1\}}, 1, 0\big) \\
&\qquad + \mu\big(n, n-1-\widetilde{k}\,I_{\{n>1\}}, \widetilde{k}\,I_{\{n>1\}}, 0, 1\big) \\
&\qquad\quad \times \mathbb{P}\big(1,0,1,0,0 \,\big|\, n, n-1-\widetilde{k}\,I_{\{n>1\}}, \widetilde{k}\,I_{\{n>1\}}, 0, 1\big)\,\Bigg] \\
&\qquad - \mathcal{F}\,\mu\big(1,0,1,0,0\big)\,\mathbb{P}\big(1,0,1,0,0 \,\big|\, 1,0,1,0,0\big)\,.
\end{aligned}$$

vi) $\mathcal{N} = (1,0,0,1,0)$

$$\text{outflow}\big(1,0,0,1,0\big) = \Big[\, 1 - \mathbb{P}\big(1,0,0,1,0 \,\big|\, 1,0,0,1,0\big) \,\Big]\, \mathcal{F}\,\mu\big(1,0,0,1,0\big)\,.$$

$$\begin{aligned}
\text{inflow}(1,0,0,1,0) = {}& \mathcal{E}\,x_e\,\mu(\Delta^1) \\
&+ \mathcal{F}\sum_{n=1}^{\overline{n}_f}\sum_{\widetilde{k}=0}^{n}\Bigg[\,\mu\big(n, n-\widetilde{k}, \widetilde{k}, 0, 0\big)\,\mathbb{P}\big(1,0,0,1,0 \,\big|\, n, n-\widetilde{k}, \widetilde{k}, 0, 0\big) \\
&\qquad + \mu\big(n, n-1-\widetilde{k}\,I_{\{n>1\}}, \widetilde{k}\,I_{\{n>1\}}, 1, 0\big)
\end{aligned}$$

$$\times\, \mathbb{P}\big(1,0,0,1,0 \,\big|\, n, n-1-\widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 1, 0\big)$$

$$+\, \mu\big(n, n-1-\widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 0, 1\big)$$

$$\times\, \mathbb{P}\big(1,0,0,1,0 \,\big|\, n, n-1-\widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 0, 1\big) \Bigg]$$

$$-\, \mathcal{F}\, \mu\big(1,0,0,1,0\big)\, \mathbb{P}\big(1,0,0,1,0 \,\big|\, 1,0,0,1,0\big)\,.$$

vii) $\mathcal{N} = (1,0,0,0,1)$

$$\mathrm{outflow}\big(1,0,0,0,1\big) = \Big[\, 1 - \mathbb{P}\big(1,0,0,0,1 \,\big|\, 1,0,0,0,1\big) \,\Big]\, \mathcal{F}\, \mu\big(1,0,0,0,1\big)\,.$$

$$\mathrm{inflow}(1,0,0,0,1) = \,\mathcal{E}\, x_e\, \mu(\Delta^2)\,(1-z^2)$$

$$+\, \mathcal{F} \sum_{n=1}^{\overline{n}_f} \sum_{\widetilde{k}=0}^{n} \Bigg[\, \mu\big(n, n-\widetilde{k}, \widetilde{k}, 0, 0\big)\, \mathbb{P}\big(1,0,0,0,1 \,\big|\, n, n-\widetilde{k}, \widetilde{k}, 0, 0\big)$$

$$+\, \mu\big(n, n-1-\widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 1, 0\big)$$

$$\times\, \mathbb{P}\big(1,0,0,0,1 \,\big|\, n, n-1-\widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 1, 0\big)$$

$$+\, \mu\big(n, n-1-\widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 0, 1\big)$$

$$\times\, \mathbb{P}\big(1,0,0,0,1 \,\big|\, n, n-1-\widetilde{k}\, I_{\{n>1\}}, \widetilde{k}\, I_{\{n>1\}}, 0, 1\big) \Bigg]$$

$$-\, \mathcal{F}\, \mu\big(1,0,0,0,1\big)\, \mathbb{P}\big(1,0,0,0,1 \,\big|\, 1,0,0,0,1\big)\,.$$

A.1.6.1 Number of points in technology gap portfolio composition distribution

Let's denote $N(n_f)$ as the number of variations for a technology gap portfolio composition with n_f product lines, $\big(n_f, n_f^1, n_f^2, n_f^3, n_f^4\big)$, where $n_f = \sum_{\ell=1}^{4} n_f^{\ell}$, $n_f^3, n_f^4 \in \{0,1\}$, and $n_f^3 = n_f^4 = 1$ is not possible.

Let's denote $\widetilde{N}(n_f)$ as the number of variations for a technology gap portfolio composition with n_f product lines with no product line that has Δ^3 or Δ^4, $\big(n_f, n_f^1, n_f^2, 0, 0\big)$. Then

$$N(n_f) = \widetilde{N}(n_f) + 2\widetilde{N}(n_f - 1)\ ,$$

as

$$\big(n_f, n_f^1, n_f^2, 1, 0\big) = \big(n_f - 1, n_f^1, n_f^2, 0, 0\big) + (1, 0, 0, 1, 0)\ ,$$

and

$$\big(n_f, n_f^1, n_f^2, 0, 1\big) = \big(n_f - 1, n_f^1, n_f^2, 0, 0\big) + (1, 0, 0, 0, 1)\ .$$

Since $\widetilde{N}(n_f) = n_f + 1$, $N(n_f) = 3\,n_f + 1$. Thus for a maximum number of product line individual firm can have, $\overline{n}_f$, total number of points in technology gap portfolio composition distribution is

$$N_{\text{total}} = \sum_{n_f=1}^{\overline{n}_f} \left(3n_f + 1\right) = \frac{\left(3\overline{n}_f + 5\right)\overline{n}_f}{2} \, .$$

A.1.7 Total Mass of Product Lines Owned by the Domestic Firms

A.1.7.1 Proof of Lemma 2

Proof. Since the optimal probability of external innovation for both domestic firms and foreign exporters are the same, the aggregate creative destruction arrival rate can be decomposed into:

$$\overline{x} = \underbrace{\mathcal{F}_d x + \mathcal{E}_d x_e}_{\overline{x}_d} + \underbrace{\mathcal{F}_{fx} x + \mathcal{E}_{fx} x_e}_{\overline{x}_{fx}} \, .$$

In any stationary equilibrium, the share of domestic incumbent firms should be equal to the share of potential domestic startups. Thus,

$$\frac{\mathcal{F}_d}{\mathcal{F}_d + \mathcal{F}_{fx}} = \frac{\mathcal{E}_d}{\mathcal{E}_d + \mathcal{E}_{fx}} \, .$$

Since all the incumbent firms are homogeneous in terms of their optimal R&D decisions, and external innovation is undirected, the share of domestic incumbent firms should be equal to s_d in an equilibrium. Then by rearranging $\overline{x}$ and multiplying it by s_d, we get

$$\begin{aligned}
s_d\overline{x} &= s_d\big(\mathcal{F}_d x + \mathcal{F}_{fx} x + \mathcal{E}_{fx} x_e + \mathcal{E}_d x_e\big) \\
&= s_d\big(\mathcal{F}_d + \mathcal{F}_{fx}\big)x + s_d\big(\mathcal{E}_d + \mathcal{E}_{fx}\big)x_e \\
&= \frac{\mathcal{F}_d}{\mathcal{F}_d + \mathcal{F}_{fx}}\big(\mathcal{F}_d + \mathcal{F}_{fx}\big)x + \frac{\mathcal{E}_d}{\mathcal{E}_d + \mathcal{E}_{fx}}\big(\mathcal{E}_d + \mathcal{E}_{fx}\big)x_e \\
&= \mathcal{F}_d x + \mathcal{E}_d x_e \\
&= \overline{x}_d \ ,
\end{aligned}$$

and $(1 - s_d)\overline{x} = \overline{x}_{fx}$. Therefore,

$$s_d = \frac{\overline{x}_d}{\overline{x}} \ .$$

■

A.2 Simple Three-Period Model

A.2.1 Proof for Proposition 3

Proof. The first part of proposition 3 follows from simple algebra. I prove the second part here. For $q_{j,1} = q_{j,0}$, we have

$$\frac{\partial z_{j,1}}{\overline{x}_o} = -\frac{\pi_{j,2}}{2\widehat{\chi}}(\lambda - 1)\left[(1 - x_{j,1}) + (1 - \overline{x}_o)\frac{\partial x_{j,1}}{\partial \overline{x}_o}\right] \ ,$$

and

$$\frac{\partial x_{j,1}}{\partial \overline{x}_o} = 0 \,.$$

Thus, we have

$$\frac{\partial z_{j,1}}{\partial \overline{x}_o} = -\frac{\pi_{j,2}}{2\widehat{\chi}}(\lambda - 1)(1 - x_{j,1}) < 0 \,.$$

For $q_{j,1} = \lambda q_{j,0}$, we have

$$\frac{\partial z_{j,1}}{\partial \overline{x}_o} = \frac{\pi_{j,2}}{2\widehat{\chi}}\left[1 - x_{j,1} + (1 - \overline{x}_o)\frac{\partial x_{j,1}}{\partial \overline{x}_o}\right] \,,$$

and

$$\frac{\partial x_{j,1}}{\partial \overline{x}_o} = -\frac{\eta \pi_{j,2}}{2\widetilde{\chi}}\frac{\partial z_{j,1}}{\partial \overline{x}_o} \,.$$

Thus, we have

$$\frac{\partial z_{j,1}}{\partial \overline{x}_o} = (1 - x_{j,1})\left[\frac{2\widehat{\chi}}{\pi_{j,2}} + \frac{\eta \pi_{j,2}}{2\widetilde{\chi}}(1 - \overline{x}_o)\right]^{-1} > 0 \,,$$

hence

$$\frac{\partial x_{j,1}}{\partial \overline{x}_o} = -\frac{\eta \pi_{j,2}}{2\widetilde{\chi}}\frac{\partial z_{j,1}}{\partial \overline{x}_o} < 0 \,.$$

For $q_{j,1} = \eta q_{j,0}$, we have

$$\frac{\partial z_{j,1}}{\partial \overline{x}_o} = \frac{\pi_{j,2}}{2\widehat{\chi}}\frac{1}{2}\left[1 - x_{j,1} + (1-\overline{x}_o)\frac{\partial x_{j,1}}{\partial \overline{x}_o}\right] ,$$

and

$$\frac{\partial x_{j,1}}{\partial \overline{x}_o} = -\frac{\eta\pi_{j,2}}{2\widetilde{\chi}}\frac{1}{2}\frac{\partial z_{j,1}}{\partial \overline{x}_o} .$$

Thus, we have

$$\frac{\partial z_{j,1}}{\partial \overline{x}_o} = (1 - x_{j,1})\left[\frac{4\widehat{\chi}}{\pi_{j,2}} + \frac{\eta\pi_{j,2}}{4\widetilde{\chi}}(1-\overline{x}_o)\right]^{-1} > 0 ,$$

hence

$$\frac{\partial x_{j,1}}{\partial \overline{x}_o} = -\frac{1}{2}\frac{\eta\pi_{j,2}}{2\widetilde{\chi}}\frac{\partial z_{j,1}}{\partial \overline{x}_o} < 0 .$$

From $x^*_{j,1}$, we see that $\frac{\eta\pi_{j,2}}{2\widetilde{\chi}} \in (0,1)$. Then, under a parameter restriction $4\widehat{\chi} > \pi_{j,2}$,

$$\frac{4\widehat{\chi}}{\pi_{j,2}} + \frac{\eta\pi_{j,2}}{4\widetilde{\chi}}(1-\overline{x}_o) > \frac{2\widehat{\chi}}{\pi_{j,2}} + \frac{\eta\pi_{j,2}}{2\widetilde{\chi}}(1-\overline{x}_o) .$$

Thus, $\left.\frac{\partial z^*_{j,1}}{\partial x^e_1}\right|_{q_{j,1}=\lambda q_{j,0}} > \left.\frac{\partial z^*_{j,1}}{\partial x^e_1}\right|_{q_{j,1}=\eta q_{j,0}}$ ■

A.2.2 Proof of Corollary 3

Proof. From $\overline{z}_1^*$, we know that

$$\frac{\partial \overline{z}_1^*}{\partial z_{1,0}} = \frac{1}{2}\left(z_{1,1}^*\big|_{q_{1,1}=\lambda q_{1,0}} \; - \; z_{1,1}^*\big|_{q_{1,1}=q_{1,0}} \right) > 0 \,,$$

and

$$\frac{\partial \overline{z}_1^*}{\partial x_{2,0}} = \frac{1}{2}\left(z_{2,1}^*\big|_{q_{2,1}=\eta q_{2,0}} - z_{2,1}^*\big|_{q_{2,1}=q_{2,0}} \right) > 0 \,,$$

where the signs of the two derivatives follow from proposition 3. Then, the results follow from proposition 3 ■

A.2.3 Proof of Corollary 4

Proof. From $\overline{x}_1^*$, we have

$$\frac{\partial \overline{x}_1^*}{\partial z_{1,0}} = \frac{1}{2}\left(x_{1,1}^*\big|_{q_{1,1}=\lambda q_{1,0}} - x_{1,1}^*\big|_{q_{1,1}=q_{1,0}} \right) < 0 \,,$$

and

$$\frac{\partial \overline{x}_1^*}{\partial x_{2,0}} = \frac{1}{2}\left(x_{2,1}^*\big|_{q_{2,1}=\eta q_{2,0}} - x_{2,1}^*\big|_{q_{2,1}=q_{2,0}} \right) < 0 \,,$$

where the signs for the two derivatives follow from proposition 4 ■

A.2.4 Proof of Proposition 5

Proof. For $q_{j,1} = q_{j,0}$,

$$\frac{\partial z_{j,1}}{\partial \pi_{j,2}} = \frac{1}{2\widehat{\chi}}(\lambda - 1)(1 - x_{j,1})(1 - \overline{x}_o) - \frac{\pi_{j,2}}{2\widehat{\chi}}(\lambda - 1)(1 - \overline{x}_o)\frac{\partial x_{j,1}}{\partial \pi_{j,2}} ,$$

and

$$\frac{\partial x_{j,1}}{\partial \pi_{j,2}} = \frac{\eta}{2\widetilde{\chi}}$$

Thus,

$$\frac{\partial z_{j,1}}{\partial \pi_{j,2}} = \frac{1}{2\widehat{\chi}}(\lambda - 1)(1 - 2x_{j,1})(1 - \overline{x}_o) ,$$

and this is positive iff $x_{j,1} < \frac{1}{2}$. $\frac{\partial x_{j,1}}{\partial \pi_{j,2}} > 0$ unambiguously.

For $q_{j,1} = \lambda q_{j,0}$,

$$\frac{\partial z_{j,1}}{\partial \pi_{j,2}} = \frac{1}{2\widehat{\chi}}\big[\lambda - (1 - x_{j,1})(1 - \overline{x}_o)\big] + \frac{\pi_{j,2}}{2\widehat{\chi}}(1 - \overline{x}_o)\frac{\partial x_{j,1}}{\partial \pi_{j,2}} ,$$

and

$$\frac{\partial x_{j,1}}{\partial \pi_{j,2}} = \frac{x_{j,1}}{\pi_{j,2}} - \frac{\eta \pi_{j,2}}{2\widetilde{\chi}}\frac{\partial z_{j,1}}{\partial \pi_{j.2}} .$$

Thus,

$$\frac{\partial z_{j,1}}{\partial \pi_{j,2}} = \left[\lambda - (1-2x_{j,1})(1-\overline{x}_o)\right]\left[2\widehat{\chi} + \frac{\eta(\pi_{j,2})^2}{2\widetilde{\chi}}(1-\overline{x}_o)\right]^{-1},$$

and this is positive unambiguously. The sign for $\frac{\partial x_{j,1}}{\partial \pi_{j,2}}$ is ambiguous.

For $q_{j,1} = \eta q_{j,0}$,

$$\frac{\partial z_{j,1}}{\partial \pi_{j,2}} = \frac{1}{2\widehat{\chi}}\left[\lambda - \frac{1}{2} - \frac{1}{2}(1-x_{j,1})(1-\overline{x}_o)\right] + \frac{\pi_{j,2}}{2\widehat{\chi}}\frac{1}{2}(1-\overline{x}_o)\frac{\partial x_{j,1}}{\partial \pi_{j,2}},$$

and

$$\frac{\partial x_{j,1}}{\partial \pi_{j,2}} = \frac{\eta}{2\widetilde{\chi}}\frac{1}{2}(1-z_{j,1}) - \frac{\eta\pi_{j,2}}{2\widetilde{\chi}}\frac{1}{2}\frac{\partial z_{j,1}}{\partial \pi_{j,2}}.$$

Thus,

$$\frac{\partial z_{j,1}}{\partial \pi_{j,2}} = \left[\lambda - \frac{1}{2} - \frac{1}{2}(1-2x_{j,1})(1-\overline{x}_o)\right]\left[2\widehat{\chi} + \frac{\eta(\pi_{j,2})^2}{2\widetilde{\chi}}\frac{1}{4}(1-\overline{x}_o)\right]^{-1},$$

and this is positive unambiguously. The sign for $\frac{\partial x_{j,1}}{\partial \pi_{j,2}}$ is ambiguous. ■

Appendix B: Chapter 2 Appendix

B.1 Data Appendix

B.1.1 Summary Statistics

Table B.1: Trade-shock related measures

	NTR gap	Dnstream NTR g.	Upstream NTR g.	NTR rate	Non-NTR r.	Export shock
Mean	0.291	0.138	0.203	0.027	0.303	1.127
(Std. dev.)	(0.127)	(0.060)	(0.073)	(0.022)	(0.134)	(0.970)
cov(, NTR gap)		0.485	0.434	0.412	0.969	0.214
cov(, Up. NTR g.)		0.204				

Table B.2: Firm-level NTR gap constructed using different weights

	NTR gap, unweighted	NTR gap, main industry
Mean	0.333	0.336
(Std. dev.)	(0.107)	(0.116)
cov(, NTR gap)	0.78	0.86
cov(, NTR gap, main industry)	0.906	

Table B.3: Technology shocks

	Past 5 years			5 years onward	
	own US shock	own foreign shock	outside f. shock	own f. shock	outside f. shock
Mean	0.388	0.342	0.188	0.344	0.257
(Std. dev.)	(0.306)	(0.299)	(0.064)	(0.304)	(0.161)
cov(, past own f.)	0.593			-0.059	
cov(, past out f.)	-0.191	0.151			-0.991
cov(, onward out f.)				0.541	

Table B.4: All patenting firms vs. regression sample patenting firms in 1992

	All patenting firms	Regression sample
Average number of patents	6.15	8.86
	(19.46)	(24.10)
Average self-citation rate	0.0434	0.0540
	(0.0899)	(0.0941)
Innovation intensity	0.055	0.093
	(0.25)	(0.33)
Number of industries operating	2.34	5.43
	(3.67)	(6.94)
Employment	511.7	1988.0
	(1869.0)	(3835.0)
Patent stock	6.45	35.22
	(26.61)	(64.37)
Employment growth	0.07	0.06
	(0.60)	(0.40)
Firm age	12.33	15.65
	(6.76)	(9.42)
7yr patent growth		-0.854
		(1.312)
7yr self-citation ratio growth		0.356
		(1.322)
Number of firms	26,500	3,100

Table B.5: Export Share of Total Value of Shipments (CMF exporters)

	1992	2002	2007
Avg. of firm-level exp/vship	4.99%	5.27%	6.41%
Avg. of firm-level CN exp/vship	0.70%	0.89%	1.17%
Aggregate-level exp/vship	7.76%	9.29%	10.46%
Aggregate-level CN exp/vship	0.19%	0.38%	0.64%

Table B.6: Share of Exporters (LBD firms)

Year	1992	2002	2007
Share of exporters	15.90%	22.10%	24.00%
Share of firms exporting to CN	0.60%	2.30%	4.00%

B.1.2 Overall and Escape-Competition Effect

Table B.7: Overall Effect

	ΔPatents (1)	ΔPatents (2)	ΔSelf-cite (3)	ΔSelf-cite (4)
NTR gap × Post	0.226	0.049	0.025	0.052
	(0.230)	(0.279)	(0.260)	(0.291)
NTR gap	-2.222***	0.569	1.104***	-0.117
	(0.372)	(0.405)	(0.317)	(0.393)
Post	-0.276***	-0.198**	-0.092	-0.021
	(0.077)	(0.082)	(0.080)	(0.084)
Past 5yr Δpat in own tech.		0.170*		0.282***
		(0.087)		(0.091)
Log employment		0.134***		0.014
		(0.013)		(0.014)
Firm age		-0.005**		-0.009***
		(0.002)		(0.002)
NTR rate		-2.273		1.222
		(1.690)		(2.267)
Observations	6,500	6,500	6,500	6,500
Fixed effects	j, p	j, p	j, p	j, p
Controls	no	full	no	full

Notes: Full controls include past 5-year U.S. patent growth in firms' own technology fields, log employment, firm age, NTR rate, dummy for publicly traded firms, dummy for firms with total imports > 0, dummy for firms with total exports > 0, and dummy for firms with imports from relative parties > 0. Estimates for industry (j) and the period (p) fixed effects as well as the constant are suppressed. Robust standard errors adjusted for clustering at the level of the firms' major industries are displayed below each coefficient. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table B.8: Escape-competition effect

	ΔPatents	ΔPatents	ΔSelf-cite	ΔSelf-cite
	(1)	(2)	(3)	(4)
NTR gap × Post	0.238	0.054	-0.075	-0.051
	(0.237)	(0.287)	(0.257)	(0.295)
× Innovation-intensity	0.077	-0.017	0.732**	0.784***
	(0.231)	(0.233)	(0.299)	(0.268)
NTR gap	-2.206***	0.593	1.101***	-0.067
	(0.375)	(0.409)	(0.315)	(0.397)
× Innovation intensity	-0.226	-0.213	-0.198	-0.379
	(0.158)	(0.175)	(0.231)	(0.231)
Post	-0.277***	-0.202**	-0.071	-0.002
	(0.078)	(0.083)	(0.080)	(0.083)
× Innovation-intensity	-0.053	0.017	-0.179*	-0.198**
	(0.070)	(0.075)	(0.095)	(0.085)
Innovation-intensity	0.080*	0.057	0.059	0.086
	(0.048)	(0.046)	(0.070)	(0.066)
NTR rate		-2.403		1.021
		(1.703)		(2.272)
× Innovation-intensity		0.593		0.539
		(0.507)		(0.484)
Observations	6,500	6,500	6,500	6,500
Fixed effects	j, p	j, p	j, p	j, p
Controls	no	full	no	full

Notes: Full controls include past 5-year U.S. patent growth in firms' own technology fields, log employment, firm age, NTR rate, dummy for publicly traded firms, dummy for firms with total imports > 0, dummy for firms with total exports > 0, and dummy for firms with imports from relative parties > 0. Estimates for industry (j) and the period (p) fixed effects as well as the constant are suppressed. Robust standard errors adjusted for clustering at the level of the firms' major industries are displayed below each coefficient. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * p < 0.1, ** p < 0.05, *** p < 0.01.

B.1.3 Import Competition

Table B.9: Effect of PNTR on US imports from China

	Δlog(CN imp) HS8-level (1)	Δlog(CN imp) NAICS6-level (2)
NTR gap	0.631***	0.846*
	(0.216)	(0.509)
Δlog(NTR rate)	-6.497**	-7.696*
	(3.210)	(4.206)
Δlog(Transport cost)	-2.638**	-2.509
	(1.119)	(1.613)
Observations	6862	490

Notes: Table reports results of OLS regressions of changes in US imports from China from 2000 to 2007 on NTR gap at the 8-digit HS level, and 6-digit NAICS level. NTR rates at the 8-digit HS level are from the United States International Trade Commission (https://dataweb.usitc.gov/tariff/annual). Data for 8-digit HS level US imports from China and transport cost is from Schott (2008) (https://sompks4.github.io/sub_data.html). NTR rates and transport costs are in their iceberg form (e.g. from 10% to log(1.1)). * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table B.10: Regression using 7-year changes in the U.S. imports from China

(a) 7-year changes in the US imports from China

	ΔPatents	ΔPatents	ΔPatents	ΔPatents	ΔSelf-cite	ΔSelf-cite	ΔSelf-cite	ΔSelf-cite
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
7yr ΔUS imports from CN	-0.273***	-0.041	-0.277***	-0.043	0.082	-0.030	0.081	-0.030
	(0.047)	(0.041)	(0.047)	(0.041)	(0.061)	(0.058)	(0.061)	(0.058)
× Innovation intensity			0.037**	0.017			0.001	-0.001
			(0.017)	(0.015)			(0.020)	(0.015)
Observations	6,500	6,500	6,500	6,500	6,500	6,500	6,500	6,500
Fixed effects	j, p	j, p	j, p	j, p	j, p	j, p	j, p	j, p
Controls	no	full	no	full	no	full	no	full

Notes: Table reports results of OLS regression results estimating the relationship between the U.S. firms' innovation and realized changes in the U.S. imports from China. Full controls include past 5-year U.S. patent growth in firms' own technology fields, log employment, firm age, dummy for publicly traded firms, dummy for firms with total imports > 0, dummy for firms with total exports > 0, and dummy for firms with imports from relative parties > 0. Estimates for industry (j) and the period (p) fixed effects as well as the constant are suppressed. Robust standard errors adjusted for clustering at the level of the firms' major industries are displayed below each coefficient. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

B.1.4 Firm Growth and Two Types of Innovation

Akcigit and Kerr (2018) show that internal innovation contributes less to firm employment growth by using the LBD. Here, I replicate their result while including firm controls for the Census years: 1982, and 1992 and construct non-overlapping five-year first differences (DHS growth) by using the LBD matched USPTO patent database. I estimate the following fixed-effect regression model:

$$\Delta Y_{ijt+5} = \beta_1 Pat_{ijt} + \beta_2 Internal_{ijt} + \mathbf{X}_{ijt}\,\gamma_1 + \delta_{jt+5} + \varepsilon_{ijt+5}$$

For firm i in industry j, ΔY_{ijt+5} is a 5-year DHS growth rate of i) firm employment growth from year t to $t+5$, and ii) number of six-digit NAICS industries added. Pat_{ijt} is a log of citation adjusted number of patents in year t, and $Internal_{ijt}$ is an citation-adjusted average self-citation ratio in year t. Firm and industry controls include firm age, and log of payroll. The regression is unweighted and standard errors are clustered on firm. Based on Akcigit and Kerr (2018) we expect β_1 to be positive while β_2 to be negative, as internal innovation contributes less to firm employment growth. I run the same regression model with the number of products (seven-digit NAICS product codes) added by using the CMF firms.

Table B.11: Real effect of innovation: employment growth, industry add, and product add

	LBD firms		CMF firms
	ΔEmployment	Log nb. of industries added	Log nb. of products added
	(1)	(2)	(3)
Log nb. of patents	0.031***	0.098***	0.078***
	(0.010)	(0.011)	(0.013)
Avg. self-citation	-0.269**	-0.154**	-0.343***
	(0.106)	(0.078)	(0.102)
Log payroll	-0.025***	0.083***	0.154***
	(0.009)	(0.006)	(0.008)
Firm age	-0.004**	-0.004**	-0.007***
	(0.002)	(0.002)	(0.002)
Innovation intensity	0.032	0.009	0.076'***
	(0.029)	(0.015)	(0.017)
Observations	5,400	5,400	5,700
Fixed effects	*jp*	*jp*	*jp*

Notes: Estimates for industry-period (*jp*) fixed effects as well as the constant are suppressed. Robust standard errors adjusted for clustering at the firm-level are displayed below each coefficient. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

B.1.5 Pre-trend and Robustness

Table B.12: Parallel pre-trend test

	ΔPatents (1)	ΔPatents (2)	ΔSelf-cite (3)	ΔSelf-cite (4)
NTR gap	-0.393	-0.379	-0.559	-0.551
	(0.487)	(0.488)	(0.403)	(0.403)
× Innovation intensity		-0.193		-0.0057
		(0.162)		(0.394)
NTR gap × $\mathcal{I}_{\{1992\}}$	0.520	0.498	0.254	0.261
	(0.355)	(0.361)	(0.294)	(0.290)
× Innovation intensity		0.092		-0.114
		(0.243)		(0.490)
Observations	5,000	5,000	5,000	5,000
Fixed effects	j, p	j, p	j, p	j, p

Notes: Full controls include past 5-year U.S. patent growth in firms' own technology fields, log employment, firm age, and dummy for publicly traded firms. Estimates for industry (j) and the period (p) fixed effects as well as the constant are suppressed. Robust standard errors adjusted for clustering at the level of the firms' major industries are displayed below each coefficient. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * p < 0.1, ** p < 0.05, *** p < 0.01.

Table B.13: Foreign competition shock with I-O

	ΔPatents (1)	ΔPatents (2)	ΔSelf-cite (3)	ΔSelf-cite (4)
NTR gap × Post	-0.111	-0.111	-0.290	-0.415
	(0.332)	(0.343)	(0.355)	(0.354)
× Innovation intensity		0.054		0.825***
		(0.319)		(0.282)
NTR gap	0.580	0.613	-0.096	-0.038
	(0.406)	(0.411)	(0.382)	(0.387)
× Innovation intensity		-0.275		-0.407
		(0.203)		(0.262)
Post	-0.254**	-0.264**	-0.145	-0.137
	(0.110)	(0.111)	(0.122)	(0.123)
× Innovation intensity		0.158		-0.098
		(0.142)		(0.139)
Innovation intensity		0.057		0.089
		(0.047)		(0.068)
NTR rate	-2.314	-2.512	1.129	0.900
	(1.670)	(1.704)	(2.237)	(2.240)
× Innovation intensity		1.027		0.666
		(0.874)		(0.765)
Downstream X Post	0.501	0.492	0.965	0.979
	(0.597)	(0.602)	(0.707)	(0.715)
× Innovation intensity		-0.241		-0.019
		(0.525)		(0.348)
Upstream X Post	0.161	0.196	0.430	0.491
	(0.443)	(0.447)	(0.480)	(0.482)
× Innovation intensity		-0.497		-0.382
		(0.381)		(0.418)
Observations	6,500	6,500	6,500	6,500
Fixed effects	j, p	j, p	j, p	j, p

Notes: Controls include past 5-year U.S. patent growth in firms' own technology fields, log employment, firm age, NTR rate, dummy for publicly traded firms, dummy for firms with total imports > 0, dummy for firms with total exports > 0, and dummy for firms with imports from relative parties > 0. Estimates for industry (j) and the period (p) fixed effects as well as the constant are suppressed. Robust standard errors adjusted for clustering at the level of the firms' major industries are displayed below each coefficient. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table B.14: Overall response: different weights for firm-level tariff measures

	ΔPatents	ΔPatents	ΔSelf-cite	ΔSelf-cite
	(1)	(2)	(3)	(4)
NTR gap × Post	-0.139	-0.017	0.133	0.091
	(0.331)	(0.247)	(0.311)	(0.260)
NTR gap	0.943**	omitted	-0.240	omited
	(0.374)		(0.349)	
Post	-0.146	-0.194***	-0.024	-0.036
	(0.107)	(0.074)	(0.106)	(0.076)
NTR rate	-1.763	-2.360	1.614	0.368
	(1.533)	(1.871)	(1.792)	(2.373)
Observations	6,500	6,500	6,500	6,500
Fixed effects	j, p	j, p	j, p	j, p
Weights for tariffs	unweighted	major indust.	unweighted	major indust.

Notes: Table reports results of OLS generalized difference-in-differences regressions in which firm-level tariff measures are constructed with different weights. Controls include past 5-year U.S. patent growth in firms' own technology fields, log employment, firm age, NTR rate, dummy for publicly traded firms, dummy for firms with total imports > 0, dummy for firms with total exports > 0, and dummy for firms with imports from relative parties > 0 (full controls). Estimates for industry (j) and the period (p) fixed effects as well as the constant are suppressed. Robust standard errors adjusted for clustering at the level of the firms' major industries are displayed below each coefficient. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * p < 0.1, ** p < 0.05, *** p < 0.01.

Table B.15: Escape-competition effect: different weights for firm-level tariff measures

	ΔPatents (1)	ΔPatents (2)	ΔSelf-cite (3)	ΔSelf-cite (4)
NTR gap × Post	-0.131	-0.015	0.017	0.021
	(0.339)	(0.251)	(0.310)	(0.260)
× Innovation intensity	0.038	0.017	0.754***	0.745***
	(0.218)	(0.218)	(0.261)	(0.263)
NTR gap	0.962**	omitted	-0.189	omitted
	(0.376)		(0.350)	
× Innovation intensity	-0.268	-0.235	-0.380*	-0.395*
	(0.168)	(0.173)	(0.228)	(0.229)
Post	-0.150	-0.197***	0.004	-0.024
	(0.109)	(0.074)	(0.105)	(0.075)
× Innovation intensity	0.002	0.008	-0.191**	-0.185**
	(0.071)	(0.071)	(0.082)	(0.083)
Innovation intensity	0.065	0.056	0.085	0.085
	(0.045)	(0.046)	(0.066)	(0.066)
NTR rate	-1.839	-2.482	1.468	0.256
	(1.541)	(1.874)	(1.795)	(2.372)
× Innovation intensity	0.583	0.584	0.576	0.666
	(0.517)	(0.525)	(0.489)	(0.477)
Observations	6,500	6,500	6,500	6,500
Fixed effects	j, p	j, p	j, p	j, p
Weights for tariffs	unweighted	major indust.	unweighted	major indust.

Notes: Table reports results of OLS generalized difference-in-differences regressions in which firm-level tariff measures are constructed with different weights. Full controls are included. Estimates for industry (j) and the period (p) fixed effects as well as the constant are suppressed. Robust standard errors adjusted for clustering at the level of the firms' major industries are displayed below each coefficient. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * p < 0.1, ** p < 0.05, *** p < 0.01.

Table B.16: Use inverse of the propensity scores to re-weight observations

	ΔPatents (1)	ΔPatents (2)	ΔSelf-cite (3)	ΔSelf-cite (4)
NTR gap × Post	-0.085	-0.058	-0.065	-0.294
	(0.417)	(0.420)	(0.362)	(0.351)
× Innovation intensity		-0.033		0.794***
		(0.271)		(0.269)
Observations	6,500	6,500	6,500	6,500
Fixed effects	j, p	j, p	j, p	j, p
Regression weights	inv. propens.	inv. propens.	inv. propens.	inv. propens.

Notes: Table reports results of OLS generalized difference-in-differences regressions in which observations are weighted by the inverse of the propensity scores from logit model (y = indicator for analysis sample). Full controls are included. Estimates for industry (j) and the period (p) fixed effects as well as the constant are suppressed. Robust standard errors adjusted for clustering at the level of the firms' major industries are displayed below each coefficient. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * p < 0.1, ** p < 0.05, *** p < 0.01.

Table B.17: Add the cumulative number of patents as a firm-level control variable

	ΔPatents	ΔPatents	ΔSelf-cite	ΔSelf-cite
	(1)	(2)	(3)	(4)
NTR gap × Post	-0.000	0.004	0.088	-0.015
	(0.279)	(0.287)	(0.290)	(0.289)
× Innovation intensity		-0.011		0.786***
		(0.231)		(0.268)
Observations	6,500	6,500	6,500	6,500
Fixed effects	j, p	j, p	j, p	j, p

Notes: Table reports results of OLS generalized difference-in-differences regressions in which firm-level cumulative number of patents are included as a control. Full controls are included. Estimates for industry (j) and the period (p) fixed effects as well as the constant are suppressed. Robust standard errors adjusted for clustering at the level of the firms' major industries are displayed below each coefficient. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table B.18: Cluster standard errors on firms

	ΔPatents	ΔPatents	ΔSelf-cite	ΔSelf-cite
	(1)	(2)	(3)	(4)
NTR gap × Post	0.004	0.010	0.103	-0.000
	(0.287)	(0.290)	(0.308)	(0.311)
× Innovation intensity		-0.012		0.784***
		(0.235)		(0.274)
Observations	6,500	6,500	6,500	6,500
Fixed effects	j, p	j, p	j, p	j, p
se. cluster	firmid	firmid	firmid	firmid

Notes: Table reports results of OLS generalized difference-in-differences regressions in which robust standard errors are adjusted for clustering at the firm-level. Full controls are included. Estimates for industry (j) and the period (p) fixed effects as well as the constant are suppressed. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table B.19: Effect of foreign competition on product add

	Log number of products added	Log number of products added
	(1)	(2)
NTR gap × Post	-0.209***	-0.208***
	(0.067)	(0.068)
× Innovation intensity		-0.554***
		(0.196)
Post × Innovation intensity		0.024
		(0.088)
Innovation intensity		0.227***
		(0.042)
Observations	497,000	497,000
Fixed effects	j, p	j, p

Notes: Controls include past 5-year U.S. patent growth in firms' own technology fields, log payroll, firm age, NTR rate and its interaction with innovation intensity, dummy for publicly traded firms, dummy for firms with total imports > 0, dummy for firms with total exports > 0, and dummy for firms with imports from relative parties > 0. Estimates for industry-period (jp) fixed effects as well as the constant are suppressed. Robust standard errors adjusted for clustering at the level of the firms' major industries are displayed below each coefficient. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

B.1.6 Technological Barrier Effect

Table B.20: Technological-barrier effect

	ΔPatents (1)	ΔPatents (2)	ΔSelf-cite (3)	ΔSelf-cite (4)
Past 5yr Δforeign patent, outside of own technology field	-5.984**	-5.209*	9.076***	8.712***
	(2.756)	(2.733)	(2.711)	(2.740)
× Innovation intensity		0.161		-0.365
		(0.240)		(0.264)
Past 5yr Δforeign patent, inside of own technology field	0.005	-0.006	0.033	0.021
	(0.079)	(0.081)	(0.081)	(0.082)
× Innovation intensity		0.048		0.047
		(0.055)		(0.059)
Observation	7,600	7,600	7,600	7,600
Fixed effects	*jp*	*jp*	*jp*	*jp*

Notes: Full controls except for the NTR rate are included. Estimates for industry-period (jp) fixed effects as well as the constant are suppressed. Robust standard errors adjusted for clustering at the firm-level are displayed below each coefficient. Observations are unweighted. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table B.21: Effect of concurrent technological shocks

	ΔPatents (1)	ΔPatents (2)	ΔSelf-cite (3)	ΔSelf-cite (4)
5yr Δforeign patent, outside of own technology field	-8.680**	-7.637**	14.15***	13.56***
	(3.546)	(3.521)	(3.540)	(3.565)
× Innovation intensity		-0.063		0.081
		(0.114)		(0.122)
5yr Δforeign patent, inside of own technology field	0.212***	0.228***	0.133*	0.118
	(0.075)	(0.077)	(0.075)	(0.076)
× Innovation intensity		-0.069		0.067
		(0.062)		(0.074)
Observation	7,600	7,600	7,600	7,600
Fixed effects	*jp*	*jp*	*jp*	*jp*

Notes: Description the same as Table B.20.

B.1.7 Industry-Level Regression

To estimate the effect of Chinese competition shock on the industry-level business dynamism statistics, I run the following regression model for the years from

1992 to 2007

$$Y_{jt} \; = \beta_1 PostPNTR \times NTRGap_j \; + \; \mathbf{X}_{jt} \, \gamma_1 \; + \; \mathbf{X}_{j0} \, \gamma_2 \; + \; \delta_j \; + \; \delta_t \; + \; \alpha \; + \; \varepsilon_{jt} \; , \tag{B.1}$$

where Y_{jt} is i) log of employment, ii) young firm share iii) startup rates, iv) exit rates, v) 90th percentile of firm employment growth rates, and vi) 10th percentile of firm employment growth rates.

Table B.22: Industry-level effect

	log(Emp)	log(Emp) by young firms	Share of young firms	Startup rate	Exit rate	P90 ΔEmp	P10 ΔEmp
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
NTR gap × Post	-0.632***	-0.809**	-0.102**	-0.036***	0.014	-0.718***	-0.014
	(0.231)	(0.364)	(0.047)	(0.012)	(0.010)	(0.195)	(0.134)
Obervations	6,200	6,200	6,200	6,200	6,200	6,200	6,200
Fixed effects	j, t	j, t	j, t	j, t	j, t	j, t	j, t
Reg. Weights	1992 emp.	1992 emp.	1992 emp.	1992 emp.	1992 emp.	1992 emp.	1992 emp.
implied impact	-20.19%	-26.54%	-3.01pp	-1.05pp		-23.24pp	

Notes: Controls include NTR rate. Robust standard errors adjusted for clustering at the industry (j) level are displayed below each coefficient. Estimates for industry and the year (t) fixed effects as well as the constant are suppressed. Observations are weighted by 1992 industry employment. Final row reports the predicted change in the dependent variable implied by the regression coefficient. Observation counts are rounded due to Census Bureau disclosure avoidance procedures. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

B.2 Theory Appendix

B.2.1 Value Function

B.2.1.1 Conditional Expectation

For

$$\Phi^f \equiv \left\{ \ \left(q_j \ , \ \Delta_j^H \ , \ \Delta_j^F \ , \ \Delta_j^G \right) \ \right\}_{j \in \mathcal{J}^f} \ ,$$

conditional expectation term, $\mathbb{E}\Big[V\left(\Phi^{f\prime} \,\middle|\, \Phi^f \ , \ \{z_j\}_{j\in\mathcal{J}^f} \ , \ x\right)\Big]$ is equal to

$$\int_{\Phi_{-j}} \sum_{\mathcal{I}^x=0}^{1} \ \sum_{\mathcal{I}_{-j}^{ZH}=0}^{1} \ \sum_{\mathcal{I}_{-j}^{ZF}=0}^{1} \ \sum_{c-t_{-j}=\text{win}}^{\text{lose}} \ \sum_{\mathcal{I}_i^Z \ , \ \ldots \ , \mathcal{I}_{n^f}^Z=0}^{1} \ \sum_{\mathcal{I}_i^{ZF} \ , \ \ldots \ , \mathcal{I}_{n^f}^{ZF}=0}^{1} \ \sum_{\mathcal{I}_i^{\overline{x}} \ , \ \ldots \ , \mathcal{I}_{n^f}^{\overline{x}}=0}^{2} \ \sum_{c-t_1 \ , \ \ldots \ , c-t_{n^f}=\text{win}}^{\text{lose}}$$

$$\mu\left(\Phi_{-j}\right) \ x^{\mathcal{I}^x}(1-x)^{1-\mathcal{I}^x} \ \left(z^H\right)^{\mathcal{I}_{-j}^{ZH}} \left(1-z^H\right)^{1-\mathcal{I}_{-j}^{ZH}} \ \left(z^F\right)^{\mathcal{I}_{-j}^{ZF}} \left(1-z^F\right)^{1-\mathcal{I}_{-j}^{ZF}} \ \frac{1}{2}$$

$$\times \prod_{i=1}^{n^f} \Big[\ \left(z_{j_i}\right)^{\mathcal{I}_i^Z} \left(1-z_{j_i}\right)^{1-\mathcal{I}_i^Z} \ \left(z_{j_i}^F\right)^{\mathcal{I}_i^{ZF}} \left(1-z_{j_i}^F\right)^{1-\mathcal{I}_i^{ZF}} \ \left(1-\overline{x}\right)^{\mathcal{I}_i^{\overline{x}0}} \ \left(\overline{x}^H\right)^{\mathcal{I}_i^{\overline{x}1}} \ \left(\overline{x}^F\right)^{\mathcal{I}_i^{\overline{x}2}} \ \Big] \left(\frac{1}{2}\right)^{n^f}$$

$$\times V\Bigg(\Bigg[\bigcup_{i=1}^{n^f} \Big[\Big\{ \left(\Delta_{j_i}^{H\prime} q_{j_i} \ , \ \Delta_{j_i}^{H\prime} \ , \ \Delta_{j_i}^{F\prime} \ , \ \Delta_{j_i}^{G\prime} \right) \ \Big| \ \left(q_{j_i} \ , \ \Delta_{j_i}^H \ , \ \Delta_{j_i}^F \ , \ \Delta_{j_i}^G \right) \ , \ \mathcal{I}_i^Z \ , \ \mathcal{I}_i^{ZF} \ , \ \mathcal{I}_i^{\overline{x}} \ , \ c-t_i \Big\} \setminus \{\mathbf{0}\} \Big] \Bigg]$$

$$\bigcup \Big[\Big\{ \left(\Delta_{-j}^{H\prime} \, q_{-j} \ , \ \Delta_{-j}^{H\prime} \ , \ \Delta_{-j}^{F\prime} \ , \ \Delta_{-j}^{G\prime} \right) \ \Big| \ \left(q_{-j} \ , \ \Delta_{-j}^H \ , \ \Delta_{-j}^F \ , \ \Delta_{-j}^G \right)$$

$$, \mathcal{I}^{x} \ , \ \mathcal{I}^{ZH}_{-j} \ , \ \mathcal{I}^{ZF}_{-j} \ , \ c - t_{-j} \Big\} \setminus \{\mathbf{0}\} \Big] \Bigg) \, \mathbf{d} \left(\Phi_{-j} \right) \ ,$$

where $\mathcal{I}_i^{\bar{x}k}$ is an indicator function equal to 1 if $\mathcal{I}_i^{\bar{x}} = k$ for $k \in \{0, 1, \}$. Note that $\Delta^{H\prime}_{-j} = \frac{1+\eta}{\Delta^{H}_{-j}}$ for the case when business takeover is successful.

B.2.1.2 Aggregate Quality Evolution

B.2.1.2.1 Proof for $\mathcal{Q}$ and $\overline{q}$ Evolution

Proof. Here, I prove Proposition 8 for a general case. Application of proper index functions provides equation (2.19). Pick any product line j with product quality q_j^H and technology gaps $\mathbf{\Delta} \equiv \left(\Delta^H \ \Delta^F \ \Delta^G \right)$, $\Delta^G \in [\underline{\Omega}, \overline{\Omega}] \cup \{\infty\}$. Then with $\mathcal{P}\left(\mathbf{\Delta'} | \mathbf{\Delta}\right)$: probability of $\mathbf{\Delta}$ becoming $\mathbf{\Delta'}$, which is described in Appendix B.3.1, the conditional expected value of q_j^H conditioning on $\mathbf{\Delta}$ next period is equal to

$$\begin{aligned}
\mathbb{E}_{\mathbf{\Delta'}} \left[\, q_j^{H\prime} \, \middle| \, \mathbf{\Delta}, \ q_j^H \, \right] &= \mathbb{E}_{\mathbf{\Delta'}} \left[\, \Delta^{H\prime} q_j^{H} \, \middle| \, \mathbf{\Delta}, \ q_j^H \, \right] \\
&= \mathbb{E}_{\mathbf{\Delta'}} \left[\, \Delta^{H\prime} \, \middle| \, \mathbf{\Delta} \, \right] q_j^H \\
&= \left[\sum_{\mathbf{\Delta'}} \Delta^{H\prime} \, \mathcal{P}\left(\mathbf{\Delta'} | \mathbf{\Delta}\right) \right] q_j^H
\end{aligned}$$

where the second equality follows from $\mathbf{\Delta} \perp q_j^H$, thus, $\mathbf{\Delta'} \perp q_j^H$ for any j. Then,

$$\mathbb{E} \left[\, q_j^{H\prime} \, \middle| \, q_j^H \, \right] = \mathbb{E}_{\mathbf{\Delta}} \Big[\, \mathbb{E}_{\mathbf{\Delta'}} \left[\, q_j^{H\prime} \, \middle| \, \mathbf{\Delta}, \ q_j^H \, \right] \Big]$$

$$= \left[\sum_{\boldsymbol{\Delta}} \sum_{\boldsymbol{\Delta}'} \Delta^{H\prime} \, \mathcal{P}\left(\boldsymbol{\Delta}'|\boldsymbol{\Delta}\right) \, \mu(\boldsymbol{\Delta}) \right] q_j^H \, . \tag{B.2}$$

Summation of (B.2) over a proper subset provides law of motion for $\mathcal{Q}_{\tilde{c}H}$ and $\overline{q}_H$. For instance, since $\mathbb{E}\left[q_j^{H\prime}\right] = \overline{q}_H$ in equilibrium, by summing up (B.2) over possible q_j^H, we have

$$\overline{q}_H' = \left[\sum_{\boldsymbol{\Delta}} \sum_{\boldsymbol{\Delta}'} \Delta^{H\prime} \, \mathcal{P}\left(\boldsymbol{\Delta}'|\boldsymbol{\Delta}\right) \, \mu(\boldsymbol{\Delta}) \right] \overline{q}_H \, ,$$

which gives us

$$q_H = \sum_{\boldsymbol{\Delta}} \sum_{\boldsymbol{\Delta}'} \Delta^{H\prime} \, \mathcal{P}\left(\boldsymbol{\Delta}'|\boldsymbol{\Delta}\right) \, \mu(\boldsymbol{\Delta}) - 1 \, .$$

The law of motion for country F can be defined symmetrically. ■

B.3 Technical Appendix

B.3.1 Technology-Gaps Evolution

Denote $\overline{x}^H$ as total external innovation intensity by domestic firms (both incumbents and startups), $\overline{x}^F$ as that of foreign counterparts, and $\overline{x} \equiv \overline{x}^H + \overline{x}^F$ as a total external innovation intensity in the economy.

B.3.1.1 (H H) case

For any q_j, the followings hold, where the last column specifies ownership change in domestic and foreign market.

$$
(H\ H),\ (\Delta^1\ \Delta^1),\ \infty \rightarrow
\begin{cases}
\text{a. no ext. innov.} & \Delta^{G\prime} = \infty & & \\
(H\ H),\ (\Delta^1\ \Delta^1) & & \text{w/ } (1-z^H)(1-\overline{x}) & \text{x x} \\
(H\ H),\ (\Delta^2\ \Delta^2) & & \text{w/ } z^H\,(1-\overline{x}) & \text{x x} \\
\text{b. H ext. innov.} & \Delta^{G\prime} = \infty & & \\
(H\ H),\ (\Delta^3\ \Delta^3) & & \text{w/ } (1-z^H)\,\overline{x}^H & \text{o o} \\
(H\ H),\ (\Delta^3\ \Delta^3) & & \text{w/ } z^H\,\overline{x}^H & \text{o o} \\
\text{c-i. F ext. innov.} & \Delta^{G\prime} = 1/\eta & \text{no int. innov.} & \\
(F\ F),\ (\Delta^3\ \Delta^3) & \text{if } \Delta^{G\prime} < \underline{\Omega} & & \text{o o} \\
(H\ F),\ (\Delta^1\ \Delta^3) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/ } (1-z^H)\,\overline{x}^F & \text{x o} \\
(H\ H),\ (\Delta^1\ \Delta^1) & \text{if } \Delta^{G\prime} > \overline{\Omega} & & \text{x x} \\
\text{c-ii. F ext. innov.} & \Delta^{G\prime} = \lambda/\eta & \text{H int. innov.} & \\
(F\ F),\ (\Delta^3\ \Delta^3) & \text{if } \Delta^{G\prime} < \underline{\Omega} & & \text{o o} \\
(H\ F),\ (\Delta^2\ \Delta^3) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/ } z^H\,\overline{x}^F & \text{x o} \\
(H\ H),\ (\Delta^2\ \Delta^2) & \text{if } \Delta^{G\prime} > \overline{\Omega} & & \text{x x}
\end{cases}
\tag{B.3}
$$

$$
(H\ H),\ (\Delta^2\ \Delta^2),\ \infty \rightarrow \begin{cases}
\text{a. no ext. innov.} & \Delta^{G\prime} = \infty & & \\
(H\ H),\ (\Delta^1\ \Delta^1) & & \text{w/}\ \ (1-z^H)(1-\overline{x}) & \text{x x} \\
(H\ H),\ (\Delta^2\ \Delta^2) & & \text{w/}\ \ z^H\,(1-\overline{x}) & \text{x x} \\
\text{b. H ext. innov.} & \Delta^{G\prime} = \infty & & \\
(H\ H),\ (\Delta^4\ \Delta^4) & & \text{w/}\ \ (1-z^H)\,\overline{x}^H & \text{o o} \\
(H\ H),\ (\Delta^2\ \Delta^2) & & \text{w/}\ \ z^H\,\overline{x}^H & \text{x x} \\
\text{c-i. F ext. innov.} & \Delta^{G\prime} = \lambda/\eta & \text{no int. innov.} & \\
(F\ F),\ (\Delta^4\ \Delta^4) & \text{if } \Delta^{G\prime} < \underline{\Omega} & & \text{o o} \\
(H\ F),\ (\Delta^1\ \Delta^4) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/}\ \ (1-z^H)\,\overline{x}^F & \text{x o} \\
(H\ H),\ (\Delta^1\ \Delta^1) & \text{if } \Delta^{G\prime} > \overline{\Omega} & & \text{x x} \\
\text{c-ii. F ext. innov.} & \Delta^{G\prime} = \lambda^2/\eta & \text{H int. innov.} & \\
(F\ F),\ (\Delta^4\ \Delta^4) & \text{if } \Delta^{G\prime} < \underline{\Omega} & & \text{o o} \\
(H\ F),\ (\Delta^2\ \Delta^4) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/}\ \ z^H\,\overline{x}^F & \text{x o} \\
(H\ H),\ (\Delta^2\ \Delta^2) & \text{if } \Delta^{G\prime} > \overline{\Omega} & & \text{x x}
\end{cases} \tag{B.4}
$$

$$
(H\ H),\ (\Delta^3\ \Delta^3),\ \infty \rightarrow \begin{cases}
\text{a. no ext. innov.} & \Delta^{G\prime} = \infty & & \\
(H\ H),\ (\Delta^1\ \Delta^1) & & \text{w/ } (1 - z^H)(1 - \overline{x}) & \text{x x} \\
(H\ H),\ (\Delta^2\ \Delta^2) & & \text{w/ } z^H\,(1 - \overline{x}) & \text{x x} \\
\text{b. H ext. innov.} & \Delta^{G\prime} = \infty & & \\
(H\ H),\ (\Delta^1\ \Delta^1) & & \text{w/ } (1 - z^H)\,\overline{x}^H & \frac{1}{2}\ \frac{1}{2} \\
(H\ H),\ (\Delta^2\ \Delta^2) & & \text{w/ } z^H\,\overline{x}^H & \text{x x} \\
\text{c-i. F ext. innov.} & \Delta^{G\prime} = 1 & \text{no int. innov.} & \\
(F\ F),\ (\Delta^1\ \Delta^1) & \text{if } \Delta^{G\prime} < \underline{\Omega} & & \text{o o} \\
(H\ F),\ (\Delta^1\ \Delta^1) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/ } (1 - z^H)\,\overline{x}^F & \text{x o} \\
(H\ H),\ (\Delta^1\ \Delta^1) & \text{if } \Delta^{G\prime} > \overline{\Omega} & & \text{x x} \\
\text{c-ii. F ext. innov.} & \Delta^{G\prime} = \lambda & \text{H int. innov.} & \\
(F\ F),\ (\Delta^1\ \Delta^1) & \text{if } \Delta^{G\prime} < \underline{\Omega} & & \text{o o} \\
(H\ F),\ (\Delta^2\ \Delta^1) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/ } z^H\,\overline{x}^F & \text{x o} \\
(H\ H),\ (\Delta^2\ \Delta^2) & \text{if } \Delta^{G\prime} > \overline{\Omega} & & \text{x x}
\end{cases} \tag{B.5}
$$

$$
(H\ H),\ (\Delta^4\ \Delta^4),\ \infty \rightarrow
\begin{cases}
\text{a. no ext. innov.} & \Delta^{G\prime} = \infty & & \\
(H\ H),\ (\Delta^1\ \Delta^1) & & \text{w/ } (1-z^H)(1-\overline{x}) & \text{x x} \\
(H\ H),\ (\Delta^2\ \Delta^2) & & \text{w/ } z^H\ (1-\overline{x}) & \text{x x} \\
\text{b. H ext. innov.} & \Delta^{G\prime} = \infty & & \\
(H\ H),\ (\Delta^2\ \Delta^2) & & \text{w/ } (1-z^H)\ \overline{x}^H & \text{o o} \\
(H\ H),\ (\Delta^2\ \Delta^2) & & \text{w/ } z^H\ \overline{x}^H & \frac{1}{2}\ \frac{1}{2} \\
\text{c-i. F ext. innov.} & \Delta^{G\prime} = 1/\lambda & \text{no int. innov.} & \\
(F\ F),\ (\Delta^2\ \Delta^2) & \text{if } \Delta^{G\prime} < \underline{\Omega} & & \text{o o} \\
(H\ F),\ (\Delta^1\ \Delta^2) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/ } (1-z^H)\ \overline{x}^F & \text{x o} \\
(H\ H),\ (\Delta^1\ \Delta^1) & \text{if } \Delta^{G\prime} > \overline{\Omega} & & \text{x x} \\
\text{c-ii. F ext. innov.} & \Delta^{G\prime} = 1 & \text{H int. innov.} & \\
(F\ F),\ (\Delta^2\ \Delta^2) & \text{if } \Delta^{G\prime} < \underline{\Omega} & & \text{o o} \\
(H\ F),\ (\Delta^2\ \Delta^2) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/ } z^H\ \overline{x}^F & \text{x o} \\
(H\ H),\ (\Delta^2\ \Delta^2) & \text{if } \Delta^{G\prime} > \overline{\Omega} & & \text{x x}
\end{cases}
\tag{B.6}
$$

B.3.1.2 (F F) case

For any q_j, the followings hold.

$$
(F\ F),\ (\Delta^1\ \Delta^1),\ -\infty \rightarrow \begin{cases}
\text{a. no ext. innov.} & \Delta^{G\prime} = -\infty & & \\
(F\ F),\ (\Delta^1\ \Delta^1) & & \text{w/ } (1-z^F)(1-\overline{x}) & \text{x x} \\
(F\ F),\ (\Delta^2\ \Delta^2) & & \text{w/ } z^F\,(1-\overline{x}) & \text{x x} \\
\text{b-i. H ext. innov.} & \Delta^{G\prime} = \eta & \text{no int. innov.} & \\
(F\ F),\ (\Delta^1\ \Delta^1) & \text{if } \Delta^{G\prime} < \underline{\Omega} & & \text{x x} \\
(H\ F),\ (\Delta^3\ \Delta^1) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/ } (1-z^F)\,\overline{x}^H & \text{o x} \\
(H\ H),\ (\Delta^3\ \Delta^3) & \text{if } \Delta^{G\prime} > \overline{\Omega} & & \text{o o} \\
\text{b-ii. H ext. innov.} & \Delta^{G\prime} = \eta/\lambda & \text{F int. innov.} & \\
(F\ F),\ (\Delta^2\ \Delta^2) & \text{if } \Delta^{G\prime} < \underline{\Omega} & & \text{x x} \\
(H\ F),\ (\Delta^3\ \Delta^2) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/ } z^F\,\overline{x}^H & \text{o x} \\
(H\ H),\ (\Delta^3\ \Delta^3) & \text{if } \Delta^{G\prime} > \overline{\Omega} & & \text{o o} \\
\text{c. F ext. innov.} & \Delta^{G\prime} = -\infty & & \\
(F\ F),\ (\Delta^3\ \Delta^3) & & \text{w/ } (1-z^F)\,\overline{x}^F & \text{o o} \\
(F\ F),\ (\Delta^3\ \Delta^3) & & \text{w/ } z^F\,\overline{x}^F & \text{o o}
\end{cases} \tag{B.7}
$$

$$
(F\,F),\ (\Delta^2\,\Delta^2),\ -\infty \rightarrow
\begin{cases}
\text{a. no ext. innov.} & \Delta^{G\prime} = -\infty & & \\
(F\,F),\ (\Delta^1\,\Delta^1) & & \text{w/ } (1-z^F)(1-\overline{x}) & \text{x x} \\
(F\,F),\ (\Delta^2\,\Delta^2) & & \text{w/ } z^F\,(1-\overline{x}) & \text{x x} \\
\text{b-i. H ext. innov.} & \Delta^{G\prime} = \eta/\lambda & \text{no int. innov.} & \\
(F\,F),\ (\Delta^1\,\Delta^1) & \text{if } \Delta^{G\prime} < \underline{\Omega} & & \text{x x} \\
(H\,F),\ (\Delta^4\,\Delta^1) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/ } (1-z^F)\,\overline{x}^H & \text{o x} \\
(H\,H),\ (\Delta^4\,\Delta^4) & \text{if } \Delta^{G\prime} > \overline{\Omega} & & \text{o o} \\
\text{b-ii. H ext. innov.} & \Delta^{G\prime} = \eta/\lambda^2 & \text{F int. innov.} & \\
(F\,F),\ (\Delta^2\,\Delta^2) & \text{if } \Delta^{G\prime} < \underline{\Omega} & & \text{x x} \\
(H\,F),\ (\Delta^4\,\Delta^2) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/ } z^F\,\overline{x}^H & \text{o x} \\
(H\,H),\ (\Delta^4\,\Delta^4) & \text{if } \Delta^{G\prime} > \overline{\Omega} & & \text{o o} \\
\text{c. F ext. innov.} & \Delta^{G\prime} = -\infty & & \\
(F\,F),\ (\Delta^4\,\Delta^4) & & \text{w/ } (1-z^F)\,\overline{x}^F & \text{o o} \\
(F\,F),\ (\Delta^2\,\Delta^2) & & \text{w/ } z^F\,\overline{x}^F & \text{x x}
\end{cases}
\tag{B.8}
$$

$$
(F\,F),\;(\Delta^3\,\Delta^3),\;-\infty \rightarrow
\begin{cases}
\text{a. no ext. innov.} & \Delta^{G\prime} = -\infty & & \\
(F\,F),\;(\Delta^1\,\Delta^1) & & \text{w/}\;\;(1-z^F)(1-\overline{x}) & \text{x x} \\
(F\,F),\;(\Delta^2\,\Delta^2) & & \text{w/}\;\;z^F\,(1-\overline{x}) & \text{x x} \\
\text{b-i. H ext. innov.} & \Delta^{G\prime} = 1 & \text{no int. innov.} & \\
(F\,F),\;(\Delta^1\,\Delta^1) & \text{if } \Delta^{G\prime} < \underline{\Omega} & & \text{x x} \\
(H\,F),\;(\Delta^1\,\Delta^1) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\,\overline{\Omega}\right] & \text{w/}\;\;(1-z^F)\,\overline{x}^H & \text{o x} \\
(H\,H),\;(\Delta^1\,\Delta^1) & \text{if } \Delta^{G\prime} > \overline{\Omega} & & \text{o o} \\
\text{b-ii. H ext. innov.} & \Delta^{G\prime} = 1/\lambda & \text{F int. innov.} & \\
(F\,F),\;(\Delta^2\,\Delta^2) & \text{if } \Delta^{G\prime} < \underline{\Omega} & & \text{x x} \\
(H\,F),\;(\Delta^1\,\Delta^2) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\,\overline{\Omega}\right] & \text{w/}\;\;z^F\,\overline{x}^H & \text{o x} \\
(H\,H),\;(\Delta^1\,\Delta^1) & \text{if } \Delta^{G\prime} > \overline{\Omega} & & \text{o o} \\
\text{c. F ext. innov.} & \Delta^{G\prime} = -\infty & & \\
(F\,F),\;(\Delta^1\,\Delta^1) & & \text{w/}\;\;(1-z^F)\,\overline{x}^F & \tfrac{1}{2}\;\tfrac{1}{2} \\
(F\,F),\;(\Delta^2\,\Delta^2) & & \text{w/}\;\;z^F\,\overline{x}^F & \text{x x}
\end{cases}
\tag{B.9}
$$

$$
(F\,F),\ (\Delta^4\,\Delta^4),\ -\infty \rightarrow \begin{cases}
\text{a. no ext. innov.} & \Delta^{G\prime} = -\infty & & \\
(F\,F),\ (\Delta^1\,\Delta^1) & & \text{w/ } (1-z^F)(1-\overline{x}) & \text{x x} \\
(F\,F),\ (\Delta^2\,\Delta^2) & & \text{w/ } z^F\,(1-\overline{x}) & \text{x x} \\
\text{b-i. H ext. innov.} & \Delta^{G\prime} = \lambda & \text{no int. innov.} & \\
(F\,F),\ (\Delta^1\,\Delta^1) & \text{if } \Delta^{G\prime} < \underline{\Omega} & & \text{x x} \\
(H\,F),\ (\Delta^2\,\Delta^1) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\, \overline{\Omega}\right] & \text{w/ } (1-z^F)\,\overline{x}^H & \text{o x} \\
(H\,H),\ (\Delta^2\,\Delta^2) & \text{if } \Delta^{G\prime} > \overline{\Omega} & & \text{o o} \\
\text{b-ii. H ext. innov.} & \Delta^{G\prime} = 1 & \text{F int. innov.} & \\
(F\,F),\ (\Delta^2\,\Delta^2) & \text{if } \Delta^{G\prime} < \underline{\Omega} & & \text{x x} \\
(H\,F),\ (\Delta^2\,\Delta^2) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\, \overline{\Omega}\right] & \text{w/ } z^F\,\overline{x}^H & \text{o x} \\
(H\,H),\ (\Delta^2\,\Delta^2) & \text{if } \Delta^{G\prime} > \overline{\Omega} & & \text{o o} \\
\text{c. F ext. innov.} & \Delta^{G\prime} = -\infty & & \\
(F\,F),\ (\Delta^2\,\Delta^2) & & \text{w/ } (1-z^F)\,\overline{x}^F & \text{o o} \\
(F\,F),\ (\Delta^2\,\Delta^2) & & \text{w/ } z^F\,\overline{x}^F & \tfrac{1}{2}\ \tfrac{1}{2}
\end{cases}
\tag{B.10}
$$

B.3.1.3 (H F) case

For any q_j, $\Delta^{D\ell}$, $\Delta^{F\ell}$, and Δ^G, the followings hold. Note that in all of below cases, $\Delta^G \in \left[\underline{\Omega},\, \overline{\Omega}\right]$. Thus, if $\Delta^{G\prime} < \Delta^G$, then $(H\,H)$ cannot be realized next period, and if $\Delta^{G\prime} > \Delta^G$, then $(F\,F)$ cannot be realized.

B.3.1.3.1 No External Innovation

$$
(H\ F), (\Delta^{D\ell}\ \Delta^{F\ell}),\ \Delta^{G} \rightarrow
$$

$$
\begin{cases}
\text{a.} & \Delta^{G\prime} = \Delta^{G} & \text{no int. innov.} & \\
(H\ F),\ (\Delta^{1}\ \Delta^{1}) & \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/}\ \ (1-z^{H})(1-z^{F})(1-\overline{x}) & \text{x x} \\
\text{b.} & \Delta^{G\prime} = \lambda\Delta^{G} & \text{H int. innov.} & \\
(H\ F),\ (\Delta^{2}\ \Delta^{1}) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/}\ \ z^{H}\,(1-z^{F})(1-\overline{x}) & \text{x x} \\
(H\ H),\ (\Delta^{2}\ \Delta^{2}) & \text{if } \Delta^{G\prime} > \overline{\Omega} & & \text{x o} \\
\text{c.} & \Delta^{G\prime} = \frac{1}{\lambda}\Delta^{G} & \text{F int. innov.} & \\
(F\ F),\ (\Delta^{2}\ \Delta^{2}) & \text{if } \Delta^{G\prime} < \underline{\Omega} & \text{w/}\ \ (1-z^{H})\,z^{F}\,(1-\overline{x}) & \text{o x} \\
(H\ F),\ (\Delta^{1}\ \Delta^{2}) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & & \text{x x} \\
\text{d.} & \Delta^{G\prime} = \Delta^{G} & \text{all int. innov.} & \\
(H\ F),\ (\Delta^{2}\ \Delta^{2}) & \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/}\ \ z^{H}\,z^{F}\,(1-\overline{x}) & \text{x x}
\end{cases}
\tag{B.11}
$$

B.3.1.3.2 External Innovation by a Domestic Firm

Since domestic firm builds its external innovation on the past-period domestic technology, technology gap in foreign market does not matter in terms of realized outcomes in this case.

$$
(H\ F), (\Delta^{1}\ \Delta^{F\ell}),\ \Delta^{G} \rightarrow
$$

$$
\begin{cases}
\text{a.} & \Delta^{G\prime} = \eta \Delta^{G} & \text{no int. innov.} & \\
(H\ F),\ (\Delta^3\ \Delta^1) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/ } (1-z^H)(1-z^F)\,\overline{x}^H & \text{o x} \\
(H\ H),\ (\Delta^3\ \Delta^3) & \text{if } \Delta^{G\prime} > \overline{\Omega} & & \text{o o} \\
\text{b.} & \Delta^{G\prime} = \eta \Delta^{G} & \text{H int. innov.} & \\
(H\ F),\ (\Delta^3\ \Delta^1) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/ } z^H\,(1-z^F)\,\overline{x}^H & \text{o x} \\
(H\ H),\ (\Delta^3\ \Delta^3) & \text{if } \Delta^{G\prime} > \overline{\Omega} & & \text{o o} \\
\text{c.} & \Delta^{G\prime} = \frac{\eta}{\lambda} \Delta^{G} & \text{F int. innov.} & \\
(H\ F),\ (\Delta^3\ \Delta^2) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/ } (1-z^H)\,z^F\,\overline{x}^H & \text{o x} \\
(H\ H),\ (\Delta^3\ \Delta^3) & \text{if } \Delta^{G\prime} > \overline{\Omega} & & \text{o o} \\
\text{d.} & \Delta^{G\prime} = \frac{\eta}{\lambda} \Delta^{G} & \text{all int. innov.} & \\
(H\ F),\ (\Delta^3\ \Delta^2) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/ } z^H\,z^F\,\overline{x}^H & \text{o x} \\
(H\ H),\ (\Delta^3\ \Delta^3) & \text{if } \Delta^{G\prime} > \overline{\Omega} & & \text{o o}
\end{cases}
\tag{B.12}
$$

$(H\ F), (\Delta^2\ \Delta^{F\ell}),\ \Delta^G \rightarrow$

$$
\begin{cases}
\text{a.} & \Delta^{G\prime} = \frac{\eta}{\lambda}\Delta^{G} & \text{no int. innov.} & \\
(H\ F),\ (\Delta^4\ \Delta^1) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/ } (1-z^H)(1-z^F)\,\overline{x}^H & \text{o x} \\
(H\ H),\ (\Delta^4\ \Delta^4) & \text{if } \Delta^{G\prime} > \overline{\Omega} & & \text{o o} \\
\text{b.} & \Delta^{G\prime} = \lambda\Delta^{G} & \text{H int. innov.} & \\
(H\ F),\ (\Delta^2\ \Delta^1) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/ } z^H\,(1-z^F)\,\overline{x}^H & \text{x x} \\
(H\ H),\ (\Delta^2\ \Delta^2) & \text{if } \Delta^{G\prime} > \overline{\Omega} & & \text{x o} \\
\text{c.} & \Delta^{G\prime} = \frac{\eta}{\lambda^2}\Delta^{G} & \text{F int. innov.} & \\
(F\ F),\ (\Delta^2\ \Delta^2) & \text{if } \Delta^{G\prime} < \underline{\Omega} & \text{w/ } (1-z^H)\,z^F\,\overline{x}^H & \text{o x} \\
(H\ F),\ (\Delta^4\ \Delta^2) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & & \text{o x} \\
\text{d.} & \Delta^{G\prime} = \Delta^{G} & \text{all int. innov.} & \\
(H\ F),\ (\Delta^2\ \Delta^2) & \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/ } z^H\,z^F\,\overline{x}^H & \text{x x}
\end{cases}
\tag{B.13}
$$

$(H\ F), (\Delta^3\ \Delta^{F\ell}),\ \Delta^G \rightarrow$

$$
\begin{cases}
\text{a.} & \Delta^{G\prime} = \Delta^{G} & \text{no int. innov.} & \\
(H\ F),\ (\Delta^1\ \Delta^1) & \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/}\ \ (1-z^H)(1-z^F)\,\overline{x}^H & \tfrac{1}{2}\ \text{x} \\
\text{b.} & \Delta^{G\prime} = \lambda\Delta^{G} & \text{H int. innov.} & \\
(H\ F),\ (\Delta^2\ \Delta^1) & \text{if}\ \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/}\ \ z^H\,(1-z^F)\,\overline{x}^H & \text{x x} \\
(H\ H),\ (\Delta^2\ \Delta^2) & \text{if}\ \Delta^{G\prime} > \overline{\Omega} & & \text{x o} \\
\text{c.} & \Delta^{G\prime} = \frac{1}{\lambda}\Delta^{G} & \text{F int. innov.} & \\
(F\ F),\ (\Delta^2\ \Delta^2) & \text{if}\ \Delta^{G\prime} < \underline{\Omega} & \text{w/}\ \ (1-z^H)\,z^F\,\overline{x}^H & \text{o x} \\
(H\ F),\ (\Delta^1\ \Delta^2) & \text{if}\ \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & & \tfrac{1}{2}\ \text{x} \\
\text{d.} & \Delta^{G\prime} = \Delta^{G} & \text{all int. innov.} & \\
(H\ F),\ (\Delta^2\ \Delta^2) & \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/}\ \ z^H\,z^F\,\overline{x}^H & \text{x x}
\end{cases}
\tag{B.14}
$$

$(H\ F), (\Delta^4\ \Delta^{F\ell}),\ \Delta^G \rightarrow$

$$
\begin{cases}
\text{a.} & \Delta^{G\prime} = \lambda\Delta^{G} & \text{no int. innov.} & \\
(H\ F),\ (\Delta^2\ \Delta^1) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/ } (1-z^H)(1-z^F)\,\overline{x}^H & \text{o x} \\
(H\ H),\ (\Delta^2\ \Delta^2) & \text{if } \Delta^{G\prime} > \overline{\Omega} & & \text{o o} \\
\text{b.} & \Delta^{G\prime} = \lambda\Delta^{G} & \text{H int. innov.} & \\
(H\ F),\ (\Delta^2\ \Delta^1) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/ } z^H\,(1-z^F)\,\overline{x}^H & \frac{1}{2}\ \text{x} \\
(H\ H),\ (\Delta^2\ \Delta^2) & \text{if } \Delta^{G\prime} > \overline{\Omega} & & \frac{1}{2}\ \text{o} \\
\text{c.} & \Delta^{G\prime} = \Delta^{G} & \text{F int. innov.} & \\
(H\ F),\ (\Delta^2\ \Delta^2) & \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/ } (1-z^H)\,z^F\,\overline{x}^H & \text{o x} \\
\text{d.} & \Delta^{G\prime} = \Delta^{G} & \text{all int. innov.} & \\
(H\ F),\ (\Delta^2\ \Delta^2) & \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/ } z^H\,z^F\,\overline{x}^H & \frac{1}{2}\ \text{x}
\end{cases}
\tag{B.15}
$$

B.3.1.3.3 External Innovation by a Foreign Firm

$(H\ F), (\Delta^{D\ell}\ \Delta^1),\ \Delta^G \rightarrow$

$$
\begin{cases}
\text{a.} & \Delta^{G\prime} = \frac{1}{\eta}\Delta^{G} & \text{no int. innov.} & \\
(F\ F),\ (\Delta^3\ \Delta^3) & \text{if } \Delta^{G\prime} < \underline{\Omega} & \text{w/ } (1-z^H)(1-z^F)\,\overline{x}^F & \text{o o} \\
(H\ F),\ (\Delta^1\ \Delta^3) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & & \text{x o} \\
\text{b.} & \Delta^{G\prime} = \frac{\lambda}{\eta}\Delta^{G} & \text{H int. innov.} & \\
(F\ F),\ (\Delta^3\ \Delta^3) & \text{if } \Delta^{G\prime} < \underline{\Omega} & \text{w/ } z^H\,(1-z^F)\,\overline{x}^F & \text{o o} \\
(H\ F),\ (\Delta^2\ \Delta^3) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & & \text{x o} \\
\text{c.} & \Delta^{G\prime} = \frac{1}{\eta}\Delta^{G} & \text{F int. innov.} & \\
(F\ F),\ (\Delta^3\ \Delta^3) & \text{if } \Delta^{G\prime} < \underline{\Omega} & \text{w/ } (1-z^H)\,z^F\,\overline{x}^F & \text{o o} \\
(H\ F),\ (\Delta^1\ \Delta^3) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & & \text{x o} \\
\text{d.} & \Delta^{G\prime} = \frac{\lambda}{\eta}\Delta^{G} & \text{all int. innov.} & \\
(F\ F),\ (\Delta^3\ \Delta^3) & \text{if } \Delta^{G\prime} < \underline{\Omega} & \text{w/ } z^H\,z^F\,\overline{x}^F & \text{o o} \\
(H\ F),\ (\Delta^2\ \Delta^3) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & & \text{x o}
\end{cases}
\tag{B.16}
$$

$(H\ F), (\Delta^{D\ell}\ \Delta^2),\ \Delta^G \rightarrow$

$$
\begin{cases}
\text{a.} & \Delta^{G\prime} = \frac{\lambda}{\eta}\Delta^{G} & \text{no int. innov.} & \\
(F\ F),\ (\Delta^4\ \Delta^4) & \text{if } \Delta^{G\prime} < \underline{\Omega} & \text{w/ } (1-z^H)(1-z^F)\,\overline{x}^F & \text{o o} \\
(H\ F),\ (\Delta^1\ \Delta^4) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & & \text{x o} \\
\text{b.} & \Delta^{G\prime} = \frac{\lambda^2}{\eta}\Delta^{G} & \text{H int. innov.} & \\
(H\ F),\ (\Delta^2\ \Delta^4) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/ } z^H\,(1-z^F)\,\overline{x}^F & \text{x o} \\
(H\ H),\ (\Delta^2\ \Delta^2) & \text{if } \Delta^{G\prime} > \overline{\Omega} & & \text{x o} \\
\text{c.} & \Delta^{G\prime} = \frac{1}{\lambda}\Delta^{G} & \text{F int. innov.} & \\
(F\ F),\ (\Delta^2\ \Delta^2) & \text{if } \Delta^{G\prime} < \underline{\Omega} & \text{w/ } (1-z^H)\,z^F\,\overline{x}^F & \text{o x} \\
(H\ F),\ (\Delta^1\ \Delta^2) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & & \text{x x} \\
\text{d.} & \Delta^{G\prime} = \Delta^{G} & \text{all int. innov.} & \\
(H\ F),\ (\Delta^2\ \Delta^2) & \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/ } z^H\,z^F\,\overline{x}^F & \text{x x}
\end{cases}
\tag{B.17}
$$

$(H\ F), (\Delta^{D\ell}\ \Delta^3),\ \Delta^G \rightarrow$

$$
\begin{cases}
\text{a.} & \Delta^{G\prime} = \Delta^{G} & \text{no int. innov.} & \\
(H\ F),\ (\Delta^1\ \Delta^1) & \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/ } (1-z^H)(1-z^F)\,\overline{x}^F & \text{x } \frac{1}{2} \\
\text{b.} & \Delta^{G\prime} = \lambda\Delta^{G} & \text{H int. innov.} & \\
(H\ F),\ (\Delta^2\ \Delta^1) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/ } z^H\,(1-z^F)\,\overline{x}^F & \text{x } \frac{1}{2} \\
(H\ H),\ (\Delta^2\ \Delta^2) & \text{if } \Delta^{G\prime} > \overline{\Omega} & & \text{x o} \\
\text{c.} & \Delta^{G\prime} = \frac{1}{\lambda}\Delta^{G} & \text{F int. innov.} & \\
(F\ F),\ (\Delta^2\ \Delta^2) & \text{if } \Delta^{G\prime} < \underline{\Omega} & \text{w/ } (1-z^H)\,z^F\,\overline{x}^F & \text{o x} \\
(H\ F),\ (\Delta^1\ \Delta^2) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & & \text{x x} \\
\text{d.} & \Delta^{G\prime} = \Delta^{G} & \text{all int. innov.} & \\
(H\ F),\ (\Delta^2\ \Delta^2) & \Delta^{G\prime} \in \left[\underline{\Omega},\ \overline{\Omega}\right] & \text{w/ } z^H\,z^F\,\overline{x}^F & \text{x x}
\end{cases}
\tag{B.18}
$$

$$
(H\,F), (\Delta^{D\ell}\,\Delta^4),\ \Delta^G \rightarrow
\begin{cases}
\text{a.} & \Delta^{G\prime} = \frac{1}{\lambda}\Delta^G & \text{no int. innov.} & \\
(F\,F),\ (\Delta^2\,\Delta^2) & \Delta^{G\prime} < \underline{\Omega} & \text{w/ } (1-z^H)(1-z^F)\,\overline{x}^F & \text{o o} \\
(H\,F),\ (\Delta^1\,\Delta^2) & \Delta^{G\prime} \in \left[\underline{\Omega},\, \overline{\Omega}\right] & & \text{x o} \\
\text{b.} & \Delta^{G\prime} = \Delta^G & \text{H int. innov.} & \\
(H\,F),\ (\Delta^2\,\Delta^2) & \Delta^{G\prime} \in \left[\underline{\Omega},\, \overline{\Omega}\right] & \text{w/ } z^H\,(1-z^F)\,\overline{x}^F & \text{x o} \\
\text{c.} & \Delta^{G\prime} = \frac{1}{\lambda}\Delta^G & \text{F int. innov.} & \\
(F\,F),\ (\Delta^2\,\Delta^2) & \text{if } \Delta^{G\prime} < \underline{\Omega} & \text{w/ } (1-z^H)\,z^F\,\overline{x}^F & \text{o } \frac{1}{2} \\
(H\,F),\ (\Delta^1\,\Delta^2) & \text{if } \Delta^{G\prime} \in \left[\underline{\Omega},\, \overline{\Omega}\right] & & \text{x } \frac{1}{2} \\
\text{d.} & \Delta^{G\prime} = \Delta^G & \text{all int. innov.} & \\
(H\,F),\ (\Delta^2\,\Delta^2) & \Delta^{G\prime} \in \left[\underline{\Omega},\, \overline{\Omega}\right] & \text{w/ } z^H\,z^F\,\overline{x}^F & \text{x } \frac{1}{2}
\end{cases}
\tag{B.19}
$$

B.3.1.4 Inflows and Outflows

$\left(\Delta^{\ell^H}\ \Delta^{\ell^F}\ \ \Delta^G\right)$ for internal innovation intensity terms are omitted for notational simplicity. They should match to that of $\mu(\cdot)$. Here, $\sum_{\Delta^G}$ is sum over $\Delta^G \in [\underline{\Omega}, \overline{\Omega}]$.

$H\ H\ \Delta^1\ \Delta^1\ \infty$ case:

$$
\begin{aligned}
Outflow\left(\Delta^1\ \Delta^1\ \ \infty\right) &= \Big[z^H(1-\overline{x}) + (1-z^H)\overline{x}^H + z^H\overline{x}^H \\
&\quad + \left(\mathcal{I}_1\left(\frac{1}{\eta}\right) + \mathcal{I}_2\left(\frac{1}{\eta}\right)\right)(1-z^H)\overline{x}^F + z^H\overline{x}^F\Big]\mu\left(\Delta^1\ \Delta^1\ \ \infty\right) \\
&= \left[1 - \left[(1-z^H)(1-\overline{x}) + \mathcal{I}_3\left(\frac{1}{\eta}\right)(1-z^H)\overline{x}^F\right]\right]\mu\left(\Delta^1\ \Delta^1\ \ \infty\right)
\end{aligned}
$$

$$
\begin{aligned}
Inflow\left(\Delta^1\ \Delta^1\ \ \infty\right) &= \left[(1-z^H)(1-\overline{x}) + \mathcal{I}_3\left(\frac{\lambda}{\eta}\right)(1-z^H)\overline{x}^F\right]\mu\left(\Delta^2\ \Delta^2\ \ \infty\right) \\
&\quad + \left[(1-z^H)(1-\overline{x}) + (1-z^H)\overline{x}^H + \mathcal{I}_3\left(1\right)(1-z^H)\overline{x}^F\right]\mu\left(\Delta^3\ \Delta^3\ \ \infty\right) \\
&\quad + \left[(1-z^H)(1-\overline{x}) + \mathcal{I}_3\left(\frac{1}{\lambda}\right)(1-z^H)\overline{x}^F\right]\mu\left(\Delta^4\ \Delta^4\ \ \infty\right) \\
&\quad + \left[\mathcal{I}_3\left(1\right)(1-z^F)\overline{x}^H + \mathcal{I}_3\left(\frac{1}{\lambda}\right)z^F\overline{x}^H\right]\mu\left(\Delta^3\ \Delta^3\ \ -\infty\right) \qquad \text{(B.20)}
\end{aligned}
$$

$H\ H\ \Delta^2\ \Delta^2\ \infty$ case:

$$
Outflow\left(\Delta^2\ \Delta^2\ \ \infty\right) = \left[1 - \left[z^H(1-\overline{x}) + z^H\overline{x}^H + \mathcal{I}_3\left(\frac{\lambda^2}{\eta}\right)z^H\overline{x}^F\right]\right]\mu\left(\Delta^2\ \Delta^2\ \ \infty\right)
$$

$$
\begin{aligned}
Inflow\left(\Delta^2\ \Delta^2\ \ \infty\right) &= \left[z^H(1-\overline{x}) + \mathcal{I}_3\left(\frac{\lambda}{\eta}\right)z^H\overline{x}^F\right]\mu\left(\Delta^1\ \Delta^1\ \ \infty\right) \\
&\quad + \left[z^H(1-\overline{x}) + z^H\overline{x}^H + \mathcal{I}_3\left(\lambda\right)z^H\overline{x}^F\right]\mu\left(\Delta^3\ \Delta^3\ \ \infty\right) \\
&\quad + \left[z^H(1-\overline{x}) + z^H\overline{x}^H + \mathcal{I}_3\left(1\right)z^H\overline{x}^F\right]\mu\left(\Delta^4\ \Delta^4\ \ \infty\right)
\end{aligned}
$$

$$+ \left[\mathcal{I}_3\left(\lambda\right)(1-z^F)\overline{x}^H + \mathcal{I}_3\left(1\right)z^F\overline{x}^H\right]\mu\left(\Delta^4\;\Delta^4\;\;-\infty\right)$$

$$+ \sum_{\Delta^H}\sum_{\Delta^F}\sum_{\Delta^G}\mathcal{I}_3\left(\lambda\Delta^G\right)z^H(1-z^F)(1-\overline{x})\mu\left(\Delta^H\;\Delta^F\;\;\Delta^G\right)$$

$$+ \sum_{\Delta^H=\Delta^2}^{\Delta^3}\sum_{\Delta^F}\sum_{\Delta^G}\mathcal{I}_3\left(\lambda\Delta^G\right)z^H(1-z^F)\overline{x}^H\mu\left(\Delta^H\;\Delta^F\;\;\Delta^G\right)$$

$$+ \sum_{\Delta^F}\sum_{\Delta^G}\mathcal{I}_3\left(\lambda\Delta^G\right)(1-z^F)\overline{x}^H\mu\left(\Delta^4\;\Delta^F\;\;\Delta^G\right)$$

$$+ \sum_{\Delta^H}\sum_{\Delta^G}\mathcal{I}_3\left(\frac{\lambda^2}{\eta}\Delta^G\right)z^H(1-z^F)\overline{x}^H\mu\left(\Delta^H\;\Delta^2\;\;\Delta^G\right)$$

$$+ \sum_{\Delta^H}\sum_{\Delta^G}\mathcal{I}_3\left(\lambda\Delta^G\right)z^H(1-z^F)\overline{x}^H\mu\left(\Delta^H\;\Delta^3\;\;\Delta^G\right) \tag{B.21}$$

$H\ H\ \Delta^3\ \Delta^3\ \infty$ case:

$$Outflow\left(\Delta^3\;\Delta^3\;\;\infty\right) = \mu\left(\Delta^3\;\Delta^3\;\;\infty\right)$$

$$Inflow\left(\Delta^3\;\Delta^3\;\;\infty\right) = \overline{x}^H\mu\left(\Delta^1\;\Delta^1\;\;\infty\right) + \left[\mathcal{I}_3\left(\eta\right)(1-z^F)\overline{x}^H + \mathcal{I}_3\left(\frac{\eta}{\lambda}\right)z^F\overline{x}^H\right]\mu\left(\Delta^1\;\Delta^1\;\;-\infty\right)$$
$$+ \sum_{\Delta^F}\sum_{\Delta^G}\left[\mathcal{I}_3\left(\eta\Delta^G\right)(1-z^F)\overline{x}^H + \mathcal{I}_3\left(\frac{\eta}{\lambda}\Delta^G\right)z^F\overline{x}^H\right]\mu\left(\Delta^1\;\Delta^F\;\;\Delta^G\right) \tag{B.22}$$

$H\ H\ \Delta^4\ \Delta^4\ \infty$ case:

$$Outflow\left(\Delta^4\;\Delta^4\;\;\infty\right) = \mu\left(\Delta^4\;\Delta^4\;\;\infty\right)$$

$$Inflow\left(\Delta^4\;\Delta^4\;\;\infty\right) = (1-z^H)\overline{x}^H\mu\left(\Delta^2\;\Delta^2\;\;\infty\right)$$
$$+ \left[\mathcal{I}_3\left(\frac{\eta}{\lambda}\right)(1-z^F)\overline{x}^H + \mathcal{I}_3\left(\frac{\eta}{\lambda^2}\right)z^F\overline{x}^H\right]\mu\left(\Delta^2\;\Delta^2\;\;-\infty\right)$$

$$+\sum_{\Delta^F}\sum_{\Delta^G}\left[\mathcal{I}_3\left(\frac{\eta}{\lambda}\Delta^G\right)(1-z^H)(1-z^F)\overline{x}^H\right]\mu\left(\Delta^2\ \Delta^F\ \ \Delta^G\right) \tag{B.23}$$

$F\ F\ \Delta^1\ \Delta^1\ \ -\infty$ case:

$$Outflow\left(\Delta^1\ \Delta^1\ \ -\infty\right)=\left[1-\left[(1-z^F)(1-\overline{x})+\mathcal{I}_1\left(\eta\right)(1-z^F)\overline{x}^H\right]\right]\mu\left(\Delta^1\ \Delta^1\ \ -\infty\right)$$

$$\begin{aligned}
Inflow\left(\Delta^1\ \Delta^1\ \ -\infty\right)&=\left[\mathcal{I}_1\left(1\right)(1-z^H)\overline{x}^F+\mathcal{I}_1\left(\lambda\right)z^H\overline{x}^F\right]\mu\left(\Delta^3\ \Delta^3\ \ \infty\right)\\
&+\left[(1-z^F)(1-\overline{x})+\mathcal{I}_1\left(\frac{\eta}{\lambda}\right)(1-z^F)\overline{x}^H\right]\mu\left(\Delta^2\ \Delta^2\ \ -\infty\right)\\
&+\left[(1-z^F)(1-\overline{x})+\mathcal{I}_1\left(1\right)(1-z^F)\overline{x}^H+(1-z^F)\overline{x}^F\right]\mu\left(\Delta^3\ \Delta^3\ \ -\infty\right)\\
&+\left[(1-z^F)(1-\overline{x})+\mathcal{I}_1\left(\lambda\right)(1-z^F)\overline{x}^H\right]\mu\left(\Delta^4\ \Delta^4\ \ -\infty\right)
\end{aligned} \tag{B.24}$$

$F\ F\ \Delta^2\ \Delta^2\ \ -\infty$ case:

$$Outflow\left(\Delta^2\ \Delta^2\ \ -\infty\right)=\left[1-\left[z^F(1-\overline{x})+\mathcal{I}_1\left(\frac{\eta}{\lambda^2}\right)z^F\overline{x}^H+z^F\overline{x}^F\right]\right]\mu\left(\Delta^2\ \Delta^2\ \ -\infty\right)$$

$$\begin{aligned}
Inflow\left(\Delta^2\ \Delta^2\ \ -\infty\right)&=\left[\mathcal{I}_1\left(\frac{1}{\lambda}\right)(1-z^H)\overline{x}^F+\mathcal{I}_1\left(1\right)z^H\overline{x}^F\right]\mu\left(\Delta^4\ \Delta^4\ \ \infty\right)\\
&+\left[z^F(1-\overline{x})+\mathcal{I}_1\left(\frac{\eta}{\lambda}\right)z^F\overline{x}^H\right]\mu\left(\Delta^1\ \Delta^1\ \ -\infty\right)\\
&+\left[z^F(1-\overline{x})+\mathcal{I}_1\left(\frac{1}{\lambda}\right)z^F\overline{x}^H+z^F\overline{x}^F\right]\mu\left(\Delta^3\ \Delta^3\ \ -\infty\right)\\
&+\left[z^F(1-\overline{x})+\mathcal{I}_1\left(1\right)z^F\overline{x}^H+\overline{x}^F\right]\mu\left(\Delta^4\ \Delta^4\ \ -\infty\right)\\
&+\sum_{\Delta^H}\sum_{\Delta^F}\sum_{\Delta^G}\left[\mathcal{I}_1\left(\frac{1}{\lambda}\Delta^G\right)(1-z^H)z^F(1-\overline{x})\right]\mu\left(\Delta^H\ \Delta^F\ \ \Delta^G\right)\\
&+\sum_{\Delta^F}\sum_{\Delta^G}\left[\mathcal{I}_1\left(\frac{\eta}{\lambda^2}\Delta^G\right)(1-z^H)z^F\overline{x}^H\right]\mu\left(\Delta^2\ \Delta^F\ \ \Delta^G\right)
\end{aligned}$$

$$+\sum_{\Delta^F}\sum_{\Delta^G}\left[\mathcal{I}_1\left(\frac{1}{\lambda}\Delta^G\right)(1-z^H)z^F\overline{x}^H\right]\mu\left(\Delta^3\ \Delta^F\ \ \Delta^G\right)$$

$$+\sum_{\Delta^F=\Delta^2}^{\Delta^3}\sum_{\Delta^H}\sum_{\Delta^G}\left[\mathcal{I}_1\left(\frac{1}{\lambda}\Delta^G\right)(1-z^H)z^F\overline{x}^F\right]\mu\left(\Delta^H\ \Delta^F\ \ \Delta^G\right)$$

$$+\sum_{\Delta^H}\sum_{\Delta^G}\left[\mathcal{I}_1\left(\frac{1}{\lambda}\Delta^G\right)(1-z^H)\overline{x}^F\right]\mu\left(\Delta^H\ \Delta^4\ \ \Delta^G\right) \tag{B.25}$$

$F\ F\ \Delta^3\ \Delta^3\ -\infty$ case:

$$Outflow\left(\Delta^3\ \Delta^3\ \ -\infty\right)=\mu\left(\Delta^3\ \Delta^3\ \ -\infty\right)$$

$$\begin{aligned}Inflow\left(\Delta^3\ \Delta^3\ \ -\infty\right)=&\left[\mathcal{I}_1\left(\frac{1}{\eta}\right)(1-z^H)\overline{x}^F+\mathcal{I}_1\left(\frac{\lambda}{\eta}\right)z^H\overline{x}^F\right]\mu\left(\Delta^1\ \Delta^1\ \ \infty\right)\\&+\overline{x}^F\mu\left(\Delta^1\ \Delta^1\ \ -\infty\right)\\&+\sum_{\Delta^H}\sum_{\Delta^G}\left[\mathcal{I}_1\left(\frac{1}{\eta}\Delta^G\right)(1-z^H)\overline{x}^F+\mathcal{I}_1\left(\frac{\lambda}{\eta}\Delta^G\right)z^H\overline{x}^F\right]\mu\left(\Delta^H\ \Delta^1\ \ \Delta^G\right)\end{aligned} \tag{B.26}$$

$F\ F\ \Delta^4\ \Delta^4\ -\infty$ case:

$$Outflow\left(\Delta^4\ \Delta^4\ \ -\infty\right)=\mu\left(\Delta^4\ \Delta^4\ \ -\infty\right)$$

$$\begin{aligned}Inflow\left(\Delta^4\ \Delta^4\ \ -\infty\right)=&\left[\mathcal{I}_1\left(\frac{\lambda}{\eta}\right)(1-z^H)\overline{x}^F+\mathcal{I}_1\left(\frac{\lambda^2}{\eta}\right)z^H\overline{x}^F\right]\mu\left(\Delta^2\ \Delta^2\ \ \infty\right)\\&+(1-z^F)\overline{x}^F\mu\left(\Delta^2\ \Delta^2\ \ -\infty\right)\\&+\sum_{\Delta^H}\sum_{\Delta^G}\left[\mathcal{I}_1\left(\frac{\lambda}{\eta}\Delta^G\right)(1-z^H)(1-z^F)\overline{x}^F\right]\mu\left(\Delta^H\ \Delta^2\ \ \Delta^G\right)\end{aligned} \tag{B.27}$$

$H\ F\ \Delta^1\ \Delta^1\ \Delta^G$ case:

$$Outflow\left(\Delta^1\ \Delta^1\ \ \Delta^G\right) = \left[1-(1-z^H)(1-z^F)(1-\overline{x})\right]\mu\left(\Delta^1\ \Delta^1\ \ \Delta^G\right)$$

$$\begin{aligned}
Inflow\left(\Delta^1\ \Delta^1\ \ \Delta^G\right) = {} & \mathcal{I}_{\{\Delta^G=1\}}\left[(1-z^H)\overline{x}^F\right]\mu\left(\Delta^3\ \Delta^3\ \ \infty\right) \\
& + \mathcal{I}_{\{\Delta^G=1\}}\left[(1-z^F)\overline{x}^H\right]\mu\left(\Delta^3\ \Delta^3\ \ -\infty\right) \\
& + \sum_{(\Delta^H\ \Delta^F)\neq(\Delta^1\ \Delta^1)}\left[(1-z^H)(1-z^F)(1-\overline{x})\right]\mu\left(\Delta^H\ \Delta^F\ \ \Delta^G\right) \\
& + \sum_{\Delta^F}\left[(1-z^H)(1-z^F)\overline{x}^H\right]\mu\left(\Delta^3\ \Delta^F\ \ \Delta^G\right) \\
& + \sum_{\Delta^H}\left[(1-z^H)(1-z^F)\overline{x}^F\right]\mu\left(\Delta^H\ \Delta^3\ \ \Delta^G\right)
\end{aligned} \tag{B.28}$$

$H\ F\ \Delta^1\ \Delta^2\ \Delta^G$ case:

$$Outflow\left(\Delta^1\ \Delta^2\ \ \Delta^G\right) = \mu\left(\Delta^1\ \Delta^2\ \ \Delta^G\right)$$

$$\begin{aligned}
Inflow\left(\Delta^1\ \Delta^2\ \ \Delta^G\right) = {} & \mathcal{I}_{\left\{\Delta^G=\frac{1}{\lambda}\right\}}\left[(1-z^H)\overline{x}^F\right]\mu\left(\Delta^4\ \Delta^4\ \ \infty\right) \\
& + \mathcal{I}_{\left\{\Delta^G=\frac{1}{\lambda}\right\}}\left[z^F\overline{x}^H\right]\mu\left(\Delta^3\ \Delta^3\ \ -\infty\right) \\
& + \sum_{\Delta^H}\sum_{\Delta^F}\left[(1-z^H)z^F(1-\overline{x})\right]\mu\left(\Delta^H\ \Delta^F\ \ \lambda\Delta^G\right) \\
& + \sum_{\Delta^F}\left[(1-z^H)z^F\overline{x}^H\right]\mu\left(\Delta^3\ \Delta^F\ \ \lambda\Delta^G\right) \\
& + \sum_{\Delta^H}\left[(1-z^H)z^F\overline{x}^F\right]\mu\left(\Delta^H\ \Delta^2\ \ \lambda\Delta^G\right) \\
& + \sum_{\Delta^H}\left[(1-z^H)z^F\overline{x}^F\right]\mu\left(\Delta^H\ \Delta^3\ \ \lambda\Delta^G\right) \\
& + \sum_{\Delta^H}\left[(1-z^H)\overline{x}^F\right]\mu\left(\Delta^H\ \Delta^4\ \ \lambda\Delta^G\right)
\end{aligned} \tag{B.29}$$

$H\ F\ \Delta^1\ \Delta^3\ \Delta^G$ case:

$$\begin{aligned}
Outflow\left(\Delta^1\ \Delta^3\ \ \Delta^G\right) &= \mu\left(\Delta^1\ \Delta^3\ \ \Delta^G\right) \\
Inflow\left(\Delta^1\ \Delta^3\ \ \Delta^G\right) &= \mathcal{I}_{\left\{\Delta^G=\frac{1}{\eta}\right\}}\left[(1-z^H)\overline{x}^F\right]\mu\left(\Delta^1\ \Delta^1\ \ \infty\right) \\
&\quad + \sum_{\Delta^H}\left[(1-z^H)\overline{x}^F\right]\mu\left(\Delta^H\ \Delta^1\ \ \eta\Delta^G\right) \qquad \text{(B.30)}
\end{aligned}$$

$H\ F\ \Delta^1\ \Delta^4\ \Delta^G$ case:

$$\begin{aligned}
Outflow\left(\Delta^1\ \Delta^4\ \ \Delta^G\right) &= \mu\left(\Delta^1\ \Delta^4\ \ \Delta^G\right) \\
Inflow\left(\Delta^1\ \Delta^4\ \ \Delta^G\right) &= \mathcal{I}_{\left\{\Delta^G=\frac{\lambda}{\eta}\right\}}\left[(1-z^H)\overline{x}^F\right]\mu\left(\Delta^2\ \Delta^2\ \ \infty\right) \\
&\quad + \sum_{\Delta^H}\left[(1-z^H)(1-z^F)\overline{x}^F\right]\mu\left(\Delta^H\ \Delta^2\ \ \frac{\eta}{\lambda}\Delta^G\right) \qquad \text{(B.31)}
\end{aligned}$$

$H\ F\ \Delta^2\ \Delta^1\ \Delta^G$ case:

$$\begin{aligned}
Outflow\left(\Delta^2\ \Delta^1\ \ \Delta^G\right) &= \mu\left(\Delta^2\ \Delta^1\ \ \Delta^G\right) \\
Inflow\left(\Delta^2\ \Delta^1\ \ \Delta^G\right) &= \mathcal{I}_{\{\Delta^G=\lambda\}}\left[z^H\overline{x}^F\right]\mu\left(\Delta^3\ \Delta^3\ \ \infty\right) \\
&\quad + \mathcal{I}_{\{\Delta^G=\lambda\}}\left[(1-z^F)\overline{x}^H\right]\mu\left(\Delta^4\ \Delta^4\ \ -\infty\right) \\
&\quad + \sum_{\Delta^H}\sum_{\Delta^F}\left[z^H(1-z^F)(1-\overline{x})\right]\mu\left(\Delta^H\ \Delta^F\ \ \frac{1}{\lambda}\Delta^G\right) \\
&\quad + \sum_{\Delta^F}\left[z^H(1-z^F)\overline{x}^H\right]\mu\left(\Delta^2\ \Delta^F\ \ \frac{1}{\lambda}\Delta^G\right)
\end{aligned}$$

$$+\sum_{\Delta^F}\left[z^H(1-z^F)\overline{x}^H\right]\mu\left(\Delta^3\ \Delta^F\ \ \frac{1}{\lambda}\Delta^G\right)$$

$$+\sum_{\Delta^F}\left[(1-z^F)\overline{x}^H\right]\mu\left(\Delta^4\ \Delta^F\ \ \frac{1}{\lambda}\Delta^G\right)$$

$$+\sum_{\Delta^H}\left[z^H(1-z^F)\overline{x}^F\right]\mu\left(\Delta^H\ \Delta^3\ \ \frac{1}{\lambda}\Delta^G\right) \tag{B.32}$$

$H\ F\ \Delta^2\ \Delta^2\ \Delta^G$ case:

$$Outflow\left(\Delta^2\ \Delta^2\ \ \Delta^G\right)=\left[1-\left[z^Hz^F(1-\overline{x})+z^Hz^F\overline{x}^H+z^Hz^F\overline{x}^F\right]\right]\mu\left(\Delta^2\ \Delta^2\ \ \Delta^G\right)$$

$$\begin{aligned}Inflow\left(\Delta^2\ \Delta^2\ \ \Delta^G\right)=&\ \mathcal{I}_{\{\Delta^G=1\}}\left[z^H\overline{x}^F\right]\mu\left(\Delta^4\ \Delta^4\ \ \infty\right)+\mathcal{I}_{\{\Delta^G=1\}}\left[z^F\overline{x}^H\right]\mu\left(\Delta^4\ \Delta^4\ \ -\infty\right)\\
&+\sum_{(\Delta^H\ \Delta^F)\neq(\Delta^2\ \Delta^2)}\left[z^Hz^F(1-\overline{x})\right]\mu\left(\Delta^H\ \Delta^F\ \ \Delta^G\right)\\
&+\sum_{\Delta^F\neq\Delta^2}\left[z^Hz^F\overline{x}^H\right]\mu\left(\Delta^2\ \Delta^F\ \ \Delta^G\right)\\
&+\sum_{\Delta^F}\left[z^Hz^F\overline{x}^H\right]\mu\left(\Delta^3\ \Delta^F\ \ \Delta^G\right)\\
&+\sum_{\Delta^F}\left[z^F\overline{x}^H\right]\mu\left(\Delta^4\ \Delta^F\ \ \Delta^G\right)\\
&+\sum_{\Delta^H\neq\Delta^2}\left[z^Hz^F\overline{x}^F\right]\mu\left(\Delta^H\ \Delta^2\ \ \Delta^G\right)\\
&+\sum_{\Delta^H}\left[z^Hz^F\overline{x}^F\right]\mu\left(\Delta^H\ \Delta^3\ \ \Delta^G\right)\\
&+\sum_{\Delta^H}\left[z^H\overline{x}^F\right]\mu\left(\Delta^H\ \Delta^4\ \ \Delta^G\right)\end{aligned} \tag{B.33}$$

$H\ F\ \Delta^2\ \Delta^3\ \Delta^G$ case:

$$Outflow\left(\Delta^2\ \Delta^3\ \ \Delta^G\right) = \mu\left(\Delta^2\ \Delta^3\ \ \Delta^G\right)$$

$$\begin{aligned} Inflow\left(\Delta^2\ \Delta^3\ \ \Delta^G\right) &= \mathcal{I}_{\left\{\Delta^G = \frac{\lambda}{\eta}\right\}}\left[z^H \overline{x}^F\right]\mu\left(\Delta^1\ \Delta^1\ \ \infty\right) \\ &+ \sum_{\Delta^H}\left[z^H \overline{x}^F\right]\mu\left(\Delta^H\ \Delta^1\ \ \frac{\eta}{\lambda}\Delta^G\right) \end{aligned} \tag{B.34}$$

$H\ F\ \Delta^2\ \Delta^4\ \Delta^G$ case:

$$Outflow\left(\Delta^2\ \Delta^4\ \ \Delta^G\right) = \mu\left(\Delta^2\ \Delta^4\ \ \Delta^G\right)$$

$$\begin{aligned} Inflow\left(\Delta^2\ \Delta^4\ \ \Delta^G\right) &= \mathcal{I}_{\left\{\Delta^G = \frac{\lambda^2}{\eta}\right\}}\left[z^H \overline{x}^F\right]\mu\left(\Delta^2\ \Delta^2\ \ \infty\right) \\ &+ \sum_{\Delta^H}\left[z^H (1-z^F) \overline{x}^F\right]\mu\left(\Delta^H\ \Delta^2\ \ \frac{\eta}{\lambda^2}\Delta^G\right) \end{aligned} \tag{B.35}$$

$H\ F\ \Delta^3\ \Delta^1\ \Delta^G$ case:

$$Outflow\left(\Delta^3\ \Delta^1\ \ \Delta^G\right) = \mu\left(\Delta^3\ \Delta^1\ \ \Delta^G\right)$$

$$\begin{aligned} Inflow\left(\Delta^3\ \Delta^1\ \ \Delta^G\right) &= \mathcal{I}_{\left\{\Delta^G = \eta\right\}}\left[(1-z^F) \overline{x}^H\right]\mu\left(\Delta^1\ \Delta^1\ \ -\infty\right) \\ &+ \sum_{\Delta^F}\left[(1-z^F) \overline{x}^H\right]\mu\left(\Delta^1\ \Delta^F\ \ \frac{1}{\eta}\Delta^G\right) \end{aligned} \tag{B.36}$$

$H\ F\ \Delta^3\ \Delta^2\ \Delta^G$ case:

$$Outflow\left(\Delta^3\,\Delta^2\ \ \Delta^G\right) = \mu\left(\Delta^3\,\Delta^2\ \ \Delta^G\right)$$

$$\begin{aligned} Inflow\left(\Delta^3\,\Delta^2\ \ \Delta^G\right) &= \mathcal{I}_{\left\{\Delta^G=\frac{\eta}{\lambda}\right\}}\left[z^F\overline{x}^H\right]\mu\left(\Delta^1\,\Delta^1\ \ -\infty\right) \\ &\quad + \sum_{\Delta^F}\left[z^F\overline{x}^H\right]\mu\left(\Delta^1\,\Delta^F\ \ \frac{\lambda}{\eta}\Delta^G\right) \end{aligned} \tag{B.37}$$

$H\ F\ \Delta^3\ \Delta^3\ \Delta^G$ case:

$$Outflow\left(\Delta^3\,\Delta^3\ \ \Delta^G\right) = \mu\left(\Delta^3\,\Delta^3\ \ \Delta^G\right)$$

$$Inflow\left(\Delta^3\,\Delta^3\ \ \Delta^G\right) = 0 \tag{B.38}$$

$H\ F\ \Delta^3\ \Delta^4\ \Delta^G$ case:

$$Outflow\left(\Delta^3\,\Delta^4\ \ \Delta^G\right) = \mu\left(\Delta^3\,\Delta^4\ \ \Delta^G\right)$$

$$Inflow\left(\Delta^3\,\Delta^4\ \ \Delta^G\right) = 0 \tag{B.39}$$

$H\ F\ \Delta^4\ \Delta^1\ \Delta^G$ case:

$$Outflow\left(\Delta^4\,\Delta^1\ \ \Delta^G\right) = \mu\left(\Delta^4\,\Delta^1\ \ \Delta^G\right)$$

$$Inflow\left(\Delta^4\,\Delta^1\ \ \Delta^G\right) = \mathcal{I}_{\left\{\Delta^G=\frac{\eta}{\lambda}\right\}}\left[(1-z^F)\overline{x}^H\right]\mu\left(\Delta^2\,\Delta^2\ \ -\infty\right)$$

$$+\sum_{\Delta^F}\left[(1-z^H)(1-z^F)\overline{x}^H\right]\mu\left(\Delta^2\;\Delta^F\;\;\frac{\lambda}{\eta}\Delta^G\right) \tag{B.40}$$

$H\;F\;\Delta^4\;\Delta^2\;\Delta^G$ case:

$$Outflow\left(\Delta^4\;\Delta^2\;\;\Delta^G\right)=\mu\left(\Delta^4\;\Delta^2\;\;\Delta^G\right)$$

$$Inflow\left(\Delta^4\;\Delta^2\;\;\Delta^G\right)=\mathcal{I}_{\left\{\Delta^G=\frac{\eta}{\lambda^2}\right\}}\left[z^F\overline{x}^H\right]\mu\left(\Delta^2\;\Delta^2\;\;-\infty\right)$$
$$+\sum_{\Delta^F}\left[(1-z^H)z^F\overline{x}^H\right]\mu\left(\Delta^2\;\Delta^F\;\;\frac{\lambda^2}{\eta}\Delta^G\right) \tag{B.41}$$

$H\;F\;\Delta^4\;\Delta^3\;\Delta^G$ case:

$$Outflow\left(\Delta^4\;\Delta^3\;\;\Delta^G\right)=\mu\left(\Delta^4\;\Delta^3\;\;\Delta^G\right)$$

$$Inflow\left(\Delta^4\;\Delta^3\;\;\Delta^G\right)=0 \tag{B.42}$$

$H\;F\;\Delta^4\;\Delta^4\;\Delta^G$ case:

$$Outflow\left(\Delta^4\;\Delta^4\;\;\Delta^G\right)=\mu\left(\Delta^4\;\Delta^4\;\;\Delta^G\right)$$

$$Inflow\left(\Delta^4\;\Delta^4\;\;\Delta^G\right)=0 \tag{B.43}$$

B.3.2 External Innovation Outcomes

B.3.2.1 Outcomes from a Successful External Innovation, Home Firm

$$\left(\, q\; H\; H\; \Delta^1\; \Delta^1\; \infty \,\right) \rightarrow \begin{cases} \left(\, \eta q\; H\; H\; \Delta^3\; \Delta^3\; \infty \,\right) & \text{w/ } 1 \end{cases} \tag{B.44}$$

$$\left(\, q\; H\; H\; \Delta^2\; \Delta^2\; \infty \,\right) \rightarrow \begin{cases} \left(\, \frac{\eta}{\lambda} q\; H\; H\; \Delta^4\; \Delta^4\; \infty \,\right) & \text{w/ } 1 - z^H \\ \mathbf{0} & \text{w/ } z^H \end{cases} \tag{B.45}$$

$$\left(\, q\; H\; H\; \Delta^3\; \Delta^3\; \infty \,\right) \rightarrow \begin{cases} \left(\, q\; H\; H\; \Delta^1\; \Delta^1\; \infty \,\right) & \text{w/ } \frac{1}{2}(1 - z^H) \\ \mathbf{0} & \text{w/ } \frac{1}{2}(1 + z^H) \end{cases} \tag{B.46}$$

$$\left(\, q\; H\; H\; \Delta^4\; \Delta^4\; \infty \,\right) \rightarrow \begin{cases} \left(\, \lambda q\; H\; H\; \Delta^2\; \Delta^2\; \infty \,\right) & \text{w/ } (1 - z^H) \\ \left(\, \lambda q\; H\; H\; \Delta^2\; \Delta^2\; \infty \,\right) & \text{w/ } \frac{1}{2} z^H \\ \mathbf{0} & \text{w/ } \frac{1}{2} z^H \end{cases} \tag{B.47}$$

$$\left(\, q\; F\; F\; \Delta^1\; \Delta^1\; -\infty \,\right) \rightarrow \begin{cases} \begin{bmatrix} \left(\, \eta q\; H\; F\; \Delta^3\; \Delta^1\; \eta \,\right) \mathcal{I}_2(\eta) \\ +\left(\, \eta q\; H\; H\; \Delta^3\; \Delta^3\; \infty \,\right) \mathcal{I}_3(\eta) \end{bmatrix} & \text{w/ } 1 - z^F \\ \begin{bmatrix} \left(\, \eta q\; H\; F\; \Delta^3\; \Delta^2\; \frac{\eta}{\lambda} \,\right) \mathcal{I}_2\left(\frac{\eta}{\lambda}\right) \\ +\left(\, \eta q\; H\; H\; \Delta^3\; \Delta^3\; \infty \,\right) \mathcal{I}_3\left(\frac{\eta}{\lambda}\right) \end{bmatrix} & \text{w/ } z^F \end{cases} \tag{B.48}$$

$$
\left(\, q\; F\; F\; \Delta^2\; \Delta^2\;\; -\infty \,\right) \rightarrow \begin{cases} \begin{bmatrix} \left(\, \frac{\eta}{\lambda} q\; H\; F\; \Delta^4\; \Delta^1\;\; \frac{\eta}{\lambda} \,\right) \mathcal{I}_2 \left(\frac{\eta}{\lambda}\right) \\ + \left(\, \frac{\eta}{\lambda} q\; H\; H\; \Delta^4\; \Delta^4\;\; \infty \,\right) \mathcal{I}_3 \left(\frac{\eta}{\lambda}\right) \end{bmatrix} & \text{w/ } 1 - z^F \\ \begin{bmatrix} \left(\, \frac{\eta}{\lambda} q\; H\; F\; \Delta^4\; \Delta^2\;\; \frac{\eta}{\lambda^2} \,\right) \mathcal{I}_2 \left(\frac{\eta}{\lambda^2}\right) \\ + \left(\, \frac{\eta}{\lambda} q\; H\; H\; \Delta^4\; \Delta^4\;\; \infty \,\right) \mathcal{I}_3 \left(\frac{\eta}{\lambda^2}\right) \end{bmatrix} & \text{w/ } z^F \end{cases} \tag{B.49}
$$

$$
\left(\, q\; F\; F\; \Delta^3\; \Delta^3\;\; -\infty \,\right) \rightarrow \begin{cases} \begin{bmatrix} \left(\, q\; H\; F\; \Delta^1\; \Delta^1\;\; 1 \,\right) \mathcal{I}_2 \left(1\right) \\ + \left(\, q\; H\; H\; \Delta^1\; \Delta^1\;\; \infty \,\right) \mathcal{I}_3 \left(1\right) \end{bmatrix} & \text{w/ } 1 - z^F \\ \begin{bmatrix} \left(\, q\; H\; F\; \Delta^1\; \Delta^2\;\; \frac{1}{\lambda} \,\right) \mathcal{I}_2 \left(\frac{1}{\lambda}\right) \\ + \left(\, q\; H\; H\; \Delta^1\; \Delta^1\;\; \infty \,\right) \mathcal{I}_3 \left(\frac{1}{\lambda}\right) \end{bmatrix} & \text{w/ } z^F \end{cases} \tag{B.50}
$$

$$
\left(\, q\; F\; F\; \Delta^4\; \Delta^4\;\; -\infty \,\right) \rightarrow \begin{cases} \begin{bmatrix} \left(\, \lambda q\; H\; F\; \Delta^2\; \Delta^1\;\; \lambda \,\right) \mathcal{I}_2 \left(\lambda\right) \\ + \left(\, \lambda q\; H\; H\; \Delta^2\; \Delta^2\;\; \infty \,\right) \mathcal{I}_3 \left(\lambda\right) \end{bmatrix} & \text{w/ } 1 - z^F \\ \begin{bmatrix} \left(\, \lambda q\; H\; F\; \Delta^2\; \Delta^2\;\; 1 \,\right) \mathcal{I}_2 \left(1\right) \\ + \left(\, \lambda q\; H\; H\; \Delta^2\; \Delta^2\;\; \infty \,\right) \mathcal{I}_3 \left(1\right) \end{bmatrix} & \text{w/ } z^F \end{cases} \tag{B.51}
$$

$$\big(\, q\; H\; F\; \Delta^1\; \Delta^{F\ell}\; \Delta^G \,\big) \to \begin{cases} \begin{bmatrix} \big(\, \eta q\; H\; F\; \Delta^3\; \Delta^1\;\; \eta\Delta^G \,\big)\, \mathcal{I}_2\left(\eta\Delta^G\right) \\ +\big(\, \eta q\; H\; H\; \Delta^3\; \Delta^3\;\; \infty \,\big)\, \mathcal{I}_3\left(\eta\Delta^G\right) \end{bmatrix} & \text{w/ } (1-z^H)(1-z^F) \\ \begin{bmatrix} \big(\, \eta q\; H\; F\; \Delta^3\; \Delta^1\;\; \eta\Delta^G \,\big)\, \mathcal{I}_2\left(\eta\Delta^G\right) \\ +\big(\, \eta q\; H\; H\; \Delta^3\; \Delta^3\;\; \infty \,\big)\, \mathcal{I}_3\left(\eta\Delta^G\right) \end{bmatrix} & \text{w/ } z^H(1-z^F) \\ \begin{bmatrix} \big(\, \eta q\; H\; F\; \Delta^3\; \Delta^2\;\; \tfrac{\eta}{\lambda}\Delta^G \,\big)\, \mathcal{I}_2\left(\tfrac{\eta}{\lambda}\Delta^G\right) \\ +\big(\, \eta q\; H\; H\; \Delta^3\; \Delta^3\;\; \infty \,\big)\, \mathcal{I}_3\left(\tfrac{\eta}{\lambda}\Delta^G\right) \end{bmatrix} & \text{w/ } (1-z^H)z^F \\ \begin{bmatrix} \big(\, \eta q\; H\; F\; \Delta^3\; \Delta^2\;\; \tfrac{\eta}{\lambda}\Delta^G \,\big)\, \mathcal{I}_2\left(\tfrac{\eta}{\lambda}\Delta^G\right) \\ +\big(\, \eta q\; H\; H\; \Delta^3\; \Delta^3\;\; \infty \,\big)\, \mathcal{I}_3\left(\tfrac{\eta}{\lambda}\Delta^G\right) \end{bmatrix} & \text{w/ } z^H z^F \end{cases} \tag{B.52}$$

$$\big(\, q\; H\; F\; \Delta^2\; \Delta^{F\ell}\; \Delta^G \,\big) \to \begin{cases} \begin{bmatrix} \big(\, \tfrac{\eta}{\lambda} q\; H\; F\; \Delta^4\; \Delta^1\;\; \tfrac{\eta}{\lambda}\Delta^G \,\big)\, \mathcal{I}_2\left(\tfrac{\eta}{\lambda}\Delta^G\right) \\ +\big(\, \tfrac{\eta}{\lambda} q\; H\; H\; \Delta^4\; \Delta^4\;\; \infty \,\big)\, \mathcal{I}_3\left(\tfrac{\eta}{\lambda}\Delta^G\right) \end{bmatrix} & \text{w/ } (1-z^H)(1-z^F) \\ \big(\, \tfrac{\eta}{\lambda} q\; H\; F\; \Delta^4\; \Delta^2\;\; \tfrac{\eta}{\lambda^2}\Delta^G \,\big)\, \mathcal{I}_2\left(\tfrac{\eta}{\lambda^2}\Delta^G\right) & \text{w/ } (1-z^H)z^F \\ \mathbf{0} & \text{w/ } z^H \end{cases} \tag{B.53}$$

$$\big(\, q\; H\; F\; \Delta^3\; \Delta^{F\ell}\; \Delta^G \,\big) \to \begin{cases} \big(\, q\; H\; F\; \Delta^1\; \Delta^1\;\; \Delta^G \,\big) & \text{w/ } \tfrac{1}{2}(1-z^H)(1-z^F) \\ \big(\, q\; H\; F\; \Delta^1\; \Delta^2\;\; \tfrac{1}{\lambda}\Delta^G \,\big)\, \mathcal{I}_2\left(\tfrac{1}{\lambda}\Delta^G\right) & \text{w/ } \tfrac{1}{2}(1-z^H)z^F \\ \mathbf{0} & \text{w/ } \tfrac{1}{2}(1+z^H) \end{cases} \tag{B.54}$$

$$
\left(\, q\; H\; F\; \Delta^4\; \Delta^{F\ell}\; \Delta^G \,\right) \rightarrow \begin{cases}
\begin{bmatrix} \left(\, \lambda q\; H\; F\; \Delta^2\; \Delta^1\;\; \lambda\Delta^G \,\right) \mathcal{I}_2\left(\lambda\Delta^G\right) \\ +\left(\, \lambda q\; H\; H\; \Delta^2\; \Delta^2\;\; \infty \,\right) \mathcal{I}_3\left(\lambda\Delta^G\right) \end{bmatrix} & \text{w/ } (1-z^H)(1-z^F) \\
\begin{bmatrix} \left(\, \lambda q\; H\; F\; \Delta^2\; \Delta^1\;\; \lambda\Delta^G \,\right) \mathcal{I}_2\left(\lambda\Delta^G\right) \\ +\left(\, \lambda q\; H\; H\; \Delta^2\; \Delta^2\;\; \infty \,\right) \mathcal{I}_3\left(\lambda\Delta^G\right) \end{bmatrix} & \text{w/ } \frac{1}{2}z^H(1-z^F) \\
\left(\, \lambda q\; H\; F\; \Delta^2\; \Delta^2\;\; \Delta^G \,\right) & \text{w/ } (1-z^H)z^F \\
\left(\, \lambda q\; H\; F\; \Delta^2\; \Delta^2\;\; \Delta^G \,\right) & \text{w/ } \frac{1}{2}z^H z^F \\
\mathbf{0} & \text{w/ } \frac{1}{2}z^H
\end{cases} \tag{B.55}
$$

B.3.2.2 Outcomes from a Successful External Innovation, Foreign Firm

$$
\left(\, q\; H\; H\; \Delta^1\; \Delta^1\; \infty \,\right) \rightarrow \begin{cases}
\begin{bmatrix} \left(\, \eta q\; F\; F\; \Delta^3\; \Delta^3\;\; -\infty \,\right) \mathcal{I}_1\left(\frac{1}{\eta}\right) \\ +\left(\, \eta q\; H\; F\; \Delta^1\; \Delta^3\;\; \frac{1}{\eta} \,\right) \mathcal{I}_2\left(\frac{1}{\eta}\right) \end{bmatrix} & \text{w/ } 1-z^H \\
\begin{bmatrix} \left(\, \eta q\; F\; F\; \Delta^3\; \Delta^3\;\; -\infty \,\right) \mathcal{I}_1\left(\frac{\lambda}{\eta}\right) \\ +\left(\, \eta q\; H\; F\; \Delta^2\; \Delta^3\;\; \frac{\lambda}{\eta} \,\right) \mathcal{I}_2\left(\frac{\lambda}{\eta}\right) \end{bmatrix} & \text{w/ } z^H
\end{cases} \tag{B.56}
$$

$$\left(\, q\; H\; H\; \Delta^2\; \Delta^2\; \infty \,\right) \rightarrow \begin{cases} \left[\begin{array}{c} \left(\, \frac{\eta}{\lambda} q\; F\; F\; \Delta^4\; \Delta^4\;\; -\infty \,\right) \mathcal{I}_1\left(\frac{\lambda}{\eta}\right) \\ +\left(\, \frac{\eta}{\lambda} q\; H\; F\; \Delta^1\; \Delta^4\;\; \frac{\lambda}{\eta} \,\right) \mathcal{I}_2\left(\frac{\lambda}{\eta}\right) \end{array} \right] & \text{w/ } 1 - z^H \\ \left[\begin{array}{c} \left(\, \frac{\eta}{\lambda} q\; F\; F\; \Delta^4\; \Delta^4\;\; -\infty \,\right) \mathcal{I}_1\left(\frac{\lambda^2}{\eta}\right) \\ +\left(\, \frac{\eta}{\lambda} q\; H\; F\; \Delta^2\; \Delta^4\;\; \frac{\lambda^2}{\eta} \,\right) \mathcal{I}_2\left(\frac{\lambda^2}{\eta}\right) \end{array} \right] & \text{w/ } z^H \end{cases} \tag{B.57}$$

$$\left(\, q\; H\; H\; \Delta^3\; \Delta^3\; \infty \,\right) \rightarrow \begin{cases} \left[\begin{array}{c} \left(\, q\; F\; F\; \Delta^1\; \Delta^1\;\; -\infty \,\right) \mathcal{I}_1\left(1\right) \\ +\left(\, q\; H\; F\; \Delta^1\; \Delta^1\;\; 1 \,\right) \mathcal{I}_2\left(1\right) \end{array} \right] & \text{w/ } 1 - z^H \\ \left[\begin{array}{c} \left(\, q\; F\; F\; \Delta^1\; \Delta^1\;\; -\infty \,\right) \mathcal{I}_1\left(\lambda\right) \\ +\left(\, q\; H\; F\; \Delta^2\; \Delta^1\;\; \lambda \,\right) \mathcal{I}_2\left(\lambda\right) \end{array} \right] & \text{w/ } z^H \end{cases} \tag{B.58}$$

$$\left(\, q\; H\; H\; \Delta^4\; \Delta^4\; \infty \,\right) \rightarrow \begin{cases} \left[\begin{array}{c} \left(\, \lambda q\; F\; F\; \Delta^2\; \Delta^2\;\; -\infty \,\right) \mathcal{I}_1\left(\frac{1}{\lambda}\right) \\ +\left(\, \lambda q\; H\; F\; \Delta^1\; \Delta^2\;\; \frac{1}{\lambda} \,\right) \mathcal{I}_2\left(\frac{1}{\lambda}\right) \end{array} \right] & \text{w/ } 1 - z^H \\ \left[\begin{array}{c} \left(\, \lambda q\; F\; F\; \Delta^2\; \Delta^2\;\; -\infty \,\right) \mathcal{I}_1\left(1\right) \\ +\left(\, \lambda q\; H\; F\; \Delta^2\; \Delta^2\;\; 1 \,\right) \mathcal{I}_2\left(1\right) \end{array} \right] & \text{w/ } z^H \end{cases} \tag{B.59}$$

$$\left(\, q\; F\; F\; \Delta^1\; \Delta^1\;\; -\infty \,\right) \rightarrow \begin{cases} \left(\, \eta q\; F\; F\; \Delta^3\; \Delta^3\;\; -\infty \,\right) & \text{w/ } 1 \end{cases} \tag{B.60}$$

$$\left(\, q\; F\; F\; \Delta^2\; \Delta^2\;\; -\infty \,\right) \rightarrow \begin{cases} \left(\, \frac{\eta}{\lambda} q\; F\; F\; \Delta^4\; \Delta^4\;\; -\infty \,\right) & \text{w/ } 1 - z^F \\ \mathbf{0} & \text{w/ } z^F \end{cases} \tag{B.61}$$

$$\big(\, q\ F\ F\ \Delta^3\ \Delta^3\ \ -\infty \,\big) \to \begin{cases} \big(\, q\ F\ F\ \Delta^1\ \Delta^1\ \ -\infty \,\big) & \text{w/ } \tfrac{1}{2}(1-z^F) \\ \mathbf{0} & \text{w/ } \tfrac{1}{2}(1+z^F) \end{cases} \tag{B.62}$$

$$\big(\, q\ F\ F\ \Delta^4\ \Delta^4\ \ -\infty \,\big) \to \begin{cases} \big(\, \lambda q\ F\ F\ \Delta^2\ \Delta^2\ \ -\infty \,\big) & \text{w/ } (1-\tfrac{1}{2}z^F) \\ \mathbf{0} & \text{w/ } \tfrac{1}{2}z^F \end{cases} \tag{B.63}$$

$$\big(\, q\ H\ F\ \Delta^{\ell^H}\ \Delta^1\ \ \Delta^G \,\big) \to \begin{cases} \left[\begin{array}{r} \big(\, \eta q\ F\ F\ \Delta^3\ \Delta^3\ \ -\infty \,\big)\, \mathcal{I}_1\left(\frac{1}{\eta}\Delta^G\right) \\ +\big(\, \eta q\ H\ F\ \Delta^1\ \Delta^3\ \ \frac{1}{\eta}\Delta^G \,\big)\, \mathcal{I}_2\left(\frac{1}{\eta}\Delta^G\right) \end{array} \right] & \text{w/ } 1-z^H \\ \left[\begin{array}{r} \big(\, \eta q\ F\ F\ \Delta^3\ \Delta^3\ \ -\infty \,\big)\, \mathcal{I}_1\left(\frac{\lambda}{\eta}\Delta^G\right) \\ +\big(\, \eta q\ H\ F\ \Delta^3\ \Delta^3\ \ \frac{\lambda}{\eta}\Delta^G \,\big)\, \mathcal{I}_2\left(\frac{\lambda}{\eta}\Delta^G\right) \end{array} \right] & \text{w/ } z^H \end{cases} \tag{B.64}$$

$$\big(\, q\ H\ F\ \Delta^{\ell^H}\ \Delta^2\ \ \Delta^G \,\big) \to \begin{cases} \left[\begin{array}{r} \big(\, \frac{\eta}{\lambda} q\ F\ F\ \Delta^4\ \Delta^4\ \ -\infty \,\big)\, \mathcal{I}_1\left(\frac{\lambda}{\eta}\Delta^G\right) \\ +\big(\, \frac{\eta}{\lambda} q\ H\ F\ \Delta^1\ \Delta^4\ \ \frac{\lambda}{\eta}\Delta^G \,\big)\, \mathcal{I}_2\left(\frac{\lambda}{\eta}\Delta^G\right) \end{array} \right] & \text{w/ } (1-z^H)(1-z^F) \\ \big(\, \frac{\eta}{\lambda} q\ H\ F\ \Delta^2\ \Delta^4\ \ \frac{\lambda^2}{\eta}\Delta^G \,\big)\, \mathcal{I}_2\left(\frac{\lambda^2}{\eta}\Delta^G\right) & \text{w/ } z^H(1-z^F) \\ \mathbf{0} & \text{w/ } z^F \end{cases} \tag{B.65}$$

$$\big(\, q\ H\ F\ \Delta^{\ell^H}\ \Delta^3\ \ \Delta^G \,\big) \to \begin{cases} \big(\, q\ H\ F\ \Delta^1\ \Delta^1\ \ \Delta^G \,\big) & \text{w/ } \tfrac{1}{2}(1-z^H)(1-z^F) \\ \big(\, q\ H\ F\ \Delta^2\ \Delta^1\ \ \lambda\Delta^G \,\big)\, \mathcal{I}_2\left(\lambda\Delta^G\right) & \text{w/ } \tfrac{1}{2}z^H(1-z^F) \\ \mathbf{0} & \text{w/ } \tfrac{1}{2}(1+z^F) \end{cases} \tag{B.66}$$

$$
\left(\, q\; H\; F\; \Delta^{\ell^H}\; \Delta^4\;\; \Delta^G \,\right) \rightarrow \begin{cases} \begin{bmatrix} \left(\, \lambda q\; F\; F\; \Delta^2\; \Delta^2\;\; - \infty \,\right) \mathcal{I}_1 \left(\frac{1}{\lambda} \Delta^G\right) \\ + \left(\, \lambda q\; H\; F\; \Delta^1\; \Delta^2\;\; \frac{1}{\lambda} \Delta^G \,\right) \mathcal{I}_2 \left(\frac{1}{\lambda} \Delta^G\right) \end{bmatrix} & \text{w/ } (1 - z^H)(1 - \frac{1}{2} z^F) \\ \left(\, \lambda q\; H\; F\; \Delta^2\; \Delta^2\;\; \Delta^G \,\right) & \text{w/ } z^H (1 - \frac{1}{2} z^F) \\ \mathbf{0} & \text{w/ } \frac{1}{2} z^F \end{cases} \tag{B.67}
$$

B.3.3 Complete Description of $\mathcal{Q}$ Evolution

$\mathcal{Q}'_{HH} =$

$$
\left\{
\begin{array}{l}
\left[\begin{array}{l}
\left[\,(1-z^H)+\lambda z^H\,\right](1-\overline{x})+\eta\overline{x}^H \\[2ex]
+\left[\,\mathcal{I}_2\left(\frac{1}{\eta}\right)+\mathcal{I}_3\left(\frac{1}{\eta}\right)\,\right](1-z^H)\overline{x}^F \\[2ex]
+\left[\,\mathcal{I}_2\left(\frac{\lambda}{\eta}\right)+\mathcal{I}_3\left(\frac{\lambda}{\eta}\right)\,\right]\lambda\, z^H\,\overline{x}^F
\end{array}\right]\mu\left(\Delta^1\;\Delta^1\;\;\infty\right) \\[6ex]
+\left[\begin{array}{l}
\left[\,(1-z^H)+\lambda z^H\,\right](1-\overline{x}) \\[2ex]
+\left[\,\frac{\eta}{\lambda}(1-z^H)+\lambda z^H\,\right]\overline{x}^H \\[2ex]
+\left[\,\mathcal{I}_2\left(\frac{\lambda}{\eta}\right)+\mathcal{I}_3\left(\frac{\lambda}{\eta}\right)\,\right](1-z^H)\overline{x}^F \\[2ex]
+\left[\,\mathcal{I}_2\left(\frac{\lambda^2}{\eta}\right)+\mathcal{I}_3\left(\frac{\lambda^2}{\eta}\right)\,\right]\lambda\, z^H\,\overline{x}^F
\end{array}\right]\mu\left(\Delta^2\;\Delta^2\;\;\infty\right) \\[6ex]
+\left[\begin{array}{l}
\left[\,(1-z^H)+\lambda z^H\,\right](1-\overline{x}) \\[2ex]
+\left[\,(1-z^H)+\lambda z^H\,\right]\overline{x}^H \\[2ex]
+\left[\,\mathcal{I}_2\left(1\right)+\mathcal{I}_3\left(1\right)\,\right](1-z^H)\overline{x}^F \\[2ex]
+\left[\,\mathcal{I}_2\left(\lambda\right)+\mathcal{I}_3\left(\lambda\right)\,\right]\lambda\, z^H\,\overline{x}^F
\end{array}\right]\mu\left(\Delta^3\;\Delta^3\;\;\infty\right) \\[6ex]
+\left[\begin{array}{l}
\left[\,(1-z^H)+\lambda z^H\,\right](1-\overline{x})+\lambda\overline{x}^H \\[2ex]
+\left[\,\mathcal{I}_2\left(\frac{1}{\lambda}\right)+\mathcal{I}_3\left(\frac{1}{\lambda}\right)\,\right](1-z^H)\overline{x}^F \\[2ex]
+\left[\,\mathcal{I}_2\left(1\right)+\mathcal{I}_3\left(1\right)\,\right]\lambda\, z^H\,\overline{x}^F
\end{array}\right]\mu\left(\Delta^4\;\Delta^4\;\;\infty\right)
\end{array}
\right\}
$$

$$
\times\,\mathcal{Q}_{HH}
$$

$$
+\left\{\sum_{\Delta^H}\sum_{\Delta^F}\sum_{\Delta^G\in[\underline{\Omega},\overline{\Omega}]}
\left[\begin{array}{l}
(1-z^H)(1-z^F) \\[2ex]
+\left[\,\mathcal{I}_2\left(\lambda\Delta^G\right)+\mathcal{I}_3\left(\lambda\Delta^G\right)\,\right]\lambda\, z^H(1-z^F) \\[2ex]
+\mathcal{I}_2\left(\frac{1}{\lambda}\Delta^G\right)(1-z^H)z^F+\lambda\, z^H z^F
\end{array}\right](1-\overline{x})\,\mu\left(\Delta^H\;\Delta^F\;\;\Delta^G\right)\right\}
$$

$$
\times\,\mathcal{Q}_{HH}
$$

$$
+\left\{\begin{array}{l}
\displaystyle\sum_{\Delta^F}\sum_{\Delta^G\in[\underline{\Omega},\overline{\Omega}]}\left[\begin{array}{l}\left[\,\mathcal{I}_2\left(\eta\Delta^G\right)+\mathcal{I}_3\left(\eta\Delta^G\right)\,\right]\,\eta\,(1-z^F)\\[2ex] +\left[\,\mathcal{I}_2\left(\frac{\eta}{\lambda}\Delta^G\right)+\mathcal{I}_3\left(\frac{\eta}{\lambda}\Delta^G\right)\,\right]\,\eta\,z^F\end{array}\right]\overline{x}^H\,\mu\left(\Delta^1\,\Delta^F\;\;\Delta^G\right)\\[6ex]
\displaystyle+\sum_{\Delta^F}\sum_{\Delta^G\in[\underline{\Omega},\overline{\Omega}]}\left[\begin{array}{l}\left[\,\mathcal{I}_2\left(\frac{\eta}{\lambda}\Delta^G\right)+\mathcal{I}_3\left(\frac{\eta}{\lambda}\Delta^G\right)\,\right]\,\frac{\eta}{\lambda}\,(1-z^H)(1-z^F)\\[2ex] +\left[\,\mathcal{I}_2\left(\lambda\Delta^G\right)+\mathcal{I}_3\left(\lambda\Delta^G\right)\,\right]\,\lambda\,z^H(1-z^F)\\[2ex] +\mathcal{I}_2\left(\frac{\eta}{\lambda^2}\Delta^G\right)\,\frac{\eta}{\lambda}\,(1-z^H)z^F+\lambda\,z^Hz^F\end{array}\right]\overline{x}^H\,\mu\left(\Delta^2\,\Delta^F\;\;\Delta^G\right)\\[8ex]
\displaystyle+\sum_{\Delta^F}\sum_{\Delta^G\in[\underline{\Omega},\overline{\Omega}]}\left[\begin{array}{l}(1-z^H)(1-z^F)\\[2ex] +\left[\,\mathcal{I}_2\left(\lambda\Delta^G\right)+\mathcal{I}_3\left(\lambda\Delta^G\right)\,\right]\,\lambda\,z^H(1-z^F)\\[2ex] +\mathcal{I}_2\left(\frac{1}{\lambda}\Delta^G\right)\,(1-z^H)z^F+\lambda\,z^Hz^F\end{array}\right]\overline{x}^H\,\mu\left(\Delta^3\,\Delta^F\;\;\Delta^G\right)\\[8ex]
\displaystyle+\sum_{\Delta^F}\sum_{\Delta^G\in[\underline{\Omega},\overline{\Omega}]}\left[\,\left[\,\mathcal{I}_2\left(\lambda\Delta^G\right)+\mathcal{I}_3\left(\lambda\Delta^G\right)\,\right]\,\lambda\,(1-z^F)+\;\lambda\,z^F\,\right]\overline{x}^H\,\mu\left(\Delta^4\,\Delta^F\;\;\Delta^G\right)
\end{array}\right\}
$$

$$\times\,\mathcal{Q}_{HH}$$

$$
+\left\{\begin{array}{l}
\displaystyle\sum_{\Delta^H}\sum_{\Delta^G\in[\underline{\Omega},\overline{\Omega}]}\left[\,\mathcal{I}_2\left(\frac{1}{\eta}\Delta^G\right)\,(1-z^H)+\mathcal{I}_2\left(\frac{\lambda}{\eta}\Delta^G\right)\,\lambda\,z^H\,\right]\overline{x}^F\,\mu\left(\Delta^H\,\Delta^1\;\;\Delta^G\right)\\[6ex]
\displaystyle+\sum_{\Delta^H}\sum_{\Delta^G\in[\underline{\Omega},\overline{\Omega}]}\left[\begin{array}{l}\mathcal{I}_2\left(\frac{\lambda}{\eta}\Delta^G\right)\,(1-z^H)(1-z^F)\\[2ex] +\left[\,\mathcal{I}_2\left(\frac{\lambda^2}{\eta}\Delta^G\right)+\mathcal{I}_3\left(\frac{\lambda^2}{\eta}\Delta^G\right)\,\right]\,\lambda\,z^H(1-z^F)\\[2ex] +\mathcal{I}_2\left(\frac{1}{\lambda}\Delta^G\right)\,(1-z^H)z^F+\lambda\,z^Hz^F\end{array}\right]\overline{x}^F\,\mu\left(\Delta^H\,\Delta^2\;\;\Delta^G\right)\\[8ex]
\displaystyle+\sum_{\Delta^H}\sum_{\Delta^G\in[\underline{\Omega},\overline{\Omega}]}\left[\begin{array}{l}(1-z^H)(1-z^F)\\[2ex] +\left[\,\mathcal{I}_2\left(\lambda\Delta^G\right)+\mathcal{I}_3\left(\lambda\Delta^G\right)\,\right]\,\lambda\,z^H(1-z^F)\\[2ex] +\mathcal{I}_2\left(\frac{1}{\lambda}\Delta^G\right)\,(1-z^H)z^F+\lambda\,z^Hz^F\end{array}\right]\overline{x}^F\,\mu\left(\Delta^H\,\Delta^3\;\;\Delta^G\right)\\[8ex]
\displaystyle+\sum_{\Delta^H}\sum_{\Delta^G\in[\underline{\Omega},\overline{\Omega}]}\left[\,\mathcal{I}_2\left(\frac{1}{\lambda}\Delta^G\right)\,(1-z^H)+\lambda\,z^H\,\right]\overline{x}^F\,\mu\left(\Delta^H\,\Delta^4\;\;\Delta^G\right)
\end{array}\right\}
$$

$$\times\,\mathcal{Q}_{HH}$$

$$
+\left\{\begin{array}{l}
\left[\begin{array}{l}
\left[\,\mathcal{I}_2\,(\eta)+\mathcal{I}_3\,(\eta)\,\right]\,\eta\,(1-z^F)\overline{x}^H \\[2ex]
+\left[\,\mathcal{I}_2\left(\frac{\eta}{\lambda}\right)+\mathcal{I}_3\left(\frac{\eta}{\lambda}\right)\,\right]\,\eta\,z^F\,\overline{x}^H
\end{array}\right]\mu\left(\Delta^1\,\Delta^1\ \ -\infty\right) \\[4ex]
+\left[\begin{array}{l}
\left[\,\mathcal{I}_2\left(\frac{\eta}{\lambda}\right)+\mathcal{I}_3\left(\frac{\eta}{\lambda}\right)\,\right]\,\frac{\eta}{\lambda}\,(1-z^F)\overline{x}^H \\[2ex]
+\left[\,\mathcal{I}_2\left(\frac{\eta}{\lambda^2}\right)+\mathcal{I}_3\left(\frac{\eta}{\lambda^2}\right)\,\right]\,\frac{\eta}{\lambda}\,z^F\,\overline{x}^H
\end{array}\right]\mu\left(\Delta^2\,\Delta^2\ \ -\infty\right) \\[4ex]
+\left[\begin{array}{l}
\left[\,\mathcal{I}_2\,(1)+\mathcal{I}_3\,(1)\,\right]\,(1-z^F)\overline{x}^H \\[2ex]
+\left[\,\mathcal{I}_2\left(\frac{1}{\lambda}\right)+\mathcal{I}_3\left(\frac{1}{\lambda}\right)\,\right]\,z^F\,\overline{x}^H
\end{array}\right]\mu\left(\Delta^3\,\Delta^3\ \ -\infty\right) \\[4ex]
+\left[\begin{array}{l}
\left[\,\mathcal{I}_2\,(\lambda)+\mathcal{I}_3\,(\lambda)\,\right]\,\lambda\,(1-z^F)\overline{x}^H \\[2ex]
+\left[\,\mathcal{I}_2\,(1)+\mathcal{I}_3\,(1)\,\right]\,\lambda\,z^F\,\overline{x}^H
\end{array}\right]\mu\left(\Delta^4\,\Delta^4\ \ -\infty\right)
\end{array}\right\}\times\mathcal{Q}_{FH}
$$

$$
\begin{aligned}
\mathcal{Q}'_{FH} = &\left\{\begin{array}{l}
\left[\ \left[\,\mathcal{I}_1\left(\frac{1}{\eta}\right)\,(1-z^H)+\mathcal{I}_1\left(\frac{\lambda}{\eta}\right)\,z^H\,\right]\,\eta\,\overline{x}^F\ \right]\mu\left(\Delta^1\,\Delta^1\ \infty\right) \\[3ex]
+\left[\ \left[\,\mathcal{I}_1\left(\frac{\lambda}{\eta}\right)\,(1-z^H)+\mathcal{I}_1\left(\frac{\lambda^2}{\eta}\right)\,z^H\,\right]\,\frac{\eta}{\lambda}\,\overline{x}^F\ \right]\mu\left(\Delta^2\,\Delta^2\ \infty\right) \\[3ex]
+\left[\ \left[\,\mathcal{I}_1\,(1)\,(1-z^H)+\mathcal{I}_1\,(\lambda)\,z^H\,\right]\,\overline{x}^F\ \right]\mu\left(\Delta^3\,\Delta^3\ \infty\right) \\[3ex]
+\left[\ \left[\,\mathcal{I}_1\left(\frac{1}{\lambda}\right)\,(1-z^H)+\mathcal{I}_1\,(1)\,z^H\,\right]\,\lambda\,\overline{x}^F\ \right]\mu\left(\Delta^4\,\Delta^4\ \infty\right)
\end{array}\right\}\times\mathcal{Q}_{HH} \\
&+\left\{\ \sum_{\Delta^H}\sum_{\Delta^F}\sum_{\Delta^G\in[\underline{\Omega},\overline{\Omega}]}\left[\ \mathcal{I}_1\left(\tfrac{1}{\lambda}\Delta^G\right)\,\lambda\,(1-z^H)\,z^F\,(1-\overline{x})\ \right]\mu\left(\Delta^H\,\Delta^F\ \ \Delta^G\right)\ \right\}\times\mathcal{Q}_{HH} \\
&+\left\{\begin{array}{l}
\displaystyle\sum_{\Delta^F}\sum_{\Delta^G\in[\underline{\Omega},\overline{\Omega}]}\left[\ \mathcal{I}_1\left(\tfrac{\eta}{\lambda^2}\Delta^G\right)\,\lambda\,(1-z^H)\,z^F\,\overline{x}^H\ \right]\mu\left(\Delta^2\,\Delta^F\ \ \Delta^G\right) \\[4ex]
\displaystyle+\sum_{\Delta^F}\sum_{\Delta^G\in[\underline{\Omega},\overline{\Omega}]}\left[\ \mathcal{I}_1\left(\tfrac{1}{\lambda}\Delta^G\right)\,\lambda\,(1-z^H)\,z^F\,\overline{x}^H\ \right]\mu\left(\Delta^3\,\Delta^F\ \ \Delta^G\right)
\end{array}\right\}\times\mathcal{Q}_{HH}
\end{aligned}
$$

$$
+\left\{
\begin{array}{l}
\displaystyle\sum_{\Delta^H}\sum_{\Delta^G\in[\underline{\Omega},\overline{\Omega}]}\left[\ \mathcal{I}_1\left(\tfrac{1}{\eta}\Delta^G\right)\ (1-z^H)+\mathcal{I}_1\left(\tfrac{\lambda}{\eta}\Delta^G\right)\ z^H\ \right]\eta\,\overline{x}^F\,\mu\left(\Delta^H\ \Delta^1\ \ \Delta^G\right)\\[2ex]
\displaystyle+\sum_{\Delta^H}\sum_{\Delta^G\in[\underline{\Omega},\overline{\Omega}]}\left[\begin{array}{l}\mathcal{I}_1\left(\tfrac{\lambda}{\eta}\Delta^G\right)\ \tfrac{\eta}{\lambda}\,(1-z^H)(1-z^F)\\[1ex] +\mathcal{I}_1\left(\tfrac{1}{\lambda}\Delta^G\right)\ \lambda\,(1-z^H)z^F\end{array}\right]\overline{x}^F\,\mu\left(\Delta^H\ \Delta^2\ \ \Delta^G\right)\\[2ex]
\displaystyle+\sum_{\Delta^H}\sum_{\Delta^G\in[\underline{\Omega},\overline{\Omega}]}\left[\ \mathcal{I}_1\left(\tfrac{1}{\lambda}\Delta^G\right)\ \lambda\,(1-z^H)z^F\ \right]\overline{x}^F\,\mu\left(\Delta^H\ \Delta^3\ \ \Delta^G\right)\\[2ex]
\displaystyle+\sum_{\Delta^H}\sum_{\Delta^G\in[\underline{\Omega},\overline{\Omega}]}\left[\ \mathcal{I}_1\left(\tfrac{1}{\lambda}\Delta^G\right)\ \lambda\,(1-z^H)\ \right]\overline{x}^F\,\mu\left(\Delta^H\ \Delta^4\ \ \Delta^G\right)
\end{array}
\right\}\times\mathcal{Q}_{HH}
$$

$$
+\left\{
\begin{array}{l}
\left[\begin{array}{l}\left[\,(1-z^F)+\lambda\,z^F\,\right](1-\overline{x})\\[1ex] +\left[\,\mathcal{I}_1\,(\eta)\,(1-z^F)+\mathcal{I}_1\left(\tfrac{\eta}{\lambda}\right)\ \lambda\,z^F\,\right]\ \overline{x}^H+\eta\,\overline{x}^F\end{array}\right]\mu\left(\Delta^1\,\Delta^1\ \ -\infty\right)\\[3ex]
+\left[\begin{array}{l}\left[\,(1-z^F)+\lambda\,z^F\,\right](1-\overline{x})\\[1ex] +\left[\,\mathcal{I}_1\left(\tfrac{\eta}{\lambda}\right)(1-z^F)+\mathcal{I}_1\left(\tfrac{\eta}{\lambda^2}\right)\ \lambda\,z^F\,\right]\ \overline{x}^H\\[1ex] +\left[\,\tfrac{\eta}{\lambda}(1-z^F)+\lambda\,z^F\,\right]\ \overline{x}^F\end{array}\right]\mu\left(\Delta^2\,\Delta^2\ \ -\infty\right)\\[3ex]
+\left[\begin{array}{l}\left[\,(1-z^F)+\lambda\,z^F\,\right](1-\overline{x})\\[1ex] +\left[\,\mathcal{I}_1\,(1)\,(1-z^F)+\mathcal{I}_1\left(\tfrac{1}{\lambda}\right)\ \lambda\,z^F\,\right]\ \overline{x}^H\\[1ex] +\left[\,(1-z^F)+\lambda\,z^F\,\right]\ \overline{x}^F\end{array}\right]\mu\left(\Delta^3\,\Delta^3\ \ -\infty\right)\\[3ex]
+\left[\begin{array}{l}\left[\,(1-z^F)+\lambda\,z^F\,\right](1-\overline{x})\\[1ex] +\left[\,\mathcal{I}_1\,(\lambda)\,(1-z^F)+\mathcal{I}_1\,(1)\ \lambda\,z^F\,\right]\ \overline{x}^H+\lambda\,\overline{x}^F\end{array}\right]\mu\left(\Delta^4\,\Delta^4\ \ -\infty\right)
\end{array}
\right\}\times\mathcal{Q}_{FH}
$$

$$
\mathcal{Q}'_{FF} = \left\{ \begin{array}{l}
\left[\begin{array}{l}
\left[\,(1-z^F)+\lambda z^F\,\right](1-\overline{x}) \\[2ex]
+\left[\,\mathcal{I}_1\left(\eta\right)+\mathcal{I}_2\left(\eta\right)\,\right](1-z^F)\overline{x}^H \\[2ex]
+\left[\,\mathcal{I}_1\left(\frac{\eta}{\lambda}\right)+\mathcal{I}_2\left(\frac{\eta}{\lambda}\right)\,\right]\ \lambda\, z^F\,\overline{x}^H+\eta\,\overline{x}^F
\end{array} \right] \mu\left(\Delta^1\,\Delta^1\ \ -\infty\right) \\[6ex]
+\left[\begin{array}{l}
\left[\,(1-z^F)+\lambda z^F\,\right](1-\overline{x}) \\[2ex]
+\left[\,\mathcal{I}_1\left(\frac{\eta}{\lambda}\right)+\mathcal{I}_2\left(\frac{\eta}{\lambda}\right)\,\right](1-z^F)\overline{x}^H \\[2ex]
+\left[\,\mathcal{I}_1\left(\frac{\eta}{\lambda^2}\right)+\mathcal{I}_2\left(\frac{\eta}{\lambda^2}\right)\,\right]\ \lambda\, z^F\,\overline{x}^H \\[2ex]
+\frac{\eta}{\lambda}\,(1-z^F)\overline{x}^F+\lambda\, z^F\,\overline{x}^F
\end{array} \right] \mu\left(\Delta^2\,\Delta^2\ \ -\infty\right) \\[6ex]
+\left[\begin{array}{l}
\left[\,(1-z^F)+\lambda z^F\,\right](1-\overline{x}) \\[2ex]
+\left[\,\mathcal{I}_1\left(1\right)+\mathcal{I}_2\left(1\right)\,\right](1-z^F)\overline{x}^H \\[2ex]
+\left[\,\mathcal{I}_1\left(\frac{1}{\lambda}\right)+\mathcal{I}_2\left(\frac{1}{\lambda}\right)\,\right]\ \lambda\, z^F\,\overline{x}^H \\[2ex]
+(1-z^F)\overline{x}^F+\lambda\, z^F\,\overline{x}^F
\end{array} \right] \mu\left(\Delta^3\,\Delta^3\ \ -\infty\right) \\[6ex]
+\left[\begin{array}{l}
\left[\,(1-z^F)+\lambda z^F\,\right](1-\overline{x}) \\[2ex]
+\left[\,\mathcal{I}_1\left(\lambda\right)+\mathcal{I}_2\left(\lambda\right)\,\right](1-z^F)\overline{x}^H \\[2ex]
+\left[\,\mathcal{I}_1\left(1\right)+\mathcal{I}_2\left(1\right)\,\right]\ \lambda\, z^F\,\overline{x}^H+\lambda\,\overline{x}^F
\end{array} \right] \mu\left(\Delta^4\,\Delta^4\ \ -\infty\right)
\end{array} \right\} \times \mathcal{Q}_{FF}
$$

$$
+\left\{ \sum_{\Delta^H}\sum_{\Delta^F}\sum_{\Delta^G\in[\underline{\Omega},\overline{\Omega}]} \left[\begin{array}{l}
(1-z^H)(1-z^F) \\[2ex]
+\,\mathcal{I}_2\left(\lambda\Delta^G\right)\ z^H(1-z^F) \\[2ex]
+\left[\mathcal{I}_1\left(\frac{1}{\lambda}\Delta^G\right)+\mathcal{I}_2\left(\frac{1}{\lambda}\Delta^G\right)\right]\ \lambda\,(1-z^H)z^F \\[2ex]
+\lambda\, z^H z^F
\end{array} \right] (1-\overline{x})\,\mu\left(\Delta^H\ \Delta^F\ \ \Delta^G\right) \right\}
$$

$$
\times\ \mathcal{Q}_{FF}
$$

$$
+\left\{\begin{array}{l}
\sum\limits_{\Delta^F}\sum\limits_{\Delta^G\in[\underline{\Omega},\overline{\Omega}]}\left[\ \mathcal{I}_2\left(\eta\Delta^G\right)\ (1-z^F)+\mathcal{I}_2\left(\frac{\eta}{\lambda}\Delta^G\right)\ \lambda\, z^F\ \right]\overline{x}^H\,\mu\left(\Delta^1\,\Delta^F\ \ \Delta^G\right) \\
+\sum\limits_{\Delta^F}\sum\limits_{\Delta^G\in[\underline{\Omega},\overline{\Omega}]}\left[\begin{array}{l}
\mathcal{I}_2\left(\frac{\eta}{\lambda}\Delta^G\right)\ (1-z^H)(1-z^F) \\
+\mathcal{I}_2\left(\lambda\Delta^G\right)\ z^H(1-z^F) \\
+\left[\mathcal{I}_1\left(\frac{\eta}{\lambda^2}\Delta^G\right)+\mathcal{I}_2\left(\frac{\eta}{\lambda^2}\Delta^G\right)\right]\ \lambda\,(1-z^H)\,z^F \\
+\lambda\,z^H\,z^F
\end{array}\right]\overline{x}^H\,\mu\left(\Delta^2\,\Delta^F\ \ \Delta^G\right) \\
+\sum\limits_{\Delta^F}\sum\limits_{\Delta^G\in[\underline{\Omega},\overline{\Omega}]}\left[\begin{array}{l}
(1-z^H)(1-z^F) \\
+\mathcal{I}_2\left(\lambda\Delta^G\right)\ z^H(1-z^F) \\
+\left[\mathcal{I}_1\left(\frac{1}{\lambda}\Delta^G\right)+\mathcal{I}_2\left(\frac{1}{\lambda}\Delta^G\right)\right]\ \lambda\,(1-z^H)\,z^F \\
+\lambda\,z^H\,z^F
\end{array}\right]\overline{x}^H\,\mu\left(\Delta^3\,\Delta^F\ \ \Delta^G\right) \\
+\sum\limits_{\Delta^F}\sum\limits_{\Delta^G\in[\underline{\Omega},\overline{\Omega}]}\left[\ \mathcal{I}_2\left(\lambda\Delta^G\right)(1-z^F)+\lambda\,z^F\ \right]\overline{x}^H\,\mu\left(\Delta^4\,\Delta^F\ \ \Delta^G\right)
\end{array}\right\}
$$

$$
\times\,\mathcal{Q}_{FF}
$$

$$
+\left\{\begin{array}{l}
\sum_{\Delta^H}\sum_{\Delta^G\in[\underline{\Omega},\overline{\Omega}]}\left[\begin{array}{l}
\left[\mathcal{I}_1\left(\frac{1}{\eta}\Delta^G\right)+\mathcal{I}_2\left(\frac{1}{\eta}\Delta^G\right)\right]\,\eta\,(1-z^H)\\
+\left[\mathcal{I}_1\left(\frac{\lambda}{\eta}\Delta^G\right)+\mathcal{I}_2\left(\frac{\lambda}{\eta}\Delta^G\right)\right]\,\eta\,z^H
\end{array}\right]\overline{x}^F\,\mu\left(\Delta^H\;\Delta^1\;\;\Delta^G\right)\\
+\sum_{\Delta^H}\sum_{\Delta^G\in[\underline{\Omega},\overline{\Omega}]}\left[\begin{array}{l}
\left[\mathcal{I}_1\left(\frac{\lambda}{\eta}\Delta^G\right)+\mathcal{I}_2\left(\frac{\lambda}{\eta}\Delta^G\right)\right]\,\frac{\eta}{\lambda}\,(1-z^H)(1-z^F)\\
+\mathcal{I}_2\left(\frac{\lambda^2}{\eta}\Delta^G\right)\frac{\eta}{\lambda}\,z^H(1-z^F)\\
+\left[\mathcal{I}_1\left(\frac{1}{\lambda}\Delta^G\right)+\mathcal{I}_2\left(\frac{1}{\lambda}\Delta^G\right)\right]\,\lambda\,(1-z^H)\,z^F\\
+\lambda\,z^H\,z^F
\end{array}\right]\overline{x}^F\,\mu\left(\Delta^H\;\Delta^2\;\;\Delta^G\right)\\
+\sum_{\Delta^H}\sum_{\Delta^G\in[\underline{\Omega},\overline{\Omega}]}\left[\begin{array}{l}
(1-z^H)(1-z^F)+\mathcal{I}_2\left(\lambda\Delta^G\right)\;z^H(1-z^F)\\
+\left[\mathcal{I}_1\left(\frac{1}{\lambda}\Delta^G\right)+\mathcal{I}_2\left(\frac{1}{\lambda}\Delta^G\right)\right]\,\lambda\,(1-z^H)\,z^F\\
+\lambda\,z^H\,z^F
\end{array}\right]\overline{x}^F\,\mu\left(\Delta^H\;\Delta^3\;\;\Delta^G\right)\\
+\sum_{\Delta^H}\sum_{\Delta^G\in[\underline{\Omega},\overline{\Omega}]}\left[\;\left[\mathcal{I}_1\left(\frac{1}{\lambda}\Delta^G\right)+\mathcal{I}_2\left(\frac{1}{\lambda}\Delta^G\right)\right]\,\lambda\,(1-z^H)+\lambda\,z^H\;\right]\overline{x}^F\,\mu\left(\Delta^H\;\Delta^4\;\;\Delta^G\right)
\end{array}\right\}
$$

$$
\times\,\mathcal{Q}_{FF}
$$

$$
+\left\{\begin{array}{l}
\left[\begin{array}{l}
\left[\,\mathcal{I}_1\left(\frac{1}{\eta}\right)+\mathcal{I}_2\left(\frac{1}{\eta}\right)\,\right]\,\eta\,(1-z^H)\overline{x}^F\\
+\left[\,\mathcal{I}_1\left(\frac{\lambda}{\eta}\right)+\mathcal{I}_2\left(\frac{\lambda}{\eta}\right)\,\right]\,\eta\,z^H\,\overline{x}^F
\end{array}\right]\mu\left(\Delta^1\,\Delta^1\;\;\infty\right)\\
+\left[\begin{array}{l}
\left[\,\mathcal{I}_1\left(\frac{\lambda}{\eta}\right)+\mathcal{I}_2\left(\frac{\lambda}{\eta}\right)\,\right]\,\frac{\eta}{\lambda}\,(1-z^H)\overline{x}^F\\
+\left[\,\mathcal{I}_1\left(\frac{\lambda^2}{\eta}\right)+\mathcal{I}_2\left(\frac{\lambda^2}{\eta}\right)\,\right]\,\frac{\eta}{\lambda}\,z^H\,\overline{x}^F
\end{array}\right]\mu\left(\Delta^2\,\Delta^2\;\;\infty\right)\\
+\left[\begin{array}{l}
\left[\,\mathcal{I}_1\left(1\right)+\mathcal{I}_2\left(1\right)\,\right]\,(1-z^H)\overline{x}^F\\
+\left[\,\mathcal{I}_1\left(\lambda\right)+\mathcal{I}_2\left(\lambda\right)\,\right]\,z^H\,\overline{x}^F
\end{array}\right]\mu\left(\Delta^3\,\Delta^3\;\;\infty\right)\\
+\left[\begin{array}{l}
\left[\,\mathcal{I}_1\left(\frac{1}{\lambda}\right)+\mathcal{I}_2\left(\frac{1}{\lambda}\right)\,\right]\,\lambda\,(1-z^H)\overline{x}^F\\
+\left[\,\mathcal{I}_1\left(1\right)+\mathcal{I}_2\left(1\right)\,\right]\,\lambda\,z^H\,\overline{x}^F
\end{array}\right]\mu\left(\Delta^4\,\Delta^4\;\;\infty\right)
\end{array}\right\}\times\,\mathcal{Q}_{HF}
$$

$$
\mathcal{Q}'_{HF} = \left\{ \begin{array}{l}
\left[\ \mathcal{I}_3\left(\eta\right) \ \eta \left(1 - z^F\right)\overline{x}^H + \mathcal{I}_3\left(\tfrac{\eta}{\lambda}\right) \ \eta \, z^F \, \overline{x}^H \ \right] \mu\left(\Delta^1 \, \Delta^1 \ \ - \infty\right) \\[2em]
+ \left[\ \mathcal{I}_3\left(\tfrac{\eta}{\lambda}\right) \ \tfrac{\eta}{\lambda} \left(1 - z^F\right)\overline{x}^H + \mathcal{I}_3\left(\tfrac{\eta}{\lambda^2}\right) \ \tfrac{\eta}{\lambda} \, z^F \, \overline{x}^H \ \right] \mu\left(\Delta^2 \, \Delta^2 \ \ - \infty\right) \\[2em]
+ \left[\ \mathcal{I}_3\left(1\right) \ \left(1 - z^F\right)\overline{x}^H + \mathcal{I}_3\left(\tfrac{1}{\lambda}\right) \ z^F \, \overline{x}^H \ \right] \mu\left(\Delta^3 \, \Delta^3 \ \ - \infty\right) \\[2em]
+ \left[\ \mathcal{I}_3\left(\lambda\right) \ \lambda \left(1 - z^F\right)\overline{x}^H + \mathcal{I}_3\left(1\right) \ \lambda \, z^F \, \overline{x}^H \ \right] \mu\left(\Delta^4 \, \Delta^4 \ \ - \infty\right)
\end{array} \right\} \times \mathcal{Q}_{FF}
$$

$$
+ \left\{ \ \sum_{\Delta^H} \sum_{\Delta^F} \sum_{\Delta^G \in [\underline{\Omega}, \overline{\Omega}]} \left[\ \mathcal{I}_3\left(\lambda \Delta^G\right) \ \lambda \, z^H \left(1 - z^F\right) \ \right] \left(1 - \overline{x}\right) \mu\left(\Delta^H \, \Delta^F \ \ \Delta^G\right) \ \right\} \times \mathcal{Q}_{FF}
$$

$$
+ \left\{ \begin{array}{l}
\sum_{\Delta^F} \sum_{\Delta^G \in [\underline{\Omega}, \overline{\Omega}]} \left[\ \mathcal{I}_3\left(\eta \Delta^G\right) \ \eta \left(1 - z^F\right) + \mathcal{I}_3\left(\tfrac{\eta}{\lambda} \Delta^G\right) \ \eta \, z^F \ \right] \overline{x}^H \, \mu\left(\Delta^1 \, \Delta^F \ \ \Delta^G\right) \\[2em]
+ \sum_{\Delta^F} \sum_{\Delta^G \in [\underline{\Omega}, \overline{\Omega}]} \left[\begin{array}{l} \mathcal{I}_3\left(\tfrac{\eta}{\lambda} \Delta^G\right) \ \tfrac{\eta}{\lambda} \left(1 - z^H\right)\left(1 - z^F\right) \\[1em] + \mathcal{I}_3\left(\lambda \Delta^G\right) \ \lambda \, z^H \left(1 - z^F\right) \end{array} \right] \overline{x}^H \, \mu\left(\Delta^2 \, \Delta^F \ \ \Delta^G\right) \\[2em]
+ \sum_{\Delta^F} \sum_{\Delta^G \in [\underline{\Omega}, \overline{\Omega}]} \left[\ \mathcal{I}_3\left(\lambda \Delta^G\right) \ \lambda \, z^H \left(1 - z^F\right) \ \right] \overline{x}^H \, \mu\left(\Delta^3 \, \Delta^F \ \ \Delta^G\right) \\[2em]
+ \sum_{\Delta^F} \sum_{\Delta^G \in [\underline{\Omega}, \overline{\Omega}]} \left[\ \mathcal{I}_3\left(\lambda \Delta^G\right) \ \lambda \left(1 - z^F\right) \ \right] \overline{x}^H \, \mu\left(\Delta^4 \, \Delta^F \ \ \Delta^G\right)
\end{array} \right\} \times \mathcal{Q}_{FF}
$$

$$
+ \left\{ \begin{array}{l}
\sum_{\Delta^H} \sum_{\Delta^G \in [\underline{\Omega}, \overline{\Omega}]} \left[\ \mathcal{I}_3\left(\tfrac{\lambda^2}{\eta} \Delta^G\right) \ \lambda \, z^H \left(1 - z^F\right) \ \right] \overline{x}^F \, \mu\left(\Delta^H \, \Delta^2 \ \ \Delta^G\right) \\[2em]
+ \sum_{\Delta^H} \sum_{\Delta^G \in [\underline{\Omega}, \overline{\Omega}]} \left[\ \mathcal{I}_3\left(\lambda \Delta^G\right) \ \lambda \, z^H \left(1 - z^F\right) \ \right] \overline{x}^F \, \mu\left(\Delta^H \, \Delta^3 \ \ \Delta^G\right)
\end{array} \right\} \times \mathcal{Q}_{FF}
$$

$$
+\left\{\begin{array}{l}
\left[\begin{array}{l}
(1-z^H)(1-\overline{x})+\lambda\, z^H(1-\overline{x})+\eta\overline{x}^H \\
+\mathcal{I}_3\left(\frac{1}{\eta}\right)\,(1-z^H)\overline{x}^F+\mathcal{I}_3\left(\frac{\lambda}{\eta}\right)\,\lambda\, z^H\,\overline{x}^F
\end{array}\right]\mu\left(\Delta^1\,\Delta^1\;\infty\right) \\
+\left[\begin{array}{l}
(1-z^H)(1-\overline{x})+\lambda\, z^H(1-\overline{x}) \\
+\frac{\eta}{\lambda}\,(1-z^H)\overline{x}^H+\lambda\, z^H\,\overline{x}^H \\
+\mathcal{I}_3\left(\frac{\lambda}{\eta}\right)\,(1-z^H)\overline{x}^F+\mathcal{I}_3\left(\frac{\lambda^2}{\eta}\right)\,\lambda\, z^H\,\overline{x}^F
\end{array}\right]\mu\left(\Delta^2\,\Delta^2\;\infty\right) \\
+\left[\begin{array}{l}
(1-z^H)(1-\overline{x})+\lambda\, z^H(1-\overline{x}) \\
+(1-z^H)\overline{x}^H+\lambda\, z^H\,\overline{x}^H \\
+\mathcal{I}_3\,(1)\,\,(1-z^H)\overline{x}^F+\mathcal{I}_3\,(\lambda)\,\,\lambda\, z^H\,\overline{x}^F
\end{array}\right]\mu\left(\Delta^3\,\Delta^3\;\infty\right) \\
+\left[\begin{array}{l}
(1-z^H)(1-\overline{x})+\lambda\, z^H(1-\overline{x})+\lambda\,\overline{x}^H \\
+\mathcal{I}_3\left(\frac{1}{\lambda}\right)\,(1-z^H)\overline{x}^F+\mathcal{I}_3\,(1)\,\,\lambda\, z^H\,\overline{x}^F
\end{array}\right]\mu\left(\Delta^4\,\Delta^4\;\infty\right)
\end{array}\right\}\times\mathcal{Q}_{HF}
$$

B.3.3.1 Proof of Proposition 6

B.3.4 Value Function

B.3.4.1 One Product Case Example

For $\Phi^f = \{(\, q\; H\; H\; \Delta^1\; \Delta^1\; \infty\,)\}$,

$$
V\big(\, q\; \Delta^1\; \Delta^1\; \infty\,\big) = (\pi^H+\pi^F)q - \widehat{\chi} z^{\widehat{\psi}} q - \overline{q}\widetilde{\chi} x^{\widetilde{\psi}}
$$

$$
+\,\widetilde{\beta}\times\left[\begin{array}{ll}
V\left(\left\{\left(q\,\Delta^1\,\Delta^1\,\infty\right)\right\}\right) & \times\ (1-z)(1-\overline{x}) \\
+\,V\left(\left\{\left(\lambda q\,\Delta^2\,\Delta^2\,\infty\right)\right\}\right) & \times\ z\,(1-\overline{x}) \\
+\,V(\varnothing) & \times\ (1-z)\,\overline{x}^H \\
+\,V(\varnothing) & \times\ z\,\overline{x}^H \\
+\left[\begin{array}{ll}
V(\varnothing) & \mathcal{I}_1\left(\frac{1}{\eta}\right) \\
+\,V\left(\left\{\left(q\,\Delta^1\,\Delta^3\ \frac{1}{\eta}\right)\right\}\right) & \mathcal{I}_2\left(\frac{1}{\eta}\right) \\
+\,V\left(\left\{\left(q\,\Delta^1\,\Delta^1\ \infty\right)\right\}\right) & \mathcal{I}_3\left(\frac{1}{\eta}\right)
\end{array}\right] & \times\ (1-z)\,\overline{x}^F \\
+\left[\begin{array}{ll}
V(\varnothing) & \mathcal{I}_1\left(\frac{\lambda}{\eta}\right) \\
+\,V\left(\left\{\left(\lambda\,q\,\Delta^2\,\Delta^3\ \frac{\lambda}{\eta}\right)\right\}\right) & \mathcal{I}_2\left(\frac{\lambda}{\eta}\right) \\
+\,V\left(\left\{\left(\lambda q\,\Delta^2\,\Delta^2\ \infty\right)\right\}\right) & \mathcal{I}_3\left(\frac{\lambda}{\eta}\right)
\end{array}\right] & \times\ z\,\overline{x}^F
\end{array}\right]\times(1-x)
$$

$$
+\,\widetilde{\beta}\times\int_{\Phi_{-j}}\ \sum_{\mathcal{I}^{ZH}_{-j}=0}^{1}\ \sum_{\mathcal{I}^{ZF}_{-j}=0}^{1}\ \sum_{c-t_{-j}=\text{win}}^{\text{lose}}\ \left(z^H\right)^{\mathcal{I}^{ZH}_{-j}}\left(1-z^H\right)^{1-\mathcal{I}^{ZH}_{-j}}\ \left(z^F\right)^{\mathcal{I}^{ZF}_{-j}}\left(1-z^F\right)^{1-\mathcal{I}^{ZF}_{-j}}\ \frac{1}{2}
$$

$$
\times\left[\begin{array}{ll}
V\left(\left\{\left(q\,\Delta^1\,\Delta^1\,\infty\right),\ \Phi'_{-j}\,|\,\Phi_{-j}\,,\,\mathcal{I}^{ZH}_{-j}\,,\mathcal{I}^{ZF}_{-j}\,,\,c-t_{-j}\right\}\right) & \times\ (1-z)(1-\overline{x}) \\
+\,V\left(\left\{\left(\lambda q\,\Delta^2\,\Delta^2\,\infty\right),\ \Phi'_{-j}\,|\,\Phi_{-j}\,,\,\mathcal{I}^{ZH}_{-j}\,,\mathcal{I}^{ZF}_{-j}\,,\,c-t_{-j}\right\}\right) & \times\ z\,(1-\overline{x}) \\
+\,V\left(\left\{\Phi'_{-j}\,|\,\Phi_{-j}\,,\,\mathcal{I}^{ZH}_{-j}\,,\mathcal{I}^{ZF}_{-j}\,,\,c-t_{-j}\right\}\right) & \times\ (1-z)\,\overline{x}^H \\
+\,V\left(\left\{\Phi'_{-j}\,|\,\Phi_{-j}\,,\,\mathcal{I}^{ZH}_{-j}\,,\mathcal{I}^{ZF}_{-j}\,,\,c-t_{-j}\right\}\right) & \times\ z\,\overline{x}^H \\
+\left[\begin{array}{ll}
V\left(\left\{\Phi'_{-j}\,|\,\Phi_{-j}\,,\,\mathcal{I}^{ZH}_{-j}\,,\mathcal{I}^{ZF}_{-j}\,,\,c-t_{-j}\right\}\right) & \mathcal{I}_1\left(\frac{1}{\eta}\right) \\
+\,V\left(\left\{\left(q\,\Delta^1\,\Delta^3\ \frac{1}{\eta}\right),\ \Phi'_{-j}\,|\,\Phi_{-j}\,,\,\mathcal{I}^{ZH}_{-j}\,,\mathcal{I}^{ZF}_{-j}\,,\,c-t_{-j}\right\}\right) & \mathcal{I}_2\left(\frac{1}{\eta}\right) \\
+\,V\left(\left\{\left(q\,\Delta^1\,\Delta^1\ \infty\right),\ \Phi'_{-j}\,|\,\Phi_{-j}\,,\,\mathcal{I}^{ZH}_{-j}\,,\mathcal{I}^{ZF}_{-j}\,,\,c-t_{-j}\right\}\right) & \mathcal{I}_3\left(\frac{1}{\eta}\right)
\end{array}\right] & \times\ (1-z)\,\overline{x}^F \\
+\left[\begin{array}{ll}
V\left(\left\{\Phi'_{-j}\,|\,\Phi_{-j}\,,\,\mathcal{I}^{ZH}_{-j}\,,\mathcal{I}^{ZF}_{-j}\,,\,c-t_{-j}\right\}\right) & \mathcal{I}_1\left(\frac{\lambda}{\eta}\right) \\
+\,V\left(\left\{\left(\lambda\,q\,\Delta^2\,\Delta^3\ \frac{\lambda}{\eta}\right),\ \Phi'_{-j}\,|\,\Phi_{-j}\,,\,\mathcal{I}^{ZH}_{-j}\,,\mathcal{I}^{ZF}_{-j}\,,\,c-t_{-j}\right\}\right) & \mathcal{I}_2\left(\frac{\lambda}{\eta}\right) \\
+\,V\left(\left\{\left(\lambda q\,\Delta^2\,\Delta^2\ \infty\right),\ \Phi'_{-j}\,|\,\Phi_{-j}\,,\,\mathcal{I}^{ZH}_{-j}\,,\mathcal{I}^{ZF}_{-j}\,,\,c-t_{-j}\right\}\right) & \mathcal{I}_3\left(\frac{\lambda}{\eta}\right)
\end{array}\right] & \times\ z\,\overline{x}^F
\end{array}\right]
$$

$$\times \mu(\Phi_{-j})\ \mathbf{d}(\Phi_{-j})\, x\,,$$

where $\mathcal{I}_1(\Delta^G)$ is an indicator equal to one if $\Delta^G < \underline{\Omega}$, $\mathcal{I}_2(\Delta^G)$ is an indicator equal to one if $\Delta^G \in \left[\, \underline{\Omega}\,,\, \overline{\Omega}\,\right]$, and $\mathcal{I}_3(\Delta^G)$ is an indicator equal to one if $\Delta^G > \overline{\Omega}$. Then, with the guessed value function, the above becomes

$$
\begin{aligned}
A\big(\,\Delta^1\ \Delta^1\ \infty\,\big)q + B\overline{q} =\ & (\pi^H + \pi^F)q - \widehat{\chi} z^{\widehat{\psi}} q - \overline{q}\widetilde{\chi} x^{\widetilde{\psi}} \\
& + \widetilde{\beta} \times \left[\begin{array}{ll}
\quad A\,(\,\Delta^1\ \Delta^1\ \infty\,)\, q & \times\ (1-z)(1-\overline{x}) \\
+\, A\,(\,\Delta^2\ \Delta^2\ \infty\,)\, \lambda\, q & \times\ z\,(1-\overline{x}) \\
+\, 0 & \times\ (1-z)\,\overline{x}^H \\
+\, 0 & \times\ z\,\overline{x}^H \\
+ \left[\begin{array}{ll}
0 & \mathcal{I}_1\left(\frac{1}{\eta}\right) \\
+\, A\left(\,\Delta^1\ \Delta^3\ \frac{1}{\eta}\,\right) q & \mathcal{I}_2\left(\frac{1}{\eta}\right) \\
+\, A\,(\,\Delta^1\ \Delta^1\ \ \infty\,)\, q & \mathcal{I}_3\left(\frac{1}{\eta}\right)
\end{array}\right] & \times\ (1-z)\,\overline{x}^F \\
+ \left[\begin{array}{ll}
0 & \mathcal{I}_1\left(\frac{\lambda}{\eta}\right) \\
+\, A\left(\,\Delta^2\ \Delta^3\ \frac{\lambda}{\eta}\,\right) \lambda\, q & \mathcal{I}_2\left(\frac{\lambda}{\eta}\right) \\
+\, A\,(\,\Delta^2\ \Delta^2\ \ \infty\,)\, \lambda\, q & \mathcal{I}_3\left(\frac{\lambda}{\eta}\right)
\end{array}\right] & \times\ z\,\overline{x}^F
\end{array}\right] \\
& + \widetilde{\beta} B(1+g)\overline{q} \\
& + \widetilde{\beta}\,\overline{q} \Bigg[\sum_{\mathcal{I}^{ZH}_{-j}=0}^{1}\ \sum_{\mathcal{I}^{ZF}_{-j}=0}^{1}\ \sum_{c-t_{-j}=\text{win}}^{\text{lose}} \left(z^H\right)^{\mathcal{I}^{ZH}_{-j}} \left(1-z^H\right)^{1-\mathcal{I}^{ZH}_{-j}} \left(z^F\right)^{\mathcal{I}^{ZF}_{-j}} \left(1-z^F\right)^{1-\mathcal{I}^{ZF}_{-j}}\ \frac{1}{2}
\end{aligned}
$$

$$\times \int_{(\,\Delta^{H}_{-j}\ \ \Delta^{F}_{-j}\ \ \Delta^{G}_{-j}\,)} A\Big(\,\Delta^{H\prime}_{-j}\ \ \Delta^{F\prime}_{-j}\ \ \Delta^{G\prime}_{-j}\ \mid\ \Delta^{H}_{-j}\,,\ \Delta^{F}_{-j}\,,\ \Delta^{G}_{-j}\,,\ \mathcal{I}^{ZH}_{-j}\,,\mathcal{I}^{ZF}_{-j}\,,\ c-t_{-j}\Big)\,\Delta^{H\prime}_{-j}$$

$$\times\,\mu\left(\Delta^{H}_{-j}\ \ \Delta^{F}_{-j}\ \ \Delta^{G}_{-j}\,\right)\ \mathbf{d}\left(\Delta^{H}_{-j}\ \ \Delta^{F}_{-j}\ \ \Delta^{G}_{-j}\,\right)\Bigg]\,x\,,$$

where $1+g=\frac{\overline{q}'}{\overline{q}}$. Define $A_{takeover}$ as the terms in the last brackets. Then from FONCs, we get

$$z^{H}(\,\Delta^{1}\ \ \Delta^{1}\ \ \infty\,)=\left(\frac{\widetilde{\beta}}{\widehat{\psi}\widehat{\chi}}\right)^{\frac{1}{\widehat{\psi}-1}}\Bigg[\ \Big[A\left(\,\Delta^{2}\ \ \Delta^{2}\ \ \infty\,\right)\ \lambda-A\left(\,\Delta^{1}\ \ \Delta^{1}\ \ \infty\,\right)\Big](1-\overline{x})$$

$$+\left[\left(A\left(\,\Delta^{2}\ \ \Delta^{3}\ \ \frac{\lambda}{\eta}\right)\,\mathcal{I}_{2}\left(\frac{\lambda}{\eta}\right)+A\left(\,\Delta^{2}\ \ \Delta^{2}\ \ \infty\,\right)\,\mathcal{I}_{3}\left(\frac{\lambda}{\eta}\right)\right)\lambda\right.$$

$$\left.-\left(A\left(\,\Delta^{1}\ \ \Delta^{3}\ \ \frac{1}{\eta}\right)\,\mathcal{I}_{2}\left(\frac{1}{\eta}\right)+A\left(\,\Delta^{1}\ \ \Delta^{1}\ \ \infty\,\right)\,\mathcal{I}_{3}\left(\frac{1}{\eta}\right)\right)\right]\overline{x}^{F}\ \ \Bigg]^{\frac{1}{\widehat{\psi}-1}}$$

and

$$x=\left(\frac{\widetilde{\beta}}{\widetilde{\psi}\widetilde{\chi}}\right)^{\frac{1}{\widetilde{\psi}-1}}\left(A_{takeover}\right)^{\frac{1}{\widetilde{\psi}-1}}\,.$$

Thus,

$$A\left(\,\Delta^{1}\ \ \Delta^{1}\ \ \infty\,\right)=\pi^{H}+\pi^{F}-\widehat{\chi}\,z\left(\,\Delta^{1}\ \ \Delta^{1}\ \ \infty\,\right)^{\widehat{\psi}}$$

$$
+\widetilde{\beta} \times \left[\begin{array}{ll} A\left(\,\Delta^1\,\Delta^1\,\infty\,\right) & \times\ \left(1 - z\left(\,\Delta^1\ \ \Delta^1\ \ \infty\,\right)\right)(1-\overline{x}) \\ +\,A\left(\,\Delta^2\,\Delta^2\,\infty\,\right)\lambda & \times\ z\left(\,\Delta^1\ \ \Delta^1\ \ \infty\,\right)\ (1-\overline{x}) \\ +\left[\begin{array}{l} A\left(\Delta^1\,\Delta^3\ \frac{1}{\eta}\right)\quad \mathcal{I}_2\left(\frac{1}{\eta}\right) \\ +\,A\left(\,\Delta^1\,\Delta^1\ \ \infty\,\right)\quad \mathcal{I}_3\left(\frac{1}{\eta}\right) \end{array} \right] & \times\ \left(1 - z\left(\,\Delta^1\ \ \Delta^1\ \ \infty\,\right)\right)\overline{x}^F \\ +\left[\begin{array}{l} A\left(\Delta^2\,\Delta^3\ \frac{\lambda}{\eta}\right)\lambda\quad \mathcal{I}_2\left(\frac{\lambda}{\eta}\right) \\ +\,A\left(\,\Delta^2\,\Delta^2\ \ \infty\,\right)\lambda\quad \mathcal{I}_3\left(\frac{\lambda}{\eta}\right) \end{array} \right] & \times\ z\left(\,\Delta^1\ \ \Delta^1\ \ \infty\,\right)\ \overline{x}^F \end{array} \right],
$$

and

$$
B = \frac{\left(\widetilde{\psi}-1\right)\ \widetilde{\chi}}{1-\widetilde{\beta}(1+g)} \left(\frac{\widetilde{\beta}}{\widetilde{\psi}\ \widetilde{\chi}}\right)^{\frac{\widetilde{\psi}}{\widetilde{\psi}-1}} \left(A_{takeover}\right)^{\frac{\widetilde{\psi}}{\widetilde{\psi}-1}}.
$$

Value function and internal innovation intensity for other cases can be derived symmetrically.

B.3.4.2 Proof of Proposition 7

$$
x = \left(\frac{\widetilde{\beta}}{\widetilde{\psi}\widetilde{\chi}}\right)^{\frac{1}{\widetilde{\psi}-1}} \left(A_{takeover}\right)^{\frac{1}{\widetilde{\psi}-1}} \tag{B.68}
$$

and

$$
B = \frac{\left(\widetilde{\psi}-1\right)\ \widetilde{\chi}}{1-\widetilde{\beta}(1+g)} \left(\frac{\widetilde{\beta}}{\widetilde{\psi}\ \widetilde{\chi}}\right)^{\frac{\widetilde{\psi}}{\widetilde{\psi}-1}} \left(A_{takeover}\right)^{\frac{\widetilde{\psi}}{\widetilde{\psi}-1}},
$$

where $A_{takeover}$ is defined below.

B.3.4.2.1 Optimal Internal Innovation Intensity, Home Firm

H superscripts from all the $A(\cdot)$ and $z(\cdot)$ are dropped for notational simplicity.

$$\begin{aligned} z^H(\,\Delta^1\ \Delta^1\ \infty\,) = \left(\frac{\widetilde{\beta}}{\widehat{\psi}\widehat{\chi}}\right)^{\frac{1}{\widehat{\psi}-1}} \Bigg[& \Big[A\left(\,\Delta^2\ \Delta^2\ \infty\,\right)\ \lambda - A\left(\,\Delta^1\ \Delta^1\ \infty\,\right)\Big](1-\overline{x}) \\ & + \Bigg[\left(A\left(\,\Delta^2\ \Delta^3\ \frac{\lambda}{\eta}\right)\ \mathcal{I}_2\left(\frac{\lambda}{\eta}\right) + A\left(\,\Delta^2\ \Delta^2\ \infty\,\right)\ \mathcal{I}_3\left(\frac{\lambda}{\eta}\right)\right)\lambda \\ & - \left(A\left(\,\Delta^1\ \Delta^3\ \frac{1}{\eta}\right)\ \mathcal{I}_2\left(\frac{1}{\eta}\right) + A\left(\,\Delta^1\ \Delta^1\ \infty\,\right)\ \mathcal{I}_3\left(\frac{1}{\eta}\right)\right)\Bigg]\overline{x}^F \Bigg]^{\frac{1}{\widehat{\psi}-1}} \end{aligned} \tag{B.69}$$

$$\begin{aligned} z^H(\,\Delta^2\ \Delta^2\ \infty\,) = \left(\frac{\widetilde{\beta}}{\widehat{\psi}\widehat{\chi}}\right)^{\frac{1}{\widehat{\psi}-1}} \Bigg[& \Big[A\left(\,\Delta^2\ \Delta^2\ \infty\,\right)\ \lambda - A\left(\,\Delta^1\ \Delta^1\ \infty\,\right)\Big](1-\overline{x}) \\ & + A\left(\,\Delta^2\ \Delta^2\ \infty\,\right)\ \lambda\,\overline{x}^H \\ & + \Bigg[\left(A\left(\,\Delta^2\ \Delta^4\ \frac{\lambda^2}{\eta}\right)\ \mathcal{I}_2\left(\frac{\lambda^2}{\eta}\right) + A\left(\,\Delta^2\ \Delta^2\ \infty\,\right)\ \mathcal{I}_3\left(\frac{\lambda^2}{\eta}\right)\right)\lambda \\ & - \left(A\left(\,\Delta^1\ \Delta^4\ \frac{\lambda}{\eta}\right)\ \mathcal{I}_2\left(\frac{\lambda}{\eta}\right) + A\left(\,\Delta^1\ \Delta^1\ \infty\,\right)\ \mathcal{I}_3\left(\frac{\lambda}{\eta}\right)\right)\Bigg]\overline{x}^F \Bigg]^{\frac{1}{\widehat{\psi}-1}} \end{aligned} \tag{B.70}$$

$$\begin{aligned} z^H(\,\Delta^3\ \Delta^3\ \infty\,) = \left(\frac{\widetilde{\beta}}{\widehat{\psi}\widehat{\chi}}\right)^{\frac{1}{\widehat{\psi}-1}} \Bigg[& \Big[A\left(\,\Delta^2\ \Delta^2\ \infty\,\right)\ \lambda - A\left(\,\Delta^1\ \Delta^1\ \infty\,\right)\Big](1-\overline{x}) \\ & + \Big[A\left(\,\Delta^2\ \Delta^2\ \infty\,\right)\ \lambda\ -\ \frac{1}{2}A\left(\,\Delta^1\ \Delta^1\ \infty\,\right)\Big]\overline{x}^H \\ & + \Bigg[\Big(A\left(\,\Delta^2\ \Delta^1\ \lambda\,\right)\ \mathcal{I}_2(\lambda) + A\left(\,\Delta^2\ \Delta^2\ \infty\,\right)\ \mathcal{I}_3(\lambda)\Big)\lambda \\ & - \Big(A\left(\,\Delta^1\ \Delta^1\ 1\,\right)\ \mathcal{I}_2(1) + A\left(\,\Delta^1\ \Delta^1\ \infty\,\right)\ \mathcal{I}_3(1)\Big)\Bigg]\overline{x}^F \Bigg]^{\frac{1}{\widehat{\psi}-1}} \end{aligned} \tag{B.71}$$

$$
\begin{aligned}
z^{H}(\,\Delta^{4}\;\;\Delta^{4}\;\;\infty\,) = \left(\frac{\widetilde{\beta}}{\widehat{\psi}\widehat{\chi}}\right)^{\frac{1}{\widehat{\psi}-1}} \Bigg[& \Big[A\left(\,\Delta^{2}\;\;\Delta^{2}\;\;\infty\,\right)\,\lambda - A\left(\,\Delta^{1}\;\;\Delta^{1}\;\;\infty\,\right)\Big](1-\overline{x}) \\
& + A\left(\,\Delta^{2}\;\;\Delta^{2}\;\;\infty\,\right)\,\lambda\,\frac{1}{2}\,\overline{x}^{H} \\
& + \Bigg[\Big(A\left(\,\Delta^{2}\;\;\Delta^{2}\;\;1\,\right)\,\mathcal{I}_{2}\,(1) + A\left(\,\Delta^{2}\;\;\Delta^{2}\;\;\infty\,\right)\,\mathcal{I}_{3}\,(1)\Big)\,\lambda \\
& - \left(A\left(\,\Delta^{1}\;\;\Delta^{2}\;\;\frac{1}{\lambda}\,\right)\,\mathcal{I}_{2}\left(\frac{1}{\lambda}\right) + A\left(\,\Delta^{1}\;\;\Delta^{1}\;\;\infty\,\right)\,\mathcal{I}_{3}\left(\frac{1}{\lambda}\right)\right)\Bigg]\,\overline{x}^{F}\;\;\Bigg]^{\frac{1}{\widehat{\psi}-1}}
\end{aligned} \tag{B.72}
$$

Here, z^{F} corresponds to the internal innovation intensity under the same state variable we consider for z^{H} and the state-variable notations are removed for notational simplicity.

For $(H\;F)$ cases, the terms corresponding to no external innovation by either home firm or foreign firm are identical up to Δ^{G} and are defined as

$$
\begin{aligned}
A^{zH}_{NE}\left(\Delta^{G}\right) \equiv & \Big[\Big(A\left(\,\Delta^{2}\;\;\Delta^{1}\;\;\lambda\Delta^{G}\,\right)\mathcal{I}_{2}\left(\lambda\Delta^{G}\right) + A\left(\,\Delta^{2}\;\;\Delta^{2}\;\;\infty\,\right)\mathcal{I}_{3}\left(\lambda\Delta^{G}\right)\Big)\,\lambda \\
& - A\left(\,\Delta^{1}\;\;\Delta^{1}\;\;\Delta^{G}\,\right)\Big](1-z^{F})(1-\overline{x}) \\
& + \left[A\left(\,\Delta^{2}\;\;\Delta^{2}\;\;\Delta^{G}\,\right)\,\lambda - A\left(\,\Delta^{1}\;\;\Delta^{2}\;\;\frac{1}{\lambda}\Delta^{G}\,\right)\mathcal{I}_{2}\left(\frac{1}{\lambda}\Delta^{G}\right)\right] z^{F}\,(1-\overline{x})\,,
\end{aligned}
$$

where z^{F} corresponds to technology gaps corresponding to the z^{H} that $A^{zH}_{NE}\left(\Delta^{G}\right)$ is used. For any Δ^{H}, terms corresponding to foreign external innovation depend only on foreign technology gap Δ^{F} and global technology gap Δ^{G}. Thus, for $A^{zH}_{H\ell}\left(\Delta^{G}\right)$

corresponding to $(\Delta^H\ \Delta^\ell\ \ \Delta^G)$,

$$A^{zH}_{H1}\left(\Delta^G\right) \equiv \left[\ A\left(\ \Delta^2\ \ \Delta^3\ \ \frac{\lambda}{\eta}\Delta^G\ \right)\ \mathcal{I}_2\left(\frac{\lambda}{\eta}\Delta^G\right)\ \lambda - A\left(\ \Delta^1\ \ \Delta^3\ \ \frac{1}{\eta}\Delta^G\ \right)\ \mathcal{I}_2\left(\frac{1}{\eta}\Delta^G\right)\ \right]\overline{x}^F$$

$$\begin{aligned}A^{zH}_{H2}\left(\Delta^G\right) \equiv &\left[\left(A\left(\ \Delta^2\ \ \Delta^4\ \ \frac{\lambda^2}{\eta}\Delta^G\ \right)\mathcal{I}_2\left(\frac{\lambda^2}{\eta}\Delta^G\right) + A\left(\ \Delta^2\ \ \Delta^2\ \ \infty\ \right)\mathcal{I}_3\left(\frac{\lambda^2}{\eta}\Delta^G\right)\ \right)\ \lambda\right.\\ &\left. - A\left(\ \Delta^1\ \ \Delta^4\ \ \frac{\lambda}{\eta}\Delta^G\ \right)\mathcal{I}_2\left(\frac{\lambda}{\eta}\Delta^G\right)\ \right](1 - z^F)\ \overline{x}^F\\ &+\ \left[A\left(\ \Delta^2\ \ \Delta^2\ \ \Delta^G\ \right)\ \lambda - A\left(\ \Delta^1\ \ \Delta^2\ \ \frac{1}{\lambda}\Delta^G\ \right)\mathcal{I}_2\left(\frac{1}{\lambda}\Delta^G\right)\ \right]\ z^F\ \overline{x}^F\end{aligned}$$

$$\begin{aligned}A^{zH}_{H3}\left(\Delta^G\right) \equiv &\left[\left(A\left(\ \Delta^2\ \ \Delta^1\ \ \lambda\Delta^G\ \right)\mathcal{I}_2\left(\lambda\Delta^G\right) + A\left(\ \Delta^2\ \ \Delta^2\ \ \infty\ \right)\mathcal{I}_3\left(\lambda\Delta^G\right)\ \right)\ \lambda\right.\\ &\left. - A\left(\ \Delta^1\ \ \Delta^1\ \ \Delta^G\ \right)\ \right](1 - z^F)\ \overline{x}^F\\ &+\ \left[A\left(\ \Delta^2\ \ \Delta^2\ \ \Delta^G\ \right)\ \lambda - A\left(\ \Delta^1\ \ \Delta^2\ \ \frac{1}{\lambda}\Delta^G\ \right)\mathcal{I}_2\left(\frac{1}{\lambda}\Delta^G\right)\ \right]\ z^F\ \overline{x}^F\end{aligned}$$

$$A^{zH}_{H4}\left(\Delta^G\right) \equiv \left[A\left(\ \Delta^2\ \ \Delta^2\ \ \Delta^G\ \right)\ \lambda\ -\ A\left(\ \Delta^1\ \ \Delta^2\ \ \frac{1}{\lambda}\Delta^G\right)\mathcal{I}_2\left(\frac{1}{\lambda}\Delta^G\right)\ \right]\overline{x}^F$$

Similarly for any Δ^F, terms corresponding to domestic external innovation depend only on domestic technology gap Δ^H and global technology gap Δ^G. Thus, for $A^{zH}_{\ell F}\left(\Delta^G\right)$ corresponding to $(\Delta^\ell\ \Delta^F\ \ \Delta^G)$,

$$\begin{aligned}A^{zH}_{2F}\left(\Delta^G\right) \equiv &\left(A\left(\ \Delta^2\ \ \Delta^1\ \ \lambda\Delta^G\ \right)\mathcal{I}_2\left(\lambda\Delta^G\right) + A\left(\ \Delta^2\ \ \Delta^2\ \ \infty\ \right)\mathcal{I}_3\left(\lambda\Delta^G\right)\right)\ \lambda\ (1 - z^F)\ \overline{x}^H\\ &+\ A\left(\ \Delta^2\ \ \Delta^2\ \ \Delta^G\ \right)\ \lambda\ z^F\ \overline{x}^H\end{aligned}$$

$$\begin{aligned}A^{zH}_{3F}\left(\Delta^G\right) \equiv &\left[\left(A\left(\ \Delta^2\ \ \Delta^1\ \ \lambda\Delta^G\ \right)\mathcal{I}_2\left(\lambda\Delta^G\right) + A\left(\ \Delta^2\ \ \Delta^2\ \ \infty\ \right)\mathcal{I}_3\left(\lambda\Delta^G\right)\ \right)\ \lambda\right.\\ &\left. - \frac{1}{2}A\left(\ \Delta^1\ \ \Delta^1\ \ \Delta^G\ \right)\ \right](1 - z^F)\ \overline{x}^H\end{aligned}$$

$$+\left[A\left(\Delta^2\ \Delta^2\ \Delta^G\right)\lambda-\frac{1}{2}A\left(\Delta^1\ \Delta^2\ \frac{1}{\lambda}\Delta^G\right)\mathcal{I}_2\left(\frac{1}{\lambda}\Delta^G\right)\right]z^F\,\overline{x}^H$$

$$A^{zH}_{4F}\left(\Delta^G\right)\equiv\Big(A\left(\Delta^2\ \Delta^1\ \lambda\Delta^G\right)\mathcal{I}_2\left(\lambda\Delta^G\right)+A\left(\Delta^2\ \Delta^2\ \infty\right)\mathcal{I}_3\left(\lambda\Delta^G\right)\Big)\,\lambda\,\frac{1}{2}\left(1-z^F\right)\overline{x}^H$$
$$+\ A\left(\Delta^2\ \Delta^2\ \Delta^G\right)\lambda\,\frac{1}{2}\,z^F\,\overline{x}^H$$

For any $\ell^F\in\{1\,,\,2\,,\,3\,,\,4\}$, the following holds:

$$z^H(\,\Delta^1\ \Delta^{\ell^F}\ \Delta^G\,)=\left(\frac{\widetilde{\beta}}{\widehat{\psi}\widehat{\chi}}\right)^{\frac{1}{\widehat{\psi}-1}}\left[\ A^{zH}_{NE}\left(\Delta^G\right)+\ A^{zH}_{H\ell^F}\left(\Delta^G\right)\ \right]^{\frac{1}{\widehat{\psi}-1}}, \tag{B.73}$$

and for any $\ell^H>1$ and ℓ^F, the following holds:

$$z^H(\,\Delta^{\ell^H}\ \Delta^{\ell^F}\ \Delta^G\,)=\left(\frac{\widetilde{\beta}}{\widehat{\psi}\widehat{\chi}}\right)^{\frac{1}{\widehat{\psi}-1}}\left[\ A^{zH}_{NE}\left(\Delta^G\right)+\ A^{zH}_{\ell^H F}\left(\Delta^G\right)+A^{zH}_{H\ell^F}\left(\Delta^G\right)\ \right]^{\frac{1}{\widehat{\psi}-1}}. \tag{B.74}$$

B.3.4.2.2 Value from Existing Product, Home Firm

H superscripts from all the $A(\cdot)$ and $z(\cdot)$ are dropped for notational simplicity.

$$A\left(\Delta^1\ \Delta^1\ \infty\right)=\pi^{HH}+\pi^{HF}-\widehat{\chi}\,z\left(\Delta^1\ \Delta^1\ \infty\right)^{\widehat{\psi}}$$

$$
+\,\widetilde{\beta}\times\left[\begin{array}{ll}
\quad A\left(\,\Delta^1\,\Delta^1\,\infty\,\right) & \times\ \left(1-z\left(\,\Delta^1\ \ \Delta^1\ \ \infty\,\right)\right)(1-\overline{x}) \\
+\,A\left(\,\Delta^2\,\Delta^2\,\infty\,\right)\lambda & \times\ z\left(\,\Delta^1\ \ \Delta^1\ \ \infty\,\right)\ (1-\overline{x}) \\
+\left[\begin{array}{ll}
\quad A\left(\,\Delta^1\,\Delta^3\ \ \frac{1}{\eta}\right) & \mathcal{I}_2\left(\frac{1}{\eta}\right) \\
+\,A\left(\,\Delta^1\,\Delta^1\ \ \infty\,\right) & \mathcal{I}_3\left(\frac{1}{\eta}\right)
\end{array}\right] & \times\ \left(1-z\left(\,\Delta^1\ \ \Delta^1\ \ \infty\,\right)\right)\overline{x}^F \\
+\left[\begin{array}{ll}
\quad A\left(\,\Delta^2\,\Delta^3\ \ \frac{\lambda}{\eta}\right) & \mathcal{I}_2\left(\frac{\lambda}{\eta}\right) \\
+\,A\left(\,\Delta^2\,\Delta^2\ \ \infty\,\right) & \mathcal{I}_3\left(\frac{\lambda}{\eta}\right)
\end{array}\right]\lambda & \times\ z\left(\,\Delta^1\ \ \Delta^1\ \ \infty\,\right)\ \overline{x}^F
\end{array}\right]
$$

$$
A\left(\,\Delta^2\ \ \Delta^2\ \ \infty\,\right)=\pi^{HH}+\pi^{HF}-\widehat{\chi}\,z\left(\,\Delta^2\ \ \Delta^2\ \ \infty\,\right)^{\widehat{\psi}}
$$

$$
+\,\widetilde{\beta}\times\left[\begin{array}{ll}
\quad A\left(\,\Delta^1\,\Delta^1\,\infty\,\right) & \times\ \left(1-z\left(\,\Delta^2\ \ \Delta^2\ \ \infty\,\right)\right)(1-\overline{x}) \\
+\,A\left(\,\Delta^2\,\Delta^2\,\infty\,\right)\lambda & \times\ z\left(\,\Delta^2\ \ \Delta^2\ \ \infty\,\right)\ (1-\overline{x}) \\
+\,A\left(\,\Delta^2\,\Delta^2\,\infty\,\right)\lambda & \times\ z\left(\,\Delta^2\ \ \Delta^2\ \ \infty\,\right)\ \overline{x}^H \\
+\left[\begin{array}{ll}
\quad A\left(\,\Delta^1\,\Delta^4\ \ \frac{\lambda}{\eta}\right) & \mathcal{I}_2\left(\frac{\lambda}{\eta}\right) \\
+\,A\left(\,\Delta^1\,\Delta^1\ \ \infty\,\right) & \mathcal{I}_3\left(\frac{\lambda}{\eta}\right)
\end{array}\right] & \times\ \left(1-z\left(\,\Delta^2\ \ \Delta^2\ \ \infty\,\right)\right)\overline{x}^F \\
+\left[\begin{array}{ll}
\quad A\left(\,\Delta^2\,\Delta^4\ \ \frac{\lambda^2}{\eta}\right) & \mathcal{I}_2\left(\frac{\lambda^2}{\eta}\right) \\
+\,A\left(\,\Delta^2\,\Delta^2\ \ \infty\,\right) & \mathcal{I}_3\left(\frac{\lambda^2}{\eta}\right)
\end{array}\right]\lambda & \times\ z\left(\,\Delta^2\ \ \Delta^2\ \ \infty\,\right)\ \overline{x}^F
\end{array}\right]
$$

$$
A\left(\,\Delta^3\ \ \Delta^3\ \ \infty\,\right)=\pi^{HH}+\pi^{HF}-\widehat{\chi}\,z\left(\,\Delta^3\ \ \Delta^3\ \ \infty\,\right)^{\widehat{\psi}}
$$

$$
+\widetilde{\beta}\times\left[\begin{array}{ll}
A\left(\Delta^1\,\Delta^1\,\infty\right) & \times\ \big(1-z\left(\Delta^3\ \Delta^3\ \infty\right)\big)(1-\overline{x}) \\
+\,A\left(\Delta^2\,\Delta^2\,\infty\right)\lambda & \times\ z\left(\Delta^3\ \Delta^3\ \infty\right)\ (1-\overline{x}) \\
+\,A\left(\Delta^1\,\Delta^1\,\infty\right) & \times\ \tfrac{1}{2}\big(1-z\left(\Delta^3\ \Delta^3\ \infty\right)\big)\,\overline{x}^H \\
+\,A\left(\Delta^2\,\Delta^2\,\infty\right)\lambda & \times\ z\left(\Delta^3\ \Delta^3\ \infty\right)\ \overline{x}^H \\
+\left[\begin{array}{ll} A\left(\Delta^1\,\Delta^1\ 1\right) & \mathcal{I}_2\,(1) \\ +\,A\left(\Delta^1\,\Delta^1\ \infty\right) & \mathcal{I}_3\,(1)\end{array}\right] & \times\ \big(1-z\left(\Delta^3\ \Delta^3\ \infty\right)\big)\,\overline{x}^F \\
+\left[\begin{array}{ll} A\left(\Delta^2\,\Delta^1\ \lambda\right) & \mathcal{I}_2\,(\lambda) \\ +\,A\left(\Delta^2\,\Delta^2\ \infty\right) & \mathcal{I}_3\,(\lambda)\end{array}\right]\lambda & \times\ z\left(\Delta^3\ \Delta^3\ \infty\right)\ \overline{x}^F
\end{array}\right]
$$

$$
A\left(\Delta^4\ \Delta^4\ \infty\right)=\pi^{HH}+\pi^{HF}-\widehat{\chi}\,z\left(\Delta^4\ \Delta^4\ \infty\right)^{\widehat{\psi}}
$$

$$
+\widetilde{\beta}\times\left[\begin{array}{ll}
A\left(\Delta^1\,\Delta^1\,\infty\right) & \times\ \big(1-z\left(\Delta^4\ \Delta^4\ \infty\right)\big)(1-\overline{x}) \\
+\,A\left(\Delta^2\,\Delta^2\,\infty\right)\lambda & \times\ z\left(\Delta^4\ \Delta^4\ \infty\right)\ (1-\overline{x}) \\
+\,A\left(\Delta^2\,\Delta^2\,\infty\right)\lambda & \times\ \tfrac{1}{2}z\left(\Delta^4\ \Delta^4\ \infty\right)\ \overline{x}^H \\
+\left[\begin{array}{ll} A\left(\Delta^1\,\Delta^2\ \tfrac{1}{\lambda}\right) & \mathcal{I}_2\left(\tfrac{1}{\lambda}\right) \\ +\,A\left(\Delta^1\,\Delta^1\ \infty\right) & \mathcal{I}_3\left(\tfrac{1}{\lambda}\right)\end{array}\right] & \times\ \big(1-z\left(\Delta^4\ \Delta^4\ \infty\right)\big)\,\overline{x}^F \\
+\left[\begin{array}{ll} A\left(\Delta^2\,\Delta^2\ 1\right) & \mathcal{I}_2\,(1) \\ +\,A\left(\Delta^2\,\Delta^2\ \infty\right) & \mathcal{I}_3\,(1)\end{array}\right]\lambda & \times\ z\left(\Delta^4\ \Delta^4\ \infty\right)\ \overline{x}^F
\end{array}\right]
$$

Both z^H and z^F are the ones corresponding to the state-variable of interest. The state-variable is removed for the notational simplicity.

$$
A^{AH}_{NE} \equiv \left[\begin{array}{ll}
A\left(\,\Delta^1\,\Delta^1\,\Delta^G\,\right) & \times\ (1-z^H)(1-z^F)(1-\overline{x}) \\
+\left[\begin{array}{ll} A\left(\,\Delta^2\,\Delta^1\ \lambda\Delta^G\,\right) & \mathcal{I}_2\left(\lambda\Delta^G\right) \\ +\,A\left(\,\Delta^2\,\Delta^2\ \infty\,\right) & \mathcal{I}_3\left(\lambda\Delta^G\right) \end{array}\right]\lambda & \times\ z^H\,(1-z^F)(1-\overline{x}) \\
+\,A\left(\,\Delta^1\,\Delta^2\,\tfrac{1}{\lambda}\Delta^G\,\right)\mathcal{I}_2\left(\tfrac{1}{\lambda}\Delta^G\right) & \times\ (1-z^H)\,z^F\,(1-\overline{x}) \\
+\,A\left(\,\Delta^2\,\Delta^2\,\Delta^G\,\right)\lambda & \times\ z^H\,z^F\,(1-\overline{x})
\end{array}\right]
$$

$$
\begin{aligned}
A^{AH}_{H1} \equiv & A\left(\,\Delta^1\,\Delta^3\ \frac{1}{\eta}\Delta^G\right)\mathcal{I}_2\left(\frac{1}{\eta}\Delta^G\right)\times\ \ (1-z^H)\,\overline{x}^F \\
& +\ A\left(\,\Delta^2\,\Delta^3\ \frac{\lambda}{\eta}\Delta^G\right)\mathcal{I}_2\left(\frac{\lambda}{\eta}\Delta^G\right)\lambda\times\ \ z^H\,\overline{x}^F
\end{aligned}
$$

$$
A^{AH}_{H2} \equiv \left[\begin{array}{ll}
A\left(\,\Delta^1\,\Delta^4\ \tfrac{\lambda}{\eta}\Delta^G\right)\mathcal{I}_2\left(\tfrac{\lambda}{\eta}\Delta^G\right) & \times\ (1-z^H)(1-z^F)\overline{x}^F \\
+\left[\begin{array}{ll} A\left(\,\Delta^2\,\Delta^4\ \tfrac{\lambda^2}{\eta}\Delta^G\,\right) & \mathcal{I}_2\left(\tfrac{\lambda^2}{\eta}\Delta^G\right) \\ +\,A\left(\,\Delta^2\,\Delta^2\ \infty\,\right) & \mathcal{I}_3\left(\tfrac{\lambda^2}{\eta}\Delta^G\right) \end{array}\right]\lambda & \times\ z^H\,(1-z^F)\overline{x}^F \\
+\,A\left(\,\Delta^1\,\Delta^2\,\tfrac{1}{\lambda}\Delta^G\,\right)\mathcal{I}_2\left(\tfrac{1}{\lambda}\Delta^G\right) & \times\ (1-z^H)\,z^F\,\overline{x}^F \\
+\,A\left(\,\Delta^2\,\Delta^2\,\Delta^G\,\right)\lambda & \times\ z^H\,z^F\,\overline{x}^F
\end{array}\right]
$$

$$
A^{AH}_{H3} \equiv \left[\begin{array}{ll}
A\left(\,\Delta^1\,\Delta^1\ \Delta^G\,\right) & \times\ (1-z^H)(1-z^F)\overline{x}^F \\
+\left[\begin{array}{ll} A\left(\,\Delta^2\,\Delta^1\ \lambda\Delta^G\,\right) & \mathcal{I}_2\left(\lambda\Delta^G\right) \\ +\,A\left(\,\Delta^2\,\Delta^2\ \infty\,\right) & \mathcal{I}_3\left(\lambda\Delta^G\right) \end{array}\right]\lambda & \times\ z^H\,(1-z^F)\overline{x}^F \\
+\,A\left(\,\Delta^1\,\Delta^2\,\tfrac{1}{\lambda}\Delta^G\,\right)\mathcal{I}_2\left(\tfrac{1}{\lambda}\Delta^G\right) & \times\ (1-z^H)\,z^F\,\overline{x}^F \\
+\,A\left(\,\Delta^2\,\Delta^2\,\Delta^G\,\right)\lambda & \times\ z^H\,z^F\,\overline{x}^F
\end{array}\right]
$$

$$A_{H4}^{AH} \equiv A\left(\Delta^1\,\Delta^2\ \frac{1}{\lambda}\Delta^G \right) \mathcal{I}_2\left(\frac{1}{\lambda}\Delta^G\right) \times\ \ (1-z^H)\,\overline{x}^F\ +\ A\left(\,\Delta^2\,\Delta^2\ \ \Delta^G\,\right)\lambda \times\ \ z^H\,\overline{x}^F$$

$$A_{2F}^{AH} \equiv \begin{bmatrix} \begin{bmatrix} A\left(\,\Delta^2\,\Delta^1\ \ \lambda\Delta^G\,\right) & \mathcal{I}_2\left(\lambda\Delta^G\right) \\ +\,A\left(\,\Delta^2\,\Delta^2\ \ \infty\,\right) & \mathcal{I}_3\left(\lambda\Delta^G\right) \end{bmatrix}\lambda & \times\ z^H\,(1-z^F)\overline{x}^H \\ +\,A\left(\,\Delta^2\,\Delta^2\,\Delta^G\,\right)\lambda & \times\ z^H\,z^F\,\overline{x}^H \end{bmatrix}$$

$$A_{3F}^{AH} \equiv \begin{bmatrix} A\left(\,\Delta^1\,\Delta^1\ \ \Delta^G\,\right) & \times\ \tfrac{1}{2}\,(1-z^H)(1-z^F)\overline{x}^H \\ +\begin{bmatrix} A\left(\,\Delta^2\,\Delta^1\ \ \lambda\Delta^G\,\right) & \mathcal{I}_2\left(\lambda\Delta^G\right) \\ +\,A\left(\,\Delta^2\,\Delta^2\ \ \infty\,\right) & \mathcal{I}_3\left(\lambda\Delta^G\right) \end{bmatrix}\lambda & \times\ z^H\,(1-z^F)\overline{x}^H \\ +\,A\left(\,\Delta^1\,\Delta^2\,\tfrac{1}{\lambda}\Delta^G\,\right)\mathcal{I}_2\left(\tfrac{1}{\lambda}\Delta^G\right) & \times\ \tfrac{1}{2}\,(1-z^H)\,z^F\,\overline{x}^H \\ +\,A\left(\,\Delta^2\,\Delta^2\,\Delta^G\,\right)\lambda & \times\ z^H\,z^F\,\overline{x}^H \end{bmatrix}$$

$$A_{4F}^{AH} \equiv \begin{bmatrix} \begin{bmatrix} A\left(\,\Delta^2\,\Delta^1\ \ \lambda\Delta^G\,\right) & \mathcal{I}_2\left(\lambda\Delta^G\right) \\ +\,A\left(\,\Delta^2\,\Delta^2\ \ \infty\,\right) & \mathcal{I}_3\left(\lambda\Delta^G\right) \end{bmatrix}\lambda & \times\ \tfrac{1}{2}\,z^H\,(1-z^F)\overline{x}^H \\ +\,A\left(\,\Delta^2\,\Delta^2\ \ \Delta^G\,\right)\lambda & \times\ \tfrac{1}{2}\,z^H\,z^F\,\overline{x}^H \end{bmatrix}$$

For any $\ell^F \in \{1\,,\,2\,,\,3\,,\,4\}$, the following holds:

$$A\left(\,\Delta^1\ \ \Delta^{\ell^F}\ \ \Delta^G\,\right) = \pi^{HH} - \widehat{\chi}\ \left(z^H\right)^{\widehat{\psi}} + \widetilde{\beta} \times \left[\ A_{NE}^{AH}\ +\ A_{H\ell^F}^{AH}\ \right],$$

and for any $\ell^H > 1$ and ℓ^F, the following holds:

$$A\left(\Delta^{\ell^H}\ \Delta^{\ell^F}\ \Delta^G\right) = \pi^{HH} - \widehat{\chi}\ \left(z^H\right)^{\widehat{\psi}} + \widetilde{\beta} \times \left[\ A_{NE}^{AH} + A_{\ell^H F}^{AH} + A_{H\ell^F}^{AH}\ \right].$$

B.3.4.2.3 Value from a New Product Line $A^H_{takeover}$ for Home Firm

$$\begin{aligned}A^H_{takeover} \equiv \Bigg[& \sum_{\mathcal{I}^{ZH}_{-j}=0}^{1} \sum_{\mathcal{I}^{ZF}_{-j}=0}^{1} \sum_{c-t_{-j}=\text{win}}^{\text{lose}} \left(z^H\right)^{\mathcal{I}^{ZH}_{-j}} \left(1-z^H\right)^{1-\mathcal{I}^{ZH}_{-j}} \left(z^F\right)^{\mathcal{I}^{ZF}_{-j}} \left(1-z^F\right)^{1-\mathcal{I}^{ZF}_{-j}} \frac{1}{2} \\ & \times \int_{\left(\Delta^H_{-j}\ \Delta^F_{-j}\ \Delta^G_{-j}\right)} A^H\left(\Delta^{H\prime}_{-j}\ \Delta^{F\prime}_{-j}\ \Delta^{G\prime}_{-j} \mid \Delta^H_{-j}, \Delta^F_{-j}, \Delta^G_{-j}, \mathcal{I}^{ZH}_{-j}, \mathcal{I}^{ZF}_{-j}, c-t_{-j}\right) \Delta^{H\prime}_{-j} \\ & \qquad \times \mu\left(\Delta^H_{-j}\ \Delta^F_{-j}\ \Delta^G_{-j}\right) \mathbf{d}\left(\Delta^H_{-j}\ \Delta^F_{-j}\ \Delta^G_{-j}\right)\Bigg],\end{aligned}$$

where the RHS is equal to

$$\begin{aligned}& A\left(\Delta^3\,\Delta^3\ \infty\right) \eta\,\mu\left(\Delta^1\,\Delta^1\ \infty\right) + A\left(\Delta^4\,\Delta^4\ \infty\right) \frac{\eta}{\lambda} \left(1 - z^H\left(\Delta^2\,\Delta^2\ \infty\right)\right) \mu\left(\Delta^2\,\Delta^2\ \infty\right) \\ & + A\left(\Delta^1\,\Delta^1\ \infty\right) \frac{1}{2} \left(1 - z^H\left(\Delta^3\,\Delta^3\ \infty\right)\right) \mu\left(\Delta^3\,\Delta^3\ \infty\right) \\ & + A\left(\Delta^2\,\Delta^2\ \infty\right) \lambda \left(1 - \frac{1}{2} z^H\left(\Delta^4\,\Delta^4\ \infty\right)\right) \mu\left(\Delta^4\,\Delta^4\ \infty\right) \\ & + \begin{bmatrix} \left[A\left(\Delta^3\,\Delta^1\ \eta\right)\mathcal{I}_2(\eta) + A\left(\Delta^3\,\Delta^3\ \infty\right)\mathcal{I}_3(\eta)\right] & \left(1 - z^F\left(\Delta^1\,\Delta^1\ -\infty\right)\right) \\ + \left[A\left(\Delta^3\,\Delta^2\ \frac{\eta}{\lambda}\right)\mathcal{I}_2\left(\frac{\eta}{\lambda}\right) + A\left(\Delta^3\,\Delta^3\ \infty\right)\mathcal{I}_3\left(\frac{\eta}{\lambda}\right)\right] & z^F\left(\Delta^1\,\Delta^1\ -\infty\right) \end{bmatrix} \eta\,\mu\left(\Delta^1\,\Delta^1\ -\infty\right) \\ & + \begin{bmatrix} \left[A\left(\Delta^4\,\Delta^1\ \frac{\eta}{\lambda}\right)\mathcal{I}_2\left(\frac{\eta}{\lambda}\right) + A\left(\Delta^4\,\Delta^4\ \infty\right)\mathcal{I}_3\left(\frac{\eta}{\lambda}\right)\right] & \left(1 - z^F\left(\Delta^2\,\Delta^2\ -\infty\right)\right) \\ + \left[A\left(\Delta^4\,\Delta^2\ \frac{\eta}{\lambda^2}\right)\mathcal{I}_2\left(\frac{\eta}{\lambda^2}\right) + A\left(\Delta^4\,\Delta^4\ \infty\right)\mathcal{I}_3\left(\frac{\eta}{\lambda^2}\right)\right] & z^F\left(\Delta^2\,\Delta^2\ -\infty\right) \end{bmatrix} \frac{\eta}{\lambda}\,\mu\left(\Delta^2\,\Delta^2\ -\infty\right) \\ & + \begin{bmatrix} \left[A\left(\Delta^1\,\Delta^1\ 1\right)\mathcal{I}_2(1) + A\left(\Delta^1\,\Delta^1\ \infty\right)\mathcal{I}_3(1)\right] & \left(1 - z^F\left(\Delta^3\,\Delta^3\ -\infty\right)\right) \\ + \left[A\left(\Delta^1\,\Delta^2\ \frac{1}{\lambda}\right)\mathcal{I}_2\left(\frac{1}{\lambda}\right) + A\left(\Delta^1\,\Delta^1\ \infty\right)\mathcal{I}_3\left(\frac{1}{\lambda}\right)\right] & z^F\left(\Delta^3\,\Delta^3\ -\infty\right) \end{bmatrix} \mu\left(\Delta^3\,\Delta^3\ -\infty\right) \\ & + \begin{bmatrix} \left[A\left(\Delta^2\,\Delta^1\ \lambda\right)\mathcal{I}_2(\lambda) + A\left(\Delta^2\,\Delta^2\ \infty\right)\mathcal{I}_3(\lambda)\right] & \left(1 - z^F\left(\Delta^4\,\Delta^4\ -\infty\right)\right) \\ + \left[A\left(\Delta^2\,\Delta^2\ 1\right)\mathcal{I}_2(1) + A\left(\Delta^2\,\Delta^2\ \infty\right)\mathcal{I}_3(1)\right] & z^F\left(\Delta^4\,\Delta^4\ -\infty\right) \end{bmatrix} \lambda\,\mu\left(\Delta^4\,\Delta^4\ -\infty\right)\end{aligned}$$

$$
+\sum_{\Delta^G}\sum_{\ell^F=1}^{4}\left[\begin{array}{l}
\left[\begin{array}{l}
\left[A\left(\,\Delta^3\,\Delta^1\;\;\eta\Delta^G\,\right)\mathcal{I}_2\left(\eta\Delta^G\right)+A\left(\,\Delta^3\,\Delta^3\;\;\infty\,\right)\mathcal{I}_3\left(\eta\Delta^G\right)\right]\\
\times\left(1-z^F\left(\Delta^1\,\Delta^{\ell^F}\;\;\Delta^G\right)\right)\\
+\left[A\left(\,\Delta^3\,\Delta^2\;\;\frac{\eta}{\lambda}\Delta^G\,\right)\mathcal{I}_2\left(\frac{\eta}{\lambda}\Delta^G\right)+A\left(\,\Delta^3\,\Delta^3\;\;\infty\,\right)\mathcal{I}_3\left(\frac{\eta}{\lambda}\Delta^G\right)\right]\\
\times z^F\left(\Delta^1\,\Delta^{\ell^F}\;\;\Delta^G\right)
\end{array}\right]\eta\mu\left(\Delta^1\,\ell^F\;\;\Delta^G\right)\\
+\left[\begin{array}{l}
\left[A\left(\,\Delta^4\,\Delta^1\;\;\frac{\eta}{\lambda}\Delta^G\,\right)\mathcal{I}_2\left(\frac{\eta}{\lambda}\Delta^G\right)+A\left(\,\Delta^4\,\Delta^4\;\;\infty\,\right)\mathcal{I}_3\left(\frac{\eta}{\lambda}\Delta^G\right)\right]\\
\times\left(1-z^H\left(\Delta^2\,\Delta^{\ell^F}\;\;\Delta^G\right)\right)\left(1-z^F\left(\Delta^2\,\Delta^{\ell^F}\;\;\Delta^G\right)\right)\\
+A\left(\,\Delta^4\,\Delta^2\;\;\frac{\eta}{\lambda^2}\Delta^G\,\right)\mathcal{I}_2\left(\frac{\eta}{\lambda^2}\Delta^G\right)\\
\times\left(1-z^H\left(\Delta^2\,\Delta^{\ell^F}\;\;\Delta^G\right)\right)z^F\left(\Delta^2\,\Delta^{\ell^F}\;\;\Delta^G\right)
\end{array}\right]\frac{\eta}{\lambda}\mu\left(\Delta^2\,\ell^F\;\;\Delta^G\right)\\
+\left[\begin{array}{l}
A\left(\,\Delta^1\,\Delta^1\;\;\Delta^G\,\right)\\
\times\frac{1}{2}\left(1-z^H\left(\Delta^3\,\Delta^{\ell^F}\;\;\Delta^G\right)\right)\left(1-z^F\left(\Delta^3\,\Delta^{\ell^F}\;\;\Delta^G\right)\right)\\
+A\left(\,\Delta^1\,\Delta^2\;\;\frac{1}{\lambda}\Delta^G\,\right)\mathcal{I}_2\left(\frac{1}{\lambda}\Delta^G\right)\\
\times\frac{1}{2}\left(1-z^H\left(\Delta^3\,\Delta^{\ell^F}\;\;\Delta^G\right)\right)z^F\left(\Delta^3\,\Delta^{\ell^F}\;\;\Delta^G\right)
\end{array}\right]\mu\left(\Delta^3\,\ell^F\;\;\Delta^G\right)\\
+\left[\begin{array}{l}
\left[A\left(\,\Delta^2\,\Delta^1\;\;\lambda\Delta^G\,\right)\mathcal{I}_2\left(\lambda\Delta^G\right)+A\left(\,\Delta^2\,\Delta^2\;\;\infty\,\right)\mathcal{I}_3\left(\lambda\Delta^G\right)\right]\\
\times\left(1-\frac{1}{2}z^H\left(\Delta^4\,\Delta^{\ell^F}\;\;\Delta^G\right)\right)\left(1-z^F\left(\Delta^4\,\Delta^{\ell^F}\;\;\Delta^G\right)\right)\\
+A\left(\,\Delta^2\,\Delta^2\;\;\Delta^G\,\right)\\
\times\left(1-\frac{1}{2}z^H\left(\Delta^4\,\Delta^{\ell^F}\;\;\Delta^G\right)\right)z^F\left(\Delta^4\,\Delta^{\ell^F}\;\;\Delta^G\right)
\end{array}\right]\lambda\mu\left(\Delta^4\,\ell^F\;\;\Delta^G\right)
\end{array}\right]
$$

B.3.4.2.4 Optimal Internal Innovation Intensity, Foreign Firm

F superscripts from all the $A(\cdot)$ and $z(\cdot)$ are dropped for notational simplicity.

$$
\begin{aligned}
z^F(\,\Delta^1\;\;\Delta^1\;\;-\infty\,)=\left(\frac{\widetilde{\beta}^F}{\widehat{\psi}\widehat{\chi}}\right)^{\frac{1}{\widehat{\psi}-1}}\Bigg[\;&\Big[A\left(\,\Delta^2\;\;\Delta^2\;\;-\infty\,\right)\;\lambda-A\left(\,\Delta^1\;\;\Delta^1\;\;-\infty\,\right)\Big](1-\overline{x})\\
&+\Bigg[\left(A\left(\,\Delta^2\;\;\Delta^2\;\;-\infty\,\right)\;\mathcal{I}_1\left(\frac{\eta}{\lambda}\right)+A\left(\,\Delta^3\;\;\Delta^2\;\;\frac{\eta}{\lambda}\,\right)\;\mathcal{I}_2\left(\frac{\eta}{\lambda}\right)\right)\lambda
\end{aligned}
$$

$$- \left(A\left(\Delta^1 \;\; \Delta^1 \;\; -\infty \right) \mathcal{I}_1 \left(\eta \right) + A\left(\Delta^3 \;\; \Delta^1 \;\; \eta \right) \mathcal{I}_2 \left(\eta \right) \right) \Bigg] \, \overline{x}^H \;\; \Bigg]^{\frac{1}{\widehat{\psi}-1}} \tag{B.75}$$

$$\begin{aligned} z^F\left(\Delta^2 \;\; \Delta^2 \;\; -\infty \right) = \left(\frac{\widetilde{\beta}^F}{\widehat{\psi}\widehat{\chi}} \right)^{\frac{1}{\widehat{\psi}-1}} \Bigg[& \Big[A\left(\Delta^2 \;\; \Delta^2 \;\; -\infty \right) \lambda - A\left(\Delta^1 \;\; \Delta^1 \;\; -\infty \right) \Big] (1-\overline{x}) \\ & + \Bigg[\left(A\left(\Delta^2 \;\; \Delta^2 \;\; -\infty \right) \mathcal{I}_1 \left(\frac{\eta}{\lambda^2} \right) + A\left(\Delta^4 \;\; \Delta^2 \;\; \frac{\eta}{\lambda^2} \right) \mathcal{I}_2 \left(\frac{\eta}{\lambda^2} \right) \right) \lambda \\ & - \left(A\left(\Delta^1 \;\; \Delta^1 \;\; -\infty \right) \mathcal{I}_1 \left(\frac{\eta}{\lambda} \right) + A\left(\Delta^4 \;\; \Delta^1 \;\; \frac{\eta}{\lambda} \right) \mathcal{I}_2 \left(\frac{\eta}{\lambda} \right) \right) \Bigg] \overline{x}^H \\ & + \; A\left(\Delta^2 \;\; \Delta^2 \;\; -\infty \right) \lambda \, \overline{x}^F \;\; \Bigg]^{\frac{1}{\widehat{\psi}-1}} \end{aligned} \tag{B.76}$$

$$\begin{aligned} z^F\left(\Delta^3 \;\; \Delta^3 \;\; -\infty \right) = \left(\frac{\widetilde{\beta}^F}{\widehat{\psi}\widehat{\chi}} \right)^{\frac{1}{\widehat{\psi}-1}} \Bigg[& \Big[A\left(\Delta^2 \;\; \Delta^2 \;\; -\infty \right) \lambda - A\left(\Delta^1 \;\; \Delta^1 \;\; -\infty \right) \Big] (1-\overline{x}) \\ & + \Bigg[\left(A\left(\Delta^2 \;\; \Delta^2 \;\; -\infty \right) \mathcal{I}_1 \left(\frac{1}{\lambda} \right) + A\left(\Delta^1 \;\; \Delta^2 \;\; \frac{1}{\lambda} \right) \mathcal{I}_2 \left(\frac{1}{\lambda} \right) \right) \lambda \\ & - \left(A\left(\Delta^1 \;\; \Delta^1 \;\; -\infty \right) \mathcal{I}_1 (1) + A\left(\Delta^1 \;\; \Delta^1 \;\; 1 \right) \mathcal{I}_2 (1) \right) \Bigg] \overline{x}^H \\ & + \; \Big[A\left(\Delta^2 \;\; \Delta^2 \;\; -\infty \right) \lambda \; - \; \frac{1}{2} A\left(\Delta^1 \;\; \Delta^1 \;\; -\infty \right) \Big] \, \overline{x}^F \;\; \Bigg]^{\frac{1}{\widehat{\psi}-1}} \end{aligned} \tag{B.77}$$

$$z^F\left(\Delta^4 \;\; \Delta^4 \;\; -\infty \right) = \left(\frac{\widetilde{\beta}^F}{\widehat{\psi}\widehat{\chi}} \right)^{\frac{1}{\widehat{\psi}-1}} \Bigg[\; \Big[A\left(\Delta^2 \;\; \Delta^2 \;\; -\infty \right) \lambda - A\left(\Delta^1 \;\; \Delta^1 \;\; -\infty \right) \Big] (1-\overline{x})$$

$$+\Bigg[\Big(A\left(\Delta^2\ \Delta^2\ -\infty\right)\mathcal{I}_1(1)+A\left(\Delta^2\ \Delta^2\ 1\right)\mathcal{I}_2(1)\Big)\lambda$$

$$-\Big(A\left(\Delta^1\ \Delta^1\ -\infty\right)\mathcal{I}_1(\lambda)+A\left(\Delta^2\ \Delta^1\ \lambda\right)\mathcal{I}_2(\lambda)\Big)\Bigg]\overline{x}^H$$

$$+\ A\left(\Delta^2\ \Delta^2\ -\infty\right)\lambda\frac{1}{2}\overline{x}^F\ \Bigg]^{\frac{1}{\hat{\psi}-1}} \tag{B.78}$$

Here, z^F corresponds to the internal innovation intensity under the same state variable we consider for z^H and the state-variable notations are removed for notational simplicity.

For $(H\ F)$ cases, the terms corresponding to no external innovation by either home firm or foreign firm are identical up tp Δ^G and are defined as

$$A^{zF}_{NE}\left(\Delta^G\right)\equiv\Bigg[\Bigg(A\left(\Delta^2\ \Delta^2\ -\infty\right)\mathcal{I}_1\left(\frac{1}{\lambda}\Delta^G\right)+A\left(\Delta^1\ \Delta^2\ \frac{1}{\lambda}\Delta^G\right)\mathcal{I}_2\left(\frac{1}{\lambda}\Delta^G\right)\Bigg)\lambda$$

$$-A\left(\Delta^1\ \Delta^1\ \Delta^G\right)\Bigg](1-z^H)(1-\overline{x})$$

$$+\Big[A\left(\Delta^2\ \Delta^2\ \Delta^G\right)\lambda-A\left(\Delta^2\ \Delta^1\ \lambda\Delta^G\right)\mathcal{I}_2\left(\lambda\Delta^G\right)\Big]z^H\left(1-\overline{x}\right).$$

For any Δ^F, terms corresponding to domestic external innovation depend only on domestic technology gap Δ^H and global technology gap Δ^G. Thus, for $A^{zF}_{\ell F}\left(\Delta^G\right)$ corresponding to $(\Delta^\ell\,\Delta^F\ \Delta^G)$,

$$A^{zF}_{1F}\left(\Delta^G\right)\equiv\Bigg[A\left(\Delta^3\ \Delta^2\ \frac{\eta}{\lambda}\Delta^G\right)\lambda\,\mathcal{I}_2\left(\frac{\eta}{\lambda}\Delta^G\right)-A\left(\Delta^3\ \Delta^1\ \eta\Delta^G\right)\mathcal{I}_2\left(\eta\Delta^G\right)\Bigg]\overline{x}^H$$

$$A^{zF}_{2F}\left(\Delta^G\right)\equiv\Bigg[\Bigg(A\left(\Delta^2\ \Delta^2\ -\infty\right)\mathcal{I}_1\left(\frac{\eta}{\lambda^2}\Delta^G\right)+A\left(\Delta^4\ \Delta^2\ \frac{\eta}{\lambda^2}\Delta^G\right)\mathcal{I}_2\left(\frac{\eta}{\lambda^2}\Delta^G\right)\Bigg)\lambda$$

$$- A\left(\Delta^4 \;\; \Delta^1 \;\; \frac{\eta}{\lambda}\Delta^G \right) \mathcal{I}_2\left(\frac{\eta}{\lambda}\Delta^G\right) \Bigg] (1 - z^H)\, \overline{x}^H$$

$$+ \left[A\left(\Delta^2 \;\; \Delta^2 \;\; \Delta^G \right) \lambda - A\left(\Delta^2 \;\; \Delta^1 \;\; \lambda\Delta^G \right) \mathcal{I}_2\left(\lambda\Delta^G\right) \right] z^H \, \overline{x}^H$$

$$A^{zF}_{3F}\left(\Delta^G\right) \equiv \Bigg[\Bigg(A\left(\Delta^2 \;\; \Delta^2 \;\; -\infty \right) \mathcal{I}_1\left(\frac{1}{\lambda}\Delta^G\right) + A\left(\Delta^1 \;\; \Delta^2 \;\; \frac{1}{\lambda}\Delta^G \right) \mathcal{I}_2\left(\frac{1}{\lambda}\Delta^G\right) \Bigg) \lambda$$

$$- A\left(\Delta^1 \;\; \Delta^1 \;\; \Delta^G \right) \Bigg] (1 - z^H)\, \overline{x}^H$$

$$+ \left[A\left(\Delta^2 \;\; \Delta^2 \;\; \Delta^G \right) \lambda - A\left(\Delta^2 \;\; \Delta^1 \;\; \lambda\Delta^G \right) \mathcal{I}_2\left(\lambda\Delta^G\right) \right] z^H \, \overline{x}^H$$

$$A^{zF}_{4F}\left(\Delta^G\right) \equiv \left[A\left(\Delta^2 \;\; \Delta^2 \;\; \Delta^G \right) \lambda \; - \; A\left(\Delta^2 \;\; \Delta^1 \;\; \lambda\Delta^G \right) \mathcal{I}_2\left(\lambda\Delta^G\right) \right] \overline{x}^H$$

Similarly for any Δ^H, terms corresponding to foreign external innovation depend only on foreign technology gap Δ^F and global technology gap Δ^G. Thus, for $A^{zH}_{H\ell}$ corresponding to $(\Delta^H \; \Delta^\ell \; \Delta^G)$,

$$A^{zF}_{H2}\left(\Delta^G\right) \equiv \left(A\left(\Delta^2 \;\; \Delta^2 \;\; -\infty \right) \mathcal{I}_1\left(\frac{1}{\lambda}\Delta^G\right) + A\left(\Delta^1 \;\; \Delta^2 \;\; \frac{1}{\lambda}\Delta^G \right) \mathcal{I}_2\left(\frac{1}{\lambda}\Delta^G\right) \right) \lambda\, (1 - z^H)\, \overline{x}^F$$

$$+ \; A\left(\Delta^2 \;\; \Delta^2 \;\; \Delta^G \right) \lambda \, z^H \, \overline{x}^F$$

$$A^{zF}_{H3}\left(\Delta^G\right) \equiv \Bigg[\Bigg(A\left(\Delta^2 \;\; \Delta^2 \;\; -\infty \right) \mathcal{I}_1\left(\frac{1}{\lambda}\Delta^G\right) + A\left(\Delta^1 \;\; \Delta^2 \;\; \frac{1}{\lambda}\Delta^G \right) \mathcal{I}_2\left(\frac{1}{\lambda}\Delta^G\right) \Bigg) \lambda$$

$$- \frac{1}{2} A\left(\Delta^1 \;\; \Delta^1 \;\; \Delta^G \right) \Bigg] (1 - z^H)\, \overline{x}^F$$

$$+ \left[A\left(\Delta^2 \;\; \Delta^2 \;\; \Delta^G \right) \lambda - \frac{1}{2} A\left(\Delta^2 \;\; \Delta^1 \;\; \lambda\Delta^G \right) \mathcal{I}_2\left(\lambda\Delta^G\right) \right] z^H \, \overline{x}^F$$

$$A^{zF}_{H4}\left(\Delta^G\right) \equiv \left(A\left(\Delta^2 \;\; \Delta^2 \;\; -\infty \right) \mathcal{I}_1\left(\frac{1}{\lambda}\Delta^G\right) + A\left(\Delta^1 \;\; \Delta^2 \;\; \frac{1}{\lambda}\Delta^G \right) \mathcal{I}_2\left(\frac{1}{\lambda}\Delta^G\right) \right) \lambda\, \frac{1}{2}\, (1 - z^H)\, \overline{x}^F$$

$$+ \; A\left(\Delta^2 \;\; \Delta^2 \;\; \Delta^G \right) \lambda \, \frac{1}{2}\, z^H \, \overline{x}^F$$

For any $\ell^H \in \{1\,,\,2\,,\,3\,,\,4\}$, the following holds:

$$z^F(\,\Delta^{\ell^H}\;\;\Delta^1\;\;\Delta^G\,) = \left(\frac{\widetilde{\beta}^F}{\widehat{\psi}\widehat{\chi}}\right)^{\frac{1}{\widehat{\psi}-1}} \Big[\;A^{zF}_{NE}\left(\Delta^G\right) + \;A^{zF}_{\ell^H F}\left(\Delta^G\right)\;\Big]^{\frac{1}{\widehat{\psi}-1}}, \tag{B.79}$$

and for any $\ell^F > 1$ and ℓ^H, the following holds:

$$z^F(\,\Delta^{\ell^H}\;\;\Delta^{\ell^F}\;\;\Delta^G\,) = \left(\frac{\widetilde{\beta}^F}{\widehat{\psi}\widehat{\chi}}\right)^{\frac{1}{\widehat{\psi}-1}} \Big[\;A^{zF}_{NE}\left(\Delta^G\right) + \;A^{zF}_{\ell^H F}\left(\Delta^G\right) + A^{zF}_{H\ell^F}\left(\Delta^G\right)\;\Big]^{\frac{1}{\widehat{\psi}-1}} \tag{B.80}$$

B.3.4.2.5 Value from Existing Product, Foreign Firm

All $A(\cdot)$ and $z(\cdot)$ without superscript are for foreign firms.

$$\begin{aligned}
&A\left(\,\Delta^1\;\Delta^1\;\;-\infty\,\right)\\
&= \pi^{FF} + \pi^{FH} - \widehat{\chi}\, z\left(\,\Delta^1\;\Delta^1\;\;-\infty\,\right)^{\widehat{\psi}}\\
&+ \widetilde{\beta}^F \times \left[\begin{array}{ll}
\quad A\,(\,\Delta^1\,\Delta^1\;-\infty\,) & \times\;\Big(1 - z\,(\,\Delta^1\;\;\Delta^1\;\;-\infty\,)\Big)(1-\overline{x})\\
+\,A\,(\,\Delta^2\,\Delta^2\;-\infty\,)\,\lambda & \times\;z\,(\,\Delta^1\;\;\Delta^1\;\;-\infty\,)\;(1-\overline{x})\\
+\left[\begin{array}{ll}\quad A\,(\,\Delta^1\,\Delta^1\;\;-\infty\,) & \mathcal{I}_1\,(\eta)\\ +\,A\,(\,\Delta^3\,\Delta^1\;\;\eta\,) & \mathcal{I}_2\,(\eta)\end{array}\right] & \times\;\Big(1 - z\,(\,\Delta^1\;\;\Delta^1\;\;-\infty\,)\Big)\,\overline{x}^H\\
+\left[\begin{array}{ll}\quad A\,(\,\Delta^2\,\Delta^2\;\;-\infty\,) & \mathcal{I}_1\left(\frac{\eta}{\lambda}\right)\\ +\,A\left(\,\Delta^3\,\Delta^2\;\;\frac{\eta}{\lambda}\,\right) & \mathcal{I}_2\left(\frac{\eta}{\lambda}\right)\end{array}\right]\lambda & \times\;z\,(\,\Delta^1\;\;\Delta^1\;\;-\infty\,)\;\overline{x}^H
\end{array}\right]
\end{aligned}$$

$$A\left(\,\Delta^2\;\;\Delta^2\;\;\;-\infty\,\right)$$

$$= \pi^{FF} + \pi^{FH} - \widehat{\chi}\, z\left(\,\Delta^2\;\;\Delta^2\;\;\;-\infty\,\right)^{\widehat{\psi}}$$

$$+\, \widetilde{\beta}^F \times \left[\begin{array}{ll}
A\,(\,\Delta^1\,\Delta^1\;-\infty\,) & \times\;\big(1 - z\,(\,\Delta^2\;\;\Delta^2\;\;\;-\infty\,)\big)(1-\overline{x}) \\
+\,A\,(\,\Delta^2\,\Delta^2\;-\infty\,)\,\lambda & \times\;z\,(\,\Delta^2\;\;\Delta^2\;\;\;-\infty\,)\;(1-\overline{x}) \\
+\left[\begin{array}{ll} A\,(\,\Delta^1\,\Delta^1\;\;-\infty\,) & \mathcal{I}_1\left(\frac{\eta}{\lambda}\right) \\ +\,A\left(\,\Delta^4\,\Delta^1\;\;\frac{\eta}{\lambda}\,\right) & \mathcal{I}_2\left(\frac{\eta}{\lambda}\right) \end{array}\right] & \times\;\big(1 - z\,(\,\Delta^2\;\;\Delta^2\;\;\;-\infty\,)\big)\;\overline{x}^H \\
+\left[\begin{array}{ll} A\,(\,\Delta^2\,\Delta^2\;\;-\infty\,) & \mathcal{I}_1\left(\frac{\eta}{\lambda^2}\right) \\ +\,A\left(\,\Delta^4\,\Delta^2\;\;\frac{\eta}{\lambda^2}\,\right) & \mathcal{I}_2\left(\frac{\eta}{\lambda^2}\right) \end{array}\right]\lambda & \times\;z\,(\,\Delta^2\;\;\Delta^2\;\;\;-\infty\,)\;\overline{x}^H \\
+\,A\,(\,\Delta^2\,\Delta^2\;-\infty\,)\,\lambda & \times\;z\,(\,\Delta^2\;\;\Delta^2\;\;\;-\infty\,)\;\overline{x}^F
\end{array}\right]$$

$$A\left(\,\Delta^3\;\;\Delta^3\;\;\;-\infty\,\right)$$

$$= \pi^{FF} + \pi^{FH} - \widehat{\chi}\, z\left(\,\Delta^3\;\;\Delta^3\;\;\;-\infty\,\right)^{\widehat{\psi}}$$

$$
+\,\widetilde{\beta}^F \times \left[\begin{array}{ll}
\quad A\,(\,\Delta^1\,\Delta^1\;-\infty\,) & \times\;\big(1 - z\,(\,\Delta^3\;\;\Delta^3\;\;-\infty\,)\big)(1-\overline{x}) \\
+\,A\,(\,\Delta^2\,\Delta^2\;-\infty\,)\,\lambda & \times\;z\,(\,\Delta^3\;\;\Delta^3\;\;-\infty\,)\;(1-\overline{x}) \\
+\left[\begin{array}{ll} \quad A\,(\,\Delta^1\,\Delta^1\;\;-\infty\,) & \mathcal{I}_1\,(1) \\ \\ +\,A\,(\,\Delta^1\,\Delta^1\;\;1\,) & \mathcal{I}_2\,(1)\end{array}\right] & \times\;\big(1-z\,(\,\Delta^3\;\;\Delta^3\;\;-\infty\,)\big)\,\overline{x}^H \\
\\
+\left[\begin{array}{ll} \quad A\,(\,\Delta^2\,\Delta^2\;\;-\infty\,) & \mathcal{I}_1\left(\frac{1}{\lambda}\right) \\ \\ +\,A\left(\,\Delta^1\,\Delta^2\;\;\frac{1}{\lambda}\,\right) & \mathcal{I}_2\left(\frac{1}{\lambda}\right)\end{array}\right]\lambda & \times\;z\,(\,\Delta^3\;\;\Delta^3\;\;-\infty\,)\;\overline{x}^H \\
+\,A\,(\,\Delta^1\,\Delta^1\;-\infty\,) & \times\;\frac{1}{2}\big(1-z\,(\,\Delta^3\;\;\Delta^3\;\;-\infty\,)\big)\,\overline{x}^F \\
+\,A\,(\,\Delta^2\,\Delta^2\;-\infty\,)\,\lambda & \times\;z\,(\,\Delta^3\;\;\Delta^3\;\;-\infty\,)\;\overline{x}^F
\end{array}\right]
$$

$$
A\left(\,\Delta^4\;\;\Delta^4\;\;-\infty\,\right)
$$

$$
=\pi^{FF}+\pi^{FH}-\widehat{\chi}\,z\left(\,\Delta^4\;\;\Delta^4\;\;-\infty\,\right)^{\widehat{\psi}}
$$

$$
+\,\widetilde{\beta}^F \times \left[\begin{array}{ll}
\quad A\,(\,\Delta^1\,\Delta^1\;-\infty\,) & \times\;\big(1 - z\,(\,\Delta^4\;\;\Delta^4\;\;-\infty\,)\big)(1-\overline{x}) \\
+\,A\,(\,\Delta^2\,\Delta^2\;-\infty\,)\,\lambda & \times\;z\,(\,\Delta^4\;\;\Delta^4\;\;-\infty\,)\;(1-\overline{x}) \\
+\left[\begin{array}{ll} \quad A\,(\,\Delta^1\,\Delta^1\;\;-\infty\,) & \mathcal{I}_1\,(\lambda) \\ \\ +\,A\,(\,\Delta^2\,\Delta^1\;\;\lambda\,) & \mathcal{I}_2\,(\lambda)\end{array}\right] & \times\;\big(1-z\,(\,\Delta^4\;\;\Delta^4\;\;-\infty\,)\big)\,\overline{x}^H \\
\\
+\left[\begin{array}{ll} \quad A\,(\,\Delta^2\,\Delta^2\;\;-\infty\,) & \mathcal{I}_1\,(1) \\ \\ +\,A\,(\,\Delta^2\,\Delta^2\;\;1\,) & \mathcal{I}_2\,(1)\end{array}\right]\lambda & \times\;z\,(\,\Delta^4\;\;\Delta^4\;\;-\infty\,)\;\overline{x}^H \\
+\,A\,(\,\Delta^2\,\Delta^2\;-\infty\,)\,\lambda & \times\;\frac{1}{2}z\,(\,\Delta^4\;\;\Delta^4\;\;-\infty\,)\;\overline{x}^F
\end{array}\right]
$$

Both z^H and z^F are the ones corresponding to the state-variable of interest. The state-variable is removed for the notational simplicity.

$$A_{NE}^{AF} \equiv \begin{bmatrix} A\left(\Delta^1\,\Delta^1\,\Delta^G\right) & \times\ (1-z^H)(1-z^F)(1-\overline{x}) \\ + \begin{bmatrix} A\,(\,\Delta^2\,\Delta^2\ \ -\infty\,) & \mathcal{I}_1\left(\frac{1}{\lambda}\Delta^G\right) \\ + A\left(\Delta^1\,\Delta^2\ \frac{1}{\lambda}\Delta^G\right) & \mathcal{I}_2\left(\frac{1}{\lambda}\Delta^G\right) \end{bmatrix} \lambda & \times\ (1-z^H)\,z^F\,(1-\overline{x}) \\ + A\left(\Delta^2\,\Delta^1\ \lambda\Delta^G\right)\mathcal{I}_2\left(\lambda\Delta^G\right) & \times\ z^H\,(1-z^F)\,(1-\overline{x}) \\ + A\left(\Delta^2\,\Delta^2\ \Delta^G\right)\lambda & \times\ z^H\,z^F\,(1-\overline{x}) \end{bmatrix}$$

$$\begin{aligned} A_{1F}^{AF} \equiv & A\left(\Delta^3\,\Delta^1\ \eta\Delta^G\right)\mathcal{I}_2\left(\eta\Delta^G\right) \times\ (1-z^F)\,\overline{x}^H \\ & +\ A\left(\Delta^3\,\Delta^2\ \frac{\eta}{\lambda}\Delta^G\right)\mathcal{I}_2\left(\frac{\eta}{\lambda}\Delta^G\right)\lambda \times\ z^F\,\overline{x}^H \end{aligned}$$

$$A_{2F}^{AF} \equiv \begin{bmatrix} A\left(\Delta^4\,\Delta^1\ \frac{\eta}{\lambda}\Delta^G\right)\mathcal{I}_2\left(\frac{\eta}{\lambda}\Delta^G\right) & \times\ (1-z^H)(1-z^F)\overline{x}^H \\ + \begin{bmatrix} A\,(\,\Delta^2\,\Delta^2\ \ -\infty\,) & \mathcal{I}_1\left(\frac{\eta}{\lambda^2}\Delta^G\right) \\ + A\left(\Delta^4\,\Delta^2\ \frac{\eta}{\lambda^2}\Delta^G\right) & \mathcal{I}_2\left(\frac{\eta}{\lambda^2}\Delta^G\right) \end{bmatrix} \lambda & \times\ (1-z^H)\,z^F\,\overline{x}^H \\ + A\left(\Delta^2\,\Delta^1\ \lambda\Delta^G\right)\mathcal{I}_2\left(\lambda\Delta^G\right) & \times\ z^H\,(1-z^F)\,\overline{x}^H \\ + A\left(\Delta^2\,\Delta^2\,\Delta^G\right)\lambda & \times\ z^H\,z^F\,\overline{x}^H \end{bmatrix}$$

$$
A^{AF}_{3F} \equiv \left[\begin{array}{ll}
A\left(\,\Delta^1\,\Delta^1\;\;\Delta^G\,\right) & \times\;\;(1-z^H)(1-z^F)\overline{x}^H \\[2ex]
+\left[\begin{array}{ll} \quad A\,(\,\Delta^2\,\Delta^2\;\;-\infty\,) & \mathcal{I}_1\left(\frac{1}{\lambda}\Delta^G\right) \\[2ex] +\,A\left(\,\Delta^1\,\Delta^2\;\;\frac{1}{\lambda}\Delta^G\,\right) & \mathcal{I}_2\left(\frac{1}{\lambda}\Delta^G\right)\end{array}\right]\lambda & \times\;\;(1-z^H)\;z^F\;\overline{x}^H \\[2ex]
+\,A\left(\,\Delta^2\,\Delta^1\;\lambda\Delta^G\,\right)\mathcal{I}_2\left(\lambda\Delta^G\right) & \times\;\;z^H\;(1-z^F)\;\overline{x}^H \\[2ex]
+\,A\left(\,\Delta^2\,\Delta^2\;\Delta^G\,\right)\lambda & \times\;\;z^H\;z^F\;\overline{x}^H
\end{array}\right]
$$

$$
A^{AF}_{4F} \equiv A\left(\,\Delta^2\,\Delta^1\;\;\lambda\Delta^G\,\right)\mathcal{I}_2\left(\lambda\Delta^G\right)\times\;\;(1-z^F)\;\overline{x}^H\;+\;A\left(\,\Delta^2\,\Delta^2\;\;\Delta^G\,\right)\lambda\times\;\;z^F\;\overline{x}^H
$$

$$
A^{AF}_{H2} \equiv \left[\begin{array}{ll}
\left[\begin{array}{ll} \quad A\,(\,\Delta^2\,\Delta^2\;\;-\infty\,) & \mathcal{I}_1\left(\frac{1}{\lambda}\Delta^G\right) \\[2ex] +\,A\left(\,\Delta^1\,\Delta^2\;\;\frac{1}{\lambda}\Delta^G\,\right) & \mathcal{I}_2\left(\frac{1}{\lambda}\Delta^G\right)\end{array}\right]\lambda & \times\;\;(1-z^H)\;z^F\;\overline{x}^F \\[2ex]
+\,A\left(\,\Delta^2\,\Delta^2\;\;\Delta^G\,\right)\lambda & \times\;\;z^H\;z^F\;\overline{x}^F
\end{array}\right]
$$

$$
A^{AF}_{H3} \equiv \left[\begin{array}{ll}
A\left(\,\Delta^1\,\Delta^1\;\;\Delta^G\,\right) & \times\;\;\frac{1}{2}\;(1-z^H)(1-z^F)\overline{x}^F \\[2ex]
+\left[\begin{array}{ll} \quad A\,(\,\Delta^2\,\Delta^2\;\;-\infty\,) & \mathcal{I}_1\left(\frac{1}{\lambda}\Delta^G\right) \\[2ex] +\,A\left(\,\Delta^1\,\Delta^2\;\;\frac{1}{\lambda}\Delta^G\,\right) & \mathcal{I}_2\left(\frac{1}{\lambda}\Delta^G\right)\end{array}\right]\lambda & \times\;\;(1-z^H)\;z^F\;\overline{x}^F \\[2ex]
+\,A\left(\,\Delta^2\,\Delta^1\;\;\lambda\Delta^G\,\right)\mathcal{I}_2\left(\lambda\Delta^G\right) & \times\;\;\frac{1}{2}\;z^H\;(1-z^F)\;\overline{x}^F \\[2ex]
+\,A\left(\,\Delta^2\,\Delta^2\;\;\Delta^G\,\right)\lambda & \times\;\;z^H\;z^F\;\overline{x}^F
\end{array}\right]
$$

$$
A^{AF}_{H4} \equiv \left[\begin{array}{ll}
\left[\begin{array}{ll} \quad A\,(\,\Delta^2\,\Delta^2\;\;-\infty\,) & \mathcal{I}_1\left(\frac{1}{\lambda}\Delta^G\right) \\[2ex] +\,A\left(\,\Delta^1\,\Delta^2\;\;\frac{1}{\lambda}\Delta^G\,\right) & \mathcal{I}_2\left(\frac{1}{\lambda}\Delta^G\right)\end{array}\right]\lambda & \times\;\;\frac{1}{2}\;(1-z^H)\;z^F\;\overline{x}^F \\[2ex]
+\,A\left(\,\Delta^2\,\Delta^2\;\;\Delta^G\,\right)\lambda & \times\;\;\frac{1}{2}\;z^H\;z^F\;\overline{x}^F
\end{array}\right]
$$

For any $\ell^F \in \{1\ ,\ 2\ ,\ 3\ ,\ 4\}$, the following holds:

$$A^F\left(\Delta^{\ell^H}\ \ \Delta^1\ \ \Delta^G\right) = \pi^{FF} - \widehat{\chi}\ \left(z^F\right)^{\widehat{\psi}} + \widetilde{\beta}^F \times \left[\,A^{AF}_{NE}\ +\ A^{AF}_{\ell^H F}\,\right],$$

and for any $\ell^F > 1$ and ℓ^H, the following holds:

$$A^F\left(\Delta^{\ell^F}\ \ \Delta^{\ell^F}\ \ \Delta^G\right) = \pi^{FF} - \widehat{\chi}\ \left(z^F\right)^{\widehat{\psi}} + \widetilde{\beta}^F \times \left[\,A^{AF}_{NE}\ +\ A^{AF}_{\ell^H F}\ +\ A^{AF}_{H\ell^F}\,\right].$$

B.3.4.2.6 Value from a New Product Line $A^F_{takeover}$ for Foreign Firm

$$\begin{aligned} A^F_{takeover} \equiv \Bigg[& \sum_{\mathcal{I}^{ZH}_{-j}=0}^{1}\ \sum_{\mathcal{I}^{ZF}_{-j}=0}^{1}\ \sum_{c-t_{-j}=\text{win}}^{\text{lose}} \left(z^H\right)^{\mathcal{I}^{ZH}_{-j}}\left(1-z^H\right)^{1-\mathcal{I}^{ZH}_{-j}}\left(z^F\right)^{\mathcal{I}^{ZF}_{-j}}\left(1-z^F\right)^{1-\mathcal{I}^{ZF}_{-j}}\ \frac{1}{2} \\ & \times \int_{\left(\Delta^H_{-j}\ \Delta^F_{-j}\ \Delta^G_{-j}\right)} A^F\Big(\Delta^{H\prime}_{-j}\ \Delta^{F\prime}_{-j}\ \Delta^{G\prime}_{-j}\ \mid\ \Delta^H_{-j}\,,\ \Delta^F_{-j}\,,\ \Delta^G_{-j}\,,\ \mathcal{I}^{ZH}_{-j}\,,\mathcal{I}^{ZF}_{-j}\,,\ c-t_{-j}\Big)\,\Delta^{H\prime}_{-j} \\ & \qquad\qquad \times \mu\left(\Delta^H_{-j}\ \Delta^F_{-j}\ \Delta^G_{-j}\right)\,\mathbf{d}\left(\Delta^H_{-j}\ \Delta^F_{-j}\ \Delta^G_{-j}\right)\Bigg], \end{aligned}$$

where the RHS is equal to

$$\left[\begin{array}{l} \left[A\left(\Delta^3\,\Delta^3\ \ -\infty\right)\mathcal{I}_1\left(\frac{1}{\eta}\right) + A\left(\Delta^1\,\Delta^3\ \ \frac{1}{\eta}\right)\mathcal{I}_2\left(\frac{1}{\eta}\right)\right] \\ \times\left(1-z^H\left(\Delta^1\,\Delta^1\ \ \infty\right)\right) \\ +\left[A\left(\Delta^3\,\Delta^3\ \ -\infty\right)\mathcal{I}_1\left(\frac{\lambda}{\eta}\right) + A\left(\Delta^2\,\Delta^3\ \ \frac{\lambda}{\eta}\right)\mathcal{I}_2\left(\frac{\lambda}{\eta}\right)\right] \\ \times z^H\left(\Delta^1\,\Delta^1\ \ \infty\right) \end{array}\right]\eta\,\mu\left(\Delta^1\,\Delta^1\ \ \infty\right)$$

$$+\left[\begin{array}{l}\left[A\left(\,\Delta^4\,\Delta^4\;\;-\infty\,\right)\mathcal{I}_1\left(\frac{\lambda}{\eta}\right)+A\left(\,\Delta^1\,\Delta^4\;\;\frac{\lambda}{\eta}\,\right)\mathcal{I}_2\left(\frac{\lambda}{\eta}\right)\right]\\ \times\left(1-z^H\left(\Delta^2\,\Delta^2\;\;\infty\right)\right)\\ +\left[A\left(\,\Delta^4\,\Delta^4\;\;-\infty\,\right)\mathcal{I}_1\left(\frac{\lambda^2}{\eta}\right)+A\left(\,\Delta^2\,\Delta^4\;\;\frac{\lambda^2}{\eta}\,\right)\mathcal{I}_2\left(\frac{\lambda^2}{\eta}\right)\right]\\ \times z^H\left(\Delta^2\,\Delta^2\;\;\infty\right)\end{array}\right]\frac{\eta}{\lambda}\,\mu\left(\Delta^2\,\Delta^2\;\;\infty\right)$$

$$+\left[\begin{array}{l}\left[A\left(\,\Delta^1\,\Delta^1\;\;-\infty\,\right)\mathcal{I}_1\left(1\right)+A\left(\,\Delta^1\,\Delta^1\;\;1\,\right)\mathcal{I}_2\left(1\right)\right]\\ \times\left(1-z^H\left(\Delta^3\,\Delta^3\;\;\infty\right)\right)\\ +\left[A\left(\,\Delta^1\,\Delta^1\;\;-\infty\,\right)\mathcal{I}_1\left(\lambda\right)+A\left(\,\Delta^2\,\Delta^1\;\;\lambda\,\right)\mathcal{I}_2\left(\lambda\right)\right]\\ \times z^H\left(\Delta^3\,\Delta^3\;\;\infty\right)\end{array}\right]\mu\left(\Delta^3\,\Delta^3\;\;\infty\right)$$

$$+\left[\begin{array}{ll}\left[A\left(\,\Delta^2\,\Delta^2\;\;-\infty\,\right)\mathcal{I}_1\left(\frac{1}{\lambda}\right)+A\left(\,\Delta^1\,\Delta^2\;\;\frac{1}{\lambda}\,\right)\mathcal{I}_2\left(\frac{1}{\lambda}\right)\right] & \left(1-z^H\left(\Delta^4\,\Delta^4\;\;\infty\right)\right)\\ +\left[A\left(\,\Delta^2\,\Delta^2\;\;-\infty\,\right)\mathcal{I}_1\left(1\right)+A\left(\,\Delta^2\,\Delta^2\;\;1\,\right)\mathcal{I}_2\left(1\right)\right] & z^H\left(\Delta^4\,\Delta^4\;\;\infty\right)\end{array}\right]\lambda\,\mu\left(\Delta^4\,\Delta^4\;\;\infty\right)$$

$$+\;A\left(\,\Delta^3\,\Delta^3\;\;-\infty\,\right)\,\eta\,\mu\left(\Delta^1\,\Delta^1\;\;-\infty\right)\;+\;A\left(\,\Delta^4\,\Delta^4\;\;-\infty\,\right)\,\frac{\eta}{\lambda}\,\left(1-z^F\left(\Delta^2\,\Delta^2\;\;-\infty\right)\right)\,\mu\left(\Delta^2\,\Delta^2\;\;-\infty\right)$$

$$+\;A\left(\,\Delta^1\,\Delta^1\;\;-\infty\,\right)\,\frac{1}{2}\,\left(1-z^F\left(\Delta^3\,\Delta^3\;\;-\infty\right)\right)\,\mu\left(\Delta^3\,\Delta^3\;\;-\infty\right)$$

$$+\;A\left(\,\Delta^2\,\Delta^2\;\;-\infty\,\right)\,\lambda\,\left(1-\frac{1}{2}\,z^F\left(\Delta^4\,\Delta^4\;\;-\infty\right)\right)\,\mu\left(\Delta^4\,\Delta^4\;\;-\infty\right)$$

$$
+\sum_{\Delta^G}\sum_{\ell^H=1}^{4}\left[\begin{array}{ll}
+\left[\begin{array}{l}
\left[A\left(\,\Delta^3\,\Delta^3\;-\infty\,\right)\mathcal{I}_1\left(\tfrac{1}{\eta}\Delta^G\right)+A\left(\,\Delta^1\,\Delta^3\,\tfrac{1}{\eta}\Delta^G\,\right)\mathcal{I}_2\left(\tfrac{1}{\eta}\Delta^G\right)\right]\\
\times\left(1-z^H\left(\Delta^{\ell^H}\,\Delta^1\,\Delta^G\right)\right)\\
+\left[A\left(\,\Delta^3\,\Delta^3\;-\infty\,\right)\mathcal{I}_1\left(\tfrac{\lambda}{\eta}\Delta^G\right)+A\left(\,\Delta^2\,\Delta^3\,\tfrac{\lambda}{\eta}\Delta^G\,\right)\mathcal{I}_2\left(\tfrac{\lambda}{\eta}\Delta^G\right)\right]\\
\times z^H\left(\Delta^{\ell^H}\,\Delta^1\,\Delta^G\right)
\end{array}\right] & \eta\mu\left(\ell^H\,\Delta^1\,\Delta^G\right)\\
+\left[\begin{array}{l}
\left[A\left(\,\Delta^4\,\Delta^4\;-\infty\,\right)\mathcal{I}_1\left(\tfrac{\lambda}{\eta}\Delta^G\right)+A\left(\,\Delta^1\,\Delta^4\,\tfrac{\lambda}{\eta}\Delta^G\,\right)\mathcal{I}_2\left(\tfrac{\lambda}{\eta}\Delta^G\right)\right]\\
\times\left(1-z^H\left(\Delta^{\ell^H}\,\Delta^2\,\Delta^G\right)\right)\left(1-z^F\left(\Delta^{\ell^H}\,\Delta^2\,\Delta^G\right)\right)\\
+A\left(\,\Delta^2\,\Delta^4\,\tfrac{\lambda^2}{\eta}\Delta^G\,\right)\mathcal{I}_2\left(\tfrac{\lambda^2}{\eta}\Delta^G\right)\\
\times z^H\left(\Delta^{\ell^H}\,\Delta^2\,\Delta^G\right)\left(1-z^F\left(\Delta^{\ell^H}\,\Delta^2\,\Delta^G\right)\right)
\end{array}\right] & \tfrac{\eta}{\lambda}\mu\left(\ell^H\,\Delta^2\,\Delta^G\right)\\
+\left[\begin{array}{l}
A\left(\,\Delta^1\,\Delta^1\,\Delta^G\,\right)\\
\times\tfrac{1}{2}\left(1-z^H\left(\Delta^{\ell^H}\,\Delta^3\,\Delta^G\right)\right)\left(1-z^F\left(\Delta^{\ell^H}\,\Delta^3\,\Delta^G\right)\right)\\
+A\left(\,\Delta^2\,\Delta^1\,\lambda\Delta^G\,\right)\mathcal{I}_2\left(\lambda\Delta^G\right)\\
\times\tfrac{1}{2}z^H\left(\Delta^{\ell^H}\,\Delta^3\,\Delta^G\right)\left(1-z^F\left(\Delta^{\ell^H}\,\Delta^3\,\Delta^G\right)\right)
\end{array}\right] & \mu\left(\ell^H\,\Delta^3\,\Delta^G\right)\\
+\left[\begin{array}{l}
\left[A\left(\,\Delta^2\,\Delta^2\;-\infty\,\right)\mathcal{I}_1\left(\tfrac{1}{\lambda}\Delta^G\right)+A\left(\,\Delta^1\,\Delta^2\,\tfrac{1}{\lambda}\Delta^G\,\right)\mathcal{I}_2\left(\tfrac{1}{\lambda}\Delta^G\right)\right]\\
\times\left(1-z^H\left(\Delta^{\ell^H}\,\Delta^4\,\Delta^G\right)\right)\left(1-\tfrac{1}{2}z^F\left(\Delta^{\ell^H}\,\Delta^4\,\Delta^G\right)\right)\\
+A\left(\,\Delta^2\,\Delta^2\,\Delta^G\,\right)\\
\times z^H\left(\Delta^{\ell^H}\,\Delta^4\,\Delta^G\right)\left(1-\tfrac{1}{2}z^F\left(\Delta^{\ell^H}\,\Delta^4\,\Delta^G\right)\right)
\end{array}\right] & \lambda\mu\left(\ell^H\,\Delta^4\,\Delta^G\right)
\end{array}\right]
$$

B.3.4.3 Potential Startup's Problem

Potential startup's ex-ante entry expected profit can be written as

$$
\begin{aligned}
\Pi_c^e &= x_e^c\,\widetilde{\beta}_c\,\mathbb{E}\Big[\;V^c\left(\left\{q_j'\,\Delta_j^{H\prime}\,\Delta_j^{F\prime}\;\Delta_j^{G\prime}\right\}\right)\;\Big]\;-\;\widetilde{\chi}^e\left(x_e^c\right)^{\widetilde{\psi}^e}\;\overline{q}_c\\
&= x_e^c\,\widetilde{\beta}_c\,\mathbb{E}\Big[\;A^c\left(\Delta_j^{H\prime}\,\Delta_j^{F\prime}\;\Delta_j^{G\prime}\right)q_j'+B^c\,\overline{q}_c'\;\Big]\;-\;\widetilde{\chi}^e\left(x_e^c\right)^{\widetilde{\psi}^e}\;\overline{q}_c\\
&= x_e^c\,\widetilde{\beta}_c\left[\;A_{takeover}^c\,\overline{q}_c+B^c\,\overline{q}_c'\;\right]\;-\;\widetilde{\chi}^e\left(x_e^c\right)^{\widetilde{\psi}^e}\;\overline{q}_c\,.
\end{aligned}
$$

Thus, the FOC gives us an optimal level of external innovation intensity for potential startup equals to

$$\frac{\partial \Pi_c^e}{\partial x_e^c} : \widetilde{\psi}^e \, \widetilde{\chi}^e \, (x_e^c)^{\widetilde{\psi}^e - 1} \; \overline{q}_c = \widetilde{\beta}_c \left[A^c_{takeover} \, \overline{q}_c + B^c \, \overline{q}'_c \right]$$

$$\Rightarrow \; x_e^c = \left(\frac{\widetilde{\beta}_c}{\widetilde{\psi}^e \, \widetilde{\chi}^e} \right)^{\frac{1}{\widetilde{\psi}^e - 1}} \left(A^c_{takeover} \; + B^c \, \frac{\overline{q}'_c}{\overline{q}_c} \right)^{\frac{1}{\widetilde{\psi}^e - 1}} \tag{B.81}$$

B.3.5 Complete List of Equations

B.3.5.1 Labor Market

$$\begin{aligned}
\frac{w_H}{\overline{q}_H} &= \theta(1-\theta)^{\frac{1-2\theta}{\theta}} \left[\left(\frac{w_H}{\overline{q}_H} \right)^{-\frac{1-\theta}{\theta}} \frac{\mathcal{Q}_{HH}}{\overline{q}_H} \; + \; \left(\tau_{FH} \frac{w_F}{\overline{q}_F} \right)^{-\frac{1-\theta}{\theta}} \frac{\mathcal{Q}_{FH}}{\overline{q}_H} \right] (P_H)^{\frac{1}{\theta}} \\
\frac{w_F}{\overline{q}_F} &= \theta(1-\theta)^{\frac{1-2\theta}{\theta}} \left[\left(\frac{w_F}{\overline{q}_F} \right)^{-\frac{1-\theta}{\theta}} \frac{\mathcal{Q}_{FF}}{\overline{q}_F} \; + \; \left(\tau_{HF} \frac{w_H}{\overline{q}_H} \right)^{-\frac{1-\theta}{\theta}} \frac{\mathcal{Q}_{HF}}{\overline{q}_F} \right] (P_F)^{\frac{1}{\theta}} \\
\widetilde{L}_H &= (1-\theta)^{\frac{1}{\theta}} \left(\frac{w_H}{\overline{q}_H} \right)^{-\frac{1}{\theta}} \left[L_H \, (P_H)^{\frac{1}{\theta}} \, \frac{\mathcal{Q}_{HH}}{\overline{q}_H} + L_F \, (P_F)^{\frac{1}{\theta}} \, (\tau_{HF})^{-\frac{1}{\theta}} \, \frac{\mathcal{Q}_{HF}}{\overline{q}_H} \right] \\
\widetilde{L}_F &= (1-\theta)^{\frac{1}{\theta}} \left(\frac{w_F}{\overline{q}_F} \right)^{-\frac{1}{\theta}} \left[L_F \, (P_F)^{\frac{1}{\theta}} \, \frac{\mathcal{Q}_{FF}}{\overline{q}_F} + L_H \, (P_H)^{\frac{1}{\theta}} \, (\tau_{FH})^{-\frac{1}{\theta}} \, \frac{\mathcal{Q}_{FH}}{\overline{q}_F} \right] \\
L_H &= \overline{L}_H - \widetilde{L}_H \\
L_F &= \overline{L}_F - \widetilde{L}_F
\end{aligned}$$

B.3.5.2 Prices and Quantities

$$p_j^H = \begin{cases} \dfrac{1}{1-\theta}\,\dfrac{w_H}{\overline{q}_H} & \text{for domestic absorption} \\[2ex] \dfrac{1}{1-\theta}\,\tau_{FH}\,\dfrac{w_F}{\overline{q}_F} & \text{for imports} \end{cases}$$

$$p_j^F = \begin{cases} \dfrac{1}{1-\theta}\,\dfrac{w_F}{\overline{q}_F} & \text{for domestic absorption} \\[2ex] \dfrac{1}{1-\theta}\,\tau_{HF}\,\dfrac{w_H}{\overline{q}_H} & \text{for imports} \end{cases}$$

$$y_j^{HH}(q_j) = (1-\theta)^{\frac{1}{\theta}}\,(P_H)^{\frac{1}{\theta}}\,L_H\left(\frac{w_H}{\overline{q}_H}\right)^{-\frac{1}{\theta}}\;q_j$$

$$y_j^{HF}(q_j) = (1-\theta)^{\frac{1}{\theta}}\,(P_F)^{\frac{1}{\theta}}\,L_F\left(\tau_{HF}\frac{w_H}{\overline{q}_H}\right)^{-\frac{1}{\theta}}\;q_j$$

$$y_j^{FF}(q_j) = (1-\theta)^{\frac{1}{\theta}}\,(P_F)^{\frac{1}{\theta}}\,L_F\left(\frac{w_F}{\overline{q}_F}\right)^{-\frac{1}{\theta}}\;q_j$$

$$y_j^{FH}(q_j) = (1-\theta)^{\frac{1}{\theta}}\,(P_H)^{\frac{1}{\theta}}\,L_H\left(\tau_{FH}\frac{w_F}{\overline{q}_F}\right)^{-\frac{1}{\theta}}\;q_j$$

$$\ell_j^{HH}(q_j) = (1-\theta)^{\frac{1}{\theta}}\,(P_H)^{\frac{1}{\theta}}\,L_H\left(\frac{w_H}{\overline{q}_H}\right)^{-\frac{1}{\theta}}\;\frac{q_j}{\overline{q}_H}$$

$$\ell_j^{HF}(q_j) = (1-\theta)^{\frac{1}{\theta}}\,(P_F)^{\frac{1}{\theta}}\,L_F\left(\tau_{HF}\frac{w_H}{\overline{q}_H}\right)^{-\frac{1}{\theta}}\;\frac{q_j}{\overline{q}_H}$$

$$\ell_j^{FF}(q_j) = (1-\theta)^{\frac{1}{\theta}}\,(P_F)^{\frac{1}{\theta}}\,L_F\left(\frac{w_F}{\overline{q}_F}\right)^{-\frac{1}{\theta}}\;\frac{q_j}{\overline{q}_F}$$

$$\ell_j^{FH}(q_j) = (1-\theta)^{\frac{1}{\theta}} (P_H)^{\frac{1}{\theta}} L_H \left(\tau_{FH} \frac{w_F}{\overline{q}_F}\right)^{-\frac{1}{\theta}} \frac{q_j}{\overline{q}_F}$$

$$\pi^{HH}(q_j) = \underbrace{\theta(1-\theta)^{\frac{1-\theta}{\theta}} L_H \left(\frac{w_H}{\overline{q}_H}\right)^{-\frac{1-\theta}{\theta}} (P_H)^{\frac{1}{\theta}}}_{\equiv \pi^{HH}} q_j$$

$$\pi^{HF}(q_j) = \underbrace{\theta(1-\theta)^{\frac{1-\theta}{\theta}} L_F \left(\tau_{HF} \frac{w_H}{\overline{q}_H}\right)^{-\frac{1-\theta}{\theta}} (P_F)^{\frac{1}{\theta}}}_{\equiv \pi^{HF}} q_j$$

$$\pi^{FF}(q_j) = \underbrace{\theta(1-\theta)^{\frac{1-\theta}{\theta}} L_F \left(\frac{w_F}{\overline{q}_F}\right)^{-\frac{1-\theta}{\theta}} (P_F)^{\frac{1}{\theta}}}_{\equiv \pi^{FF}} q_j$$

$$\pi^{FH}(q_j) = \underbrace{\theta(1-\theta)^{\frac{1-\theta}{\theta}} L_H \left(\tau_{FH} \frac{w_F}{\overline{q}_F}\right)^{-\frac{1-\theta}{\theta}} (P_H)^{\frac{1}{\theta}}}_{\equiv \pi^{FH}} q_j$$

$$Y_H = (1-\theta)^{\frac{1-2\theta}{\theta}} (P_H)^{\frac{1-\theta}{\theta}} L_H \left[\left(\frac{w_H}{\overline{q}_H}\right)^{-\frac{1-\theta}{\theta}} \mathcal{Q}_{HH} + \left(\tau_{FH} \frac{w_F}{\overline{q}_F}\right)^{-\frac{1-\theta}{\theta}} \mathcal{Q}_{FH}\right]$$

$$Y_F = (1-\theta)^{\frac{1-2\theta}{\theta}} (P_F)^{\frac{1-\theta}{\theta}} L_F \left[\left(\frac{w_F}{\overline{q}_F}\right)^{-\frac{1-\theta}{\theta}} \mathcal{Q}_{FF} + \left(\tau_{HF} \frac{w_H}{\overline{q}_H}\right)^{-\frac{1-\theta}{\theta}} \mathcal{Q}_{HF}\right]$$

$$P_H = P_F = 1 \quad \text{(under Assumption 2)}$$

B.3.5.3 Aggregate External Innovation Intensity

For $c \in \{H, F\}$,

$$\overline{x}^c = x^c \, \mathcal{F}_c + x_e^c \, \mathcal{E}_c \,,$$

and

$$\overline{x} = \overline{x}^H + \overline{x}^F \,.$$

B.3.5.4 International Trade

B.3.5.4.1 Value of Trade

Value of differentiated goods imported from foreign country to home country:

$$\int_{j \in \mathcal{J}_{FH}} p_j^H y_j^{FH} dj = (1-\theta)^{\frac{1-\theta}{\theta}} \left(P_H\right)^{\frac{1}{\theta}} L_H \left(\tau_{FH} \frac{w_F}{\overline{q}_F}\right)^{-\frac{1-\theta}{\theta}} \mathcal{Q}_{FH}$$

Value of differentiated goods exported from home country to foreign country:

$$\frac{P_H}{P_F} \int_{j \in \mathcal{J}_{HF}} p_j^F y_j^{HF} dj = \frac{P_H}{P_F} (1-\theta)^{\frac{1-\theta}{\theta}} \left(P_F\right)^{\frac{1}{\theta}} L_F \left(\tau_{HF} \frac{w_H}{\overline{q}_H}\right)^{-\frac{1-\theta}{\theta}} \mathcal{Q}_{HF}$$

Value of final good traded (in a perspective of home country):

$$P_H X_H = (1-\theta)^{\frac{1-\theta}{\theta}} \left[\left(P_H\right)^{\frac{1}{\theta}} L_H \left(\tau_{FH} \frac{w_F}{\overline{q}_F}\right)^{-\frac{1-\theta}{\theta}} \mathcal{Q}_{FH} - \frac{P_H}{P_F} \left(P_F\right)^{\frac{1}{\theta}} L_F \left(\tau_{HF} \frac{w_H}{\overline{q}_H}\right)^{-\frac{1-\theta}{\theta}} \mathcal{Q}_{HF} \right]$$

$$\Leftrightarrow X_H = (1-\theta)^{\frac{1-\theta}{\theta}} \left[(P_H)^{\frac{1-\theta}{\theta}} L_H \left(\tau_{FH} \frac{w_F}{\overline{q}_F} \right)^{-\frac{1-\theta}{\theta}} \mathcal{Q}_{FH} - (P_F)^{\frac{1-\theta}{\theta}} L_F \left(\tau_{HF} \frac{w_H}{\overline{q}_H} \right)^{-\frac{1-\theta}{\theta}} \mathcal{Q}_{HF} \right],$$

$$X_F = -X_H$$

Recall: $\overline{q}_H = \mathcal{Q}_{HH} + \mathcal{Q}_{FH}$, and $\overline{q}_F = \mathcal{Q}_{FF} + \mathcal{Q}_{HF}$. In a BGP, $\mathcal{Q}_{cc}$ and $\mathcal{Q}_{c'c}$ should grow at the same rate as $\overline{q}_c$ or one should completely dominate another. We can see this from the analytic expression for equilibrium wages w_H and w_F.

B.3.5.4.2 Trade Cutoffs

$$\underline{\Omega} \equiv \left(\frac{1}{\tau_{FH}} \right)^{\frac{1-\theta}{\theta}} \left[\frac{w_H}{\overline{q}_H} \left(\frac{w_F}{\overline{q}_F} \right)^{-1} \right]^{\frac{1-\theta}{\theta}}$$

$$\overline{\Omega} \equiv (\tau_{HF})^{\frac{1-\theta}{\theta}} \left[\frac{w_H}{\overline{q}_H} \left(\frac{w_F}{\overline{q}_F} \right)^{-1} \right]^{\frac{1-\theta}{\theta}}$$

B.3.5.5 Other Macroeconomic Variables

$$C_H = w_H \overline{L}_H + \Pi_H + \widetilde{\Pi}_H + G_H$$

$$C_F = w_F \overline{L}_F + \Pi_F + \widetilde{\Pi}_F + G_F$$

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